A study of the Delta Normal Method of Measuring VaR

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Abstract

This thesis describes the Delta-Normal method of computing Value-at-Risk. The advantages and disadvantages of the Delta-Normal method compared to the Historical and Monte Carlo method of computing Value-at-Risk are discussed. The Delta-Normal method of computing Value-at-Risk is compared with the Historical Simulation method of Value-at-Risk using an implementation of portfolio consisting of ten stocks for 400 time intervals.

Based on the normality of the distribution of the portfolio risk factors, Delta-Normal would be suitable if the distribution is normal and Historical Simulation method of calculating Value-at-Risk would be ideally suited if the distribution is non-normal.

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1. Introduction & Background

Risk measurement is a classical problem in finance. Harry Markowitz's work [2] was the first that gave a clear mathematical definition to "risk" in portfolio analysis. Markowitz did not actually use the word "risk" in his original paper but he only said that the variance (or standard deviation) in return on the portfolio is the quantity that an investor would like to minimize while maximizing the return on the portfolio. The intuitive definition of risk is the probability of suffering harm or loss. Any mathematical definition of risk must capture and quantify the idea that return is a random variable and risk is the probability or possibility of loss (References [4, 5, and 6]).

1.1 Mean Variance Analysis

Given a portfolio of N assets, the investor chooses to invest a fraction of total wealth h_i in each of N assets with (random) return R_i. The expected return on the portfolio is the weighted average of the individual expected returns:

$$\mu_P = E[R_p] = \sum_{i=1}^{N} h_i E[R_i] = \sum_{i=1}^{N} h_i \mu_i = h^T \mu .$$

The risk associated with the portfolio is the variance (or standard deviation) of the return on the portfolio (see [2]) :

$$\sigma_P^2 = Var[R_p] = \sum_{j=1}^{N} \sum_{i=1}^{N} h_i \sigma_{ij} h_j = h^T C h$$

Where C is the N x N covariance matrix with entries

$$\sigma_{ij} = E[(R_i - \mu_i)(R_j - \mu_j)]$$

To say that the return on a portfolio is a *random variable* means that the (future) return is not known in advance but the analyst has some way of modeling the distribution of possible returns and their associated probabilities.

1.2 Alternate Risk Measures

In some applications, variance may not be the best measure of risk for a stock or portfolio. There are other risk measures that are easier to interpret and easier to explain to a client. These other risk measures have been developed and applied (see, for example, Chapter 3 in [3]). These risk measures include:

- Semivariance (also called downside risk or downside variance);
- Target semivariance;
- Shortfall probability;
- Value at Risk.

Semivariance simply assumes that the investor only cares about large shifts in the price of a stock if the large shifts are below the mean. If the distribution is symmetric, then semivariance is simply a multiple of variance and so no new information is recorded. If the distribution is not symmetric, then semivariance does capture useful information which variance would miss.

Target semivariance goes one step further and records only drops in price larger than a certain (target) threshold. It is a generalization of semivariance that focuses on returns below a target, such as zero or the risk free rate instead of just below the mean.

Shortfall probability records the part of the distribution in returns that is below a certain threshold. It answers the question "What is the probability that returns will be below X?" for a specified X.

One of the most used alternative risk measure is *Value at Risk (VaR)*. It records the actual loss that would occur if the returns were below a certain probability threshold of the distribution. Note that, as with the semivariance, when the distribution of returns is normal, then the value at risk is a multiple of the variance. Even in this situation where the two measures really provide the same information, some clients will demand a report of value at risk for a portfolio.

1.3 Need for Value-at-Risk

The concept and use of Value-at-Risk is recent. Value-at-Risk was first used by major financial firms in the late 1980's to measure the risk of their trading portfolios. Since that time, the use of Value-at-Risk has exploded.

Value-at-Risk is now a widely used quantitative tool to measure market risk. "VaR answers the question: how much can one lose with X % probability over a pre-set horizon" [8]. More precisely *VaR* is an amount (say *V* dollars), where the probability of losing more than *V* dollars is p^* over some future time interval, T days. Value at Risk measures the amount of risk in dollars. Investors can then decide whether they feel comfortable with this level of risk.

Value-at-Risk asks the simple question "How bad can things get?" All managers would like this question to be answered. Value-at-Risk has become widely used by corporate treasurers and fund managers as well as by financial institutions. Value-at-Risk is used by bank regulators in determining how much capital a bank should possess to reflect the market risks it is bearing [8]. Today, many banks, brokerage firms and investment funds use similar methods to gauge their financial risk. A 1995 *Institutional Investor* survey found that 32% of all firms use VaR as a measure of market risk, and

60% of pension funds responding to a survey by the New York University Stern School of Business reported using VaR (chapter 9, [9]) .

1.4 Criticism of Value-at-Risk

The widespread adoption of VaR has been accompanied by frequent criticism of VaR as a measure of risk. Any attempt to summarize a distribution in a single number is open to criticism, but VaR has a particular deficiency. Combining two portfolios into a single portfolio may result in a VaR that is larger than the sum of the VaRs for the two original portfolios. This fact contradicts the idea that diversification reduces the risk [7].

VaR assumes that the sigma and covariance matrix do not change. VaR fails when you need it the most i.e, it is uninformative about extreme tails. One good example is Long Term Capital Management (LTCM). Due to its shortcomings, it should not be used as a standalone risk measure, but one of many risk measures to be considered in firm wide risk management

1.5 Overview of this Report

Our goal is to study in detail the Delta-Normal Method of computing Value-at-Risk. This project report focuses on computing Value-at-Risk for a portfolio of ten stocks using the Delta-Normal Method and the Historical Simulation Method. The next section gives the definition of Value-at-Risk and the steps involved in computing it. We then give an overview of the different methods used to compute Value-at-Risk. We then turn to the details of computing Value-at-Risk using the Delta-Normal method. The final section provides a complete implementation analysis of computing Value-at-Risk (in dollars) of a portfolio of ten stocks using the Delta-Normal and Historical Simulation Methods.

2 Value-at-Risk Definition

Value-at-risk, as defined by Phillipe Jorion is “the worst loss over a target horizon with a given level of target probability” (See chapter 5 in [9]).

From a mathematical point of view, Value-at-Risk is just a quantile of a return distribution function. The portfolio’s Value-at-Risk (VaR) is a percentile of its return distribution over a fixed horizon Δt. For example, the value-at-risk for a target probability of 99% is a point x_{p^*} satisfying

Δt = risk-measurement time horizon

S = vector of m market prices

ΔS = Change in S over time horizon Δt

$V(S,t)$ = portfolio value at time t and market prices S

L = loss over time horizon Δt = $-\Delta V = V(S,t) - V(S+\Delta S, t+\Delta t)$

$$1 - F_L(x_{p^*}) = P(L > x_{p^*}) = p^* \text{ with } p^* = 0.01$$

The number m of relevant risk factors could be very large, potentially reaching the hundreds or thousands. The risk factors represent market variables such as prices, interest rates, spreads or implied volatilities. We therefore focus on the more fundamental issue of measuring the tail of the loss distribution, particularly at large losses- i.e., on finding $P(L > Y)$ for large thresholds Y. A target probability provides a simple way of summarizing information about the tail of the loss distribution, and this particular value of the target probability is often interpreted as a reasonable worst-case loss level. The significance of Value-at-Risk lies in its focus on the tail of the loss distribution.

Value-at-Risk has two important parameters. These are Δt, the time horizon, and p^*, the target probability. These two major parameters should be chosen in a way appropriate to the overall goal of risk measurement. For example, in bank supervision the interval Δt is usually quite short, with regulatory agencies requiring measurement over a two-week horizon. In other areas of market risk, such as asset-liability management for pension funds and insurance companies, the relevant time horizon is far longer than two weeks. These parameters can change depending upon the risk manager's tolerance for loss, the particular asset whose risk is being measured, or the business division's contribution to the firm's overall operations.

There are several different methods for calculating Value-at-Risk, which can be distinguished by their two main assumptions, the probability distribution for risk factors and the valuation methods. Probability distribution for the risk factors is discussed first followed by the second assumption i.e, the valuation methods.

Probability distribution for the risk factors: The distribution is either normal distribution or a nonnormal distribution (i.e., an asymmetric distribution). The normal distribution has (at least) two more crucial properties:

- The distribution is symmetric about the mean;
- Two parameters, the mean μ and the variance $\sigma^2 : N(\mu, \sigma^2)$. The first parameter represents the location and the second parameter represents the dispersion.

For completeness, we should also mention two other moments i.e., *Skewness* and *Kurtosis*. Skewness is a parameter that describes asymmetry in a random variable's probability distribution. Its value is 0 for a normal distribution. The skewness of a random variable X defined by

$$skew(X) = \frac{E\left[(X-\mu)^3\right]}{\sigma^3}$$

Where μ and σ are the mean and standard deviation of X. Both the probability density functions in Figure 1 have the same expectation and variance.

Figure 1: Positive Vs Negative Skewness

Figure 1 (obtained from [12]) shows the positive and negative skewness of the probability distribution of X. The one on the left is positively skewed ($skew(X) > 0$) and the one on the right is negatively skewed ($skew(X) < 0$).

Kurtosis is a parameter that describes the “flatness” of a random variable’s probability distribution. The kurtosis of a normal distribution is 3. The kurtosis of a random variable X is given by.

$$kurt(X) = \frac{E\left[(X-\mu)^4\right]}{\sigma^4}$$

The shapes of the two probability distribution functions in Figure 2 (obtained from [12]) illustrate kurtosis:

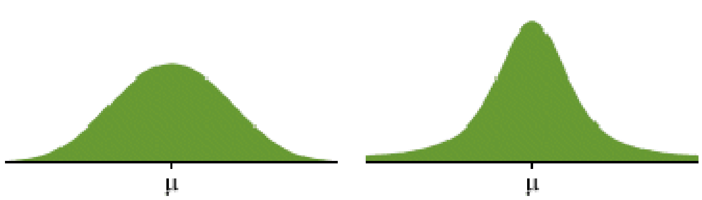

Figure 2: Low vs. Higher Kurtosis

The probability distribution function of X on the right is more peaked than one on the left and it has fatter tails. The distribution on the right has a greater kurtosis than the distribution on the left. The distributions that are both peaked and have fat tails at the same time have a $kurt(X) > 3$. The distribution that are less peaked and have thinner tails at the same time have a $kurt(X) < 3$ [12]. Skewness and Kurtosis can be used to check whether the given sample distribution is close to normal distribution or not.

The probability density function of a normal distribution:

$$f(x \mid \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\left[-(x-\mu)^2/(2\sigma^2)\right]}$$

for $\sigma > 0$, $-\infty < x < \infty$ *and* $-\infty < \mu < \infty$

The *Student t* distribution and the *generalized error distribution* (GED) are examples of a nonnormal distribution. The probability density function of a student t distribution:

$$f(x \mid \nu) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)} \frac{1}{\sqrt{\nu\pi}} \frac{1}{\left(1+\left(\frac{x^2}{\nu}\right)\right)^{\left(\nu+1/2\right)}}$$

for $-\infty < x < \infty$, $\nu = 1, \ldots\ldots$

Where Γ is the gamma function and ν is the shape defining parameter known as the degrees of freedom [11]. The Student t distributions with $\nu = 6$ have a probability distribution close to the normal distribution.

The probability density function of a generalized error distribution (GED) is given as follows:

$$f(x|\nu) = \frac{\nu}{\lambda 2^{\left(1+\frac{1}{\nu}\right)}\Gamma\left(\frac{1}{\nu}\right)} e^{\left(-\frac{1}{2}\left|\frac{x}{\lambda}\right|^{\nu}\right)}, \lambda = \left[2^{-\left(\frac{2}{\nu}\right)} \Gamma\left(\frac{1}{\nu}\right) \Big/ \Gamma\left(\frac{3}{\nu}\right) \right]^{\frac{1}{2}}$$

where ν is a shape-defining parameter. The pdf of the generalized error distribution includes the normal pdf as a special case with $\nu = 2.$ The pdf of the generalized error distribution has fatter tails for $\nu < 2.$ The generalized error distribution with $\nu = 1.3$ have a probability distribution close to the normal distribution. Both the student t distribution and the generalized error distribution have fatter tails than the normal distribution. This feature may be important when determining the potential losses using Value-at-Risk because Value-at-Risk quantifies the tail loss. Fatter tails indicate higher potential loss for a given investment.

Linear vs. Full valuation: Valuation is the process of estimating the value of an asset. For example, the single-index model is the simplest valuation model which states that the return of a portfolio (r_P) is the sum of an assets Beta, or systematic risk to movement in the market, plus an error term, referred to as idiosyncratic or residual risk

$$r_P = \beta_P r_m + \theta_P$$

Linear valuation approximates the exposure to risk factors by a linear model. The delta normal method is an example of a linear valuation method. The *full valuation method* is potentially the most accurate because it accounts for nonlinearities, income payments, and even time-decay effects that are usually ignored in the delta-normal approach. For portfolios with substantial option components (such as mortgages) or longer horizons, a full-valuation method may be required. The *Monte Carlo simulation approach* and the *historical simulation approach* are examples of full-valuation methods.

2.1 Computation of Value-at-Risk

The Value-at-Risk is computed using the following procedure:

The Portfolio's current value is denoted as p and it is known. The Portfolio's future value is not known in advance and it is a random variable denoted by P. We need to estimate the distribution of P to calculate VaR. If we assume a standard distribution such as a normal distribution, the problem reduces from one of estimating an entire distribution to that of estimating the parameters necessary to specify that distribution. The risk factors such as prices, interest rates, spreads or implied volatilities being considered are then specified. R is an N dimensional vector which contains the values of these risk factors in future. We need to make sure that the historical data is available for these risk factors. Based on the historical data, we can characterize the distribution of R. We then need to convert that characterization of the distribution of R into a characterization of the distribution of P. This is achieved by the portfolio mapping function. Portfolio's future value can be expressed in terms of R by using a function θ called the portfolio mapping function.

$$P = \theta(R)$$

This relationship is called portfolio mapping. Portfolio mapping function θ maps the N-dimensional space of the risk factors to the one-dimensional space of the portfolio's future market value.

If R holds the prices of the different stocks then it is a very simple portfolio mapping. However, if R holds many different risk factors such as prices, interest rates and implied volatilities, then the portfolio mapping function will be complicated. So, we need to apply the portfolio mapping function θ to the entire distribution of R to obtain the entire distribution of P.

If θ is a linear polynomial and P is normally distributed then all we need to do is to calculate μ_P and σ_P for the portfolio. If we assume that R contains the prices of a set of stocks, then the portfolio's standard deviation can be computed from the asset level:

$$\sigma_P = \sqrt{hCh^T} = \sqrt{\sum_i \sum_j h_i h_j \sigma_{ij}} = \sqrt{\sum_i \sum_j h_i h_j \rho_{ij} \sigma_i \sigma_j} \tag{1}$$

$h =$ N x1 vector of asset weights,

C = N x N covariance matrix for the asset returns, and

$\sigma_{ij} = \rho_{ij}\sigma_i\sigma_j$ introduces the correlation coefficient.

For any linear portfolio, we are able to compute its risk if we know the weights and the covariance matrix of the assets.

A linear mapping function θ is applied to a normal vector R. This is illustrated in the Figure 3 (obtained from [12]) intuitively by mapping evenly spaced values for R through the mapping function θ. The output values for P after the mapping are also evenly spaced, indicating that the portfolio mapping does not cause any distortion. Therefore, since R is normal, P now is normally distributed.

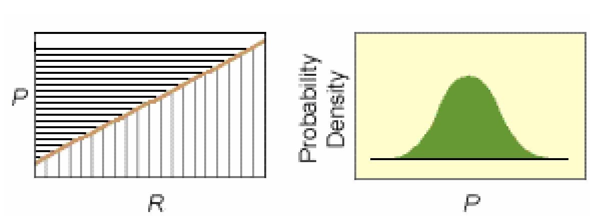

Figure 3: Linear Portfolio

If θ, the portfolio mapping function, is not a linear polynomial. A portfolio of options is one such example where θ is given by the Black Scholes's option pricing formula. This is a non-linear case, so we cannot compute σ_P using (1). Therefore, P cannot be assumed to be normally distributed. Options limit the downside risk, hence they skew the probability distribution of P.

A nonlinear mapping function θ is now applied to a normal vector R. This is illustrated in Figure 4 intuitively by mapping evenly spaced values for R through the mapping function θ. The corresponding output values for P after the mapping are not evenly spaced, indicating how the portfolio mapping distorts the distribution of P. Therefore, P now has a non-normal distribution. The left graph in the Figure 4 (obtained from [12]) depicts the familiar "hockey stick" price function of a call option.

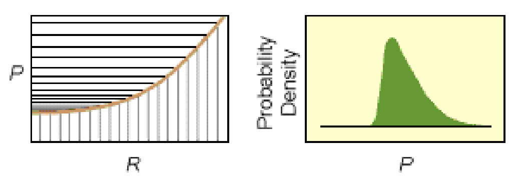

Figure 4: Nonlinear Portfolio

The mapping procedure accepts a portfolio's composition as an input and its output is the mapping function θ that defines P as a function of R. The inference procedure accepts historical data of the corresponding risk factors of the N-dimensional vector R as its input. The purpose of the inference procedure is to characterize the probability distribution of R based on its input. The output of the inference is the characterization of the distribution of R. The transformation procedure then combines the outputs from the mapping procedure and the inference procedure and uses them to characterize the distribution of P. Based on the distribution of P and the current portfolio value p, the transformation procedure then determines the value of VaR. A

Schematic representation of how the Value-at-Risk is calculated is shown in Figure 5[12].

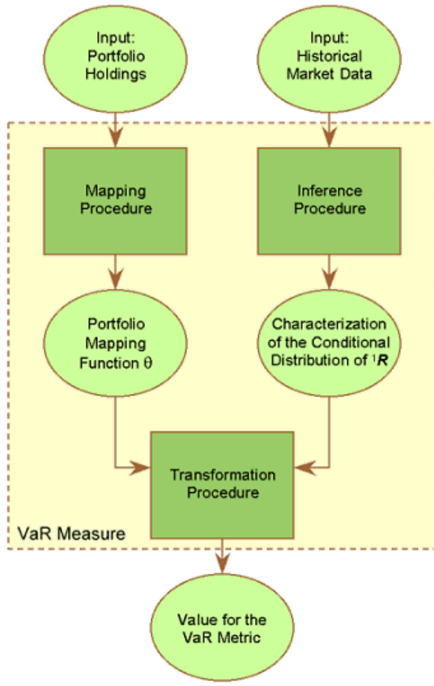

Figure 5: Schematic representation of a VaR calculation.

3 Value-at-Risk Methods

The different methods used to compute value-at-risk are discussed in detail in this section.

3.1 Historical Simulation Method

The Historical simulation method is a popular method of estimating VaR. It involves using past data in a very direct way as a guide to what might happen in the future. We apply the current weights to the historical asset returns by going back in time such as over the last 100 days. The current portfolio weights are computed using standard mathematical optimization.

$$R_P = \sum_{i=1}^{N} h_i R_i = \vec{h}^T \vec{R}$$

A distribution of portfolio returns is obtained. These portfolio returns are then sorted and depending on the target probability the corresponding quantile of the distribution is taken. This gives us the 1-day VaR using Historical Simulation method.

Hypothetical portfolios can also be generated using the current portfolio weights and the historical asset returns. This approach is called bootstrapping. An another procedure of generating scenarios for tomorrow for the market variables (such as equity prices, interest rates and so on) based on their today's values is discussed in (chapter 16, [8]).

3.1.1 Advantages:

Historical simulation method is relatively simple to implement if the past data is readily available for estimating Value-at-Risk. Historical simulation method allows nonlinearities and nonnormal distribution by relying on the actual prices. It does not rely on underlying stochastic structure of the market or any specific assumptions about valuation models. Historical simulation method does not rely on valuation models and is not subjected to the risk that the models are wrong [9].

3.1.2 Disadvantages:

The Historical Simulation method assumes the availability of sufficient historical price data. This is a drawback because some of the assets may have a short history or in some cases no history at all. There is also an assumption that the past represents the immediate future which is not always true. The Historical Simulation method quickly becomes cumbersome for large portfolios with complicated structures.

3.2 Monte Carlo Simulation Method:

The Monte Carlo simulation method can be briefly summarized in two steps. In the first step, a stochastic process is specified for the financial variables. In the second step, fictitious price paths are simulated for all financial variables of interest. Each of these “pseudo” realizations is then used to compile a distribution of returns from which a Value-at-Risk (VaR) figure can be measured**.**

3.2.1 Advantages:

The Monte Carlo method can incorporate nonlinear positions, nonnormal distributions, implied parameters, and even user-defined scenarios. As the price of computing power continues to fall, this method is bound to take on increasing importance [9].

3.2.2 Disadvantages:

The biggest disadvantage of the Monte Carlo method is its computational time. If 1000 sample paths are generated with a portfolio of 1000 assets, the total number of valuations amounts to 1 million. In addition, if the valuation of assets on the target date involves itself a simulation, the method requires "simulation within a simulation." Therefore, computer and data requirements are much higher than that required by the other approaches.

The method is the most expensive to implement in terms of systems infrastructure. Another potential weakness of the Monte Carlo method is that it is subject to the risk that the models are wrong. The Monte Carlo method relies on specific stochastic processes for the underlying risk factors as well as the pricing models for securities such as options or mortgages. Simulation results should be complemented with some sensitivity analysis to check if the results are robust to changes in the model.

3.3 Delta-Normal Method

The Delta-normal method is the best method to compute VaR for portfolios with linear positions and whose distributions are close to the normal probability density function. The Delta-Normal method may not be appropriate for portfolios with non linear positions such as options and nonnormal distributions. In such cases, one should use Monte Carlo method to calculate the Value-at-Risk of the portfolio.

Using Delta-Normal method, Value-at-Risk would be relatively easy to compute, fast, and accurate. In addition, it is not too prone to model risk (due to faulty assumptions or computations). Because the method is analytical, it allows easy analysis of the VaR results using marginal and component VaR measures.

3.3.1 Advantages:

The Delta-Normal method is easy to implement because it involves a simple matrix multiplication. It is also computationally fast, even with a large number of assets, because it replaces each position by its linear exposure. Portfolios that are linear combinations of normally distributed risk factors are themselves normally distributed. It only requires the market values and exposures of current positions, combined with risk data. Also, in many situations, the delta-normal method provides adequate measurement of market risks. As a parametric approach, VaR is easily amenable to analysis, since measures of marginal and incremental risk are a by-product of the VaR computation. This method is important not only for its own sake but also because it illustrates the "mapping" principle in risk management.

3.3.2 Disadvantages:

A first problem is the existence of fat tails in the distribution of returns on most financial assets. These fat tails are particularly worrisome precisely because VaR attempts to capture the behavior of the portfolio return in the left tail. In this situation, a model based on a normal distribution would underestimate the proportion of outliers and hence the true Value-at-Risk.

Another problem is that the method inadequately measures the risk of nonlinear instruments, such as options or mortgages. Under the delta normal method, options positions are represented by their "deltas" relative to the underlying asset. Asymmetry in the distribution of options is not captured by the delta-normal VaR.

All of these methods present some advantages. The Monte Carlo method is the most comprehensive approach to measuring market risk if modeling is done correctly.

The method can even handle credit risks. A recent survey by Britain's Financial Services Authority has revealed that 42 percent of banks use the Delta-Normal approach, 31 percent use Historical Simulation, and 23 percent use the Monte Carlo approach. The Monte Carlo analysis of linear positions with normal returns, for instance should yield the same result as the Delta-Normal method [9].

3.3.3 Implementation of Delta-Normal Method:

This implementation is a special case of the previous algorithm mentioned in the computation of Value-at-Risk with the following key differences. In this thesis, the risk factors consist of the prices of the stocks. The Portfolio's current return is denoted as p and it is known. The Portfolio's future return (or forecasted return) is not known advance and it is a random variable denoted by P. We need to estimate the distribution of P to calculate VaR. Now since the delta-normal method assumes a standard normal distribution, we assume a standard distribution such as a normal distribution for P. The problem reduces from one of estimating an entire distribution to that of estimating the parameters necessary to specify that distribution μ_P and σ_P. R is an $N-$dimensional vector which contains the values of these risk factors. Based on the historical data, we can characterize the distribution of R.

We then need to convert that characterization of the distribution of R into a characterization of the distribution of P. This is achieved by the portfolio mapping function. Portfolio's future value can be expressed in terms of R by using a function θ called the portfolio mapping function. Portfolio mapping function θ maps the N-dimensional space of the returns of the stocks to the one-dimensional space of the portfolio's future market value where N corresponds to the number of stocks chosen.

$$P = \theta(R)$$

This relationship is called portfolio mapping. Now, since R holds the prices of the different stocks then it is a very simple portfolio mapping. So, we need to apply the portfolio mapping function θ to the entire distribution of R to obtain the entire distribution of P.

θ is a linear polynomial and P is normally distributed and then all we need to do is calculate μ_P and σ_P for the portfolio. If we assume that R contains the prices of a set of stocks, then the portfolio's risk can be computed from the asset level:

$$\sigma_P = \sqrt{hCh^T} = \sqrt{\sum_i \sum_j h_i h_j \sigma_{ij}} = \sqrt{\sum_i \sum_j h_i h_j \rho_{ij} \sigma_i \sigma_j}$$

$h =$ Nx1 vector of asset weights,

C = NxN covariance matrix for the asset returns, and

$\sigma_{ij} = \rho_{ij} \sigma_i \sigma_j$ introduces the correlation coefficient.

The output of the mapping procedure in the delta-normal method is a linear mapping function θ that is applied to a normal vector R. The output values for P after the mapping are also evenly spaced, indicating that the portfolio mapping does not cause any distortion. Therefore, since R is normal, P now is normally distributed. The inference procedure accepts historical data of the stock returns of the N-dimensional vector R as its input. Since the returns of the stocks are normally distributed, a linear combination of these is also normally distributed.

The output of the inference procedure is that the characterization of the distribution of R is a normal distribution. The transformation procedure then combines the outputs from the mapping procedure and the inference procedure and uses them to characterize the distribution of P. In the delta-normal method, the transformation procedure determines that the distribution of P is a normal distribution. Based on the distribution of P and the current portfolio value p, the transformation procedure then determines the value of VaR. Since P is normally distributed then the VaR for a target probability p^* is calculated:

$$VaR(p^*) = Z_{1-p^*}\sigma_P + (p - \mu_P)$$

With Z_{1-p^*} is equal to 1.645 for a target probability of 95%. The other values of target probabilities i.e., p^* are 90%, 97.5% and 99%. Over a short time horizon, such as a day, it is reasonable to assume the portfolio's forecasted return equals to its current return. In such cases, VaR is calculated:

$$VaR(p^*) = Z_{1-p^*}\sigma_P$$

3.3.4 Proof of the VaR formula using the VaR definition:

The portfolio's Value-at-Risk (VaR) is a percentile of its loss distribution over a fixed horizon Δt. For example, the value-at-risk for a target probability of 99% is a point x_{p^*} satisfying

Δt = risk-measurement time horizon

S = vector of m market prices

ΔS = Change in S over time horizon Δt

$V(S,t)$ = portfolio value at time t and market prices S

L = loss over time horizon Δt = $-\Delta V = V(S,t) - V(S+\Delta S, t+\Delta t)$

$$1 - F_L(x_{p^*}) = P(L > x_{p^*}) = p^* \text{ with } p^* = 0.01$$

The probability $P(L > x_{p^*}) = p^*$ can be computed in terms of the standard random variable Z ,

$$P\left(\frac{L-\mu_L}{\sigma_L} > \frac{x_{p^*}-\mu_L}{\sigma_L}\right) = p^*$$

$$P\left(Z > \frac{x_{p^*}-\mu_L}{\sigma_L}\right) = p^*$$

The Central Limit Theorem is applied to L assuming that L is normally distributed and that it is a sum of independent and identically distributed random variables. Therefore, the above expression can then be written as:

$$\left(\frac{x_{p^*}-\mu_L}{\sigma_L}\right) = Z_{1-p^*}$$

So, x_{p^*} is given by:

$$x_{p^*} = \mu_L + \sigma_L Z_{1-p^*}$$

Where μ_L is the expected value of the portfolio's loss over the risk-measurement time horizon Δt. $\mu_L = E(L)$

μ_L = Current Portfolio Return – Forecasted Portfolio Return = $p - \mu_P$

σ_L = Standard deviation of the forecasted portfolio of stocks at the end of the risk-measurement time horizon Δt.

The value of x_{p^*} gives the Value-at-Risk for a particular target probability $1-p^*$.

Therefore, the Value-at- Risk for the target probability p^* is computed:

$$x_{p^*} = \mu_L + \sigma_L Z_{1-p^*}$$

For example, for p^*=0.95 the VaR is computed:

$$x_{p^*} = \mu_L + \sigma_L Z_{0.05} = \mu_L + \sigma_L 1.645$$

The value-at-risk computed here is a daily VaR. Over a short time horizon such as one day, the daily VaR is calculated:

$$x_{p^* daily} = \sigma_{Ldaily} Z_{1-p^*} \tag{2}$$

If σ_{Ldaily} is the daily return standard deviation of the portfolio, we convert this to 100-day standard deviation if the risk-measurement time horizon Δt is considered to be 100 days through:

$$\sigma_{L100days} = \sqrt{100}\sigma_{L\,daily}$$

Now, the 100-day Value-at-Risk is calculated:

$$x_{p^* 100days} = \sqrt{100}\sigma_{Ldaily} Z_{1-p^*} + \mu_{Ldaily}$$

Over a short time horizon, the current portfolio return is almost equal to the forecasted portfolio return. So, we assume that $\mu_{Ldaily} = 0$. In that case, the 100-day Value-at-Risk is calculated:

$$x_{p^* 100days} = \sqrt{100}\sigma_{Ldaily} Z_{1-p^*} \tag{3}$$

Using (2), equation (3) can be written in terms of the daily VaR as:

$$x_{p^* 100days} = \sqrt{100} x_{p^* daily}$$

The above formula is true only when the current portfolio return is equal to the forecasted portfolio return. This is equivalent to the following formula (see chapter 16, [8])

$$N - dayVaR = 1 - dayVaR * \sqrt{N}$$

3.3.4 Optimization:

Markowitz defined *efficient* portfolios as portfolios that minimized risk for a given level of return and maximized return for a given level of risk. The set of all efficient (feasible) portfolios is called the *efficient frontier* ([2, 4, 6]). Markowitz had also developed computer algorithms that could efficiently find the efficient frontier. The search for good portfolios is reduced to a standard mathematical optimization problem and this problem can be formulated in several equivalent forms.

(1) Minimize: σ_P^2 = Var(R_p) = $\sum_{i=1}^{N}\sum_{j=1}^{N} h_i \sigma_{ij} h_j = h^T C h$,subject to the constraint

μ_P = E[R_p] = $\sum_{i=1}^{N} h_i \mu_i = h^T \mu$ is equal to a specified level of return,

and $\sum_{i=1}^{N} h_i = 1$

(2) Maximize μ_P = E[R_p] = $\sum_{i=1}^{N} h_i \mu_i = h^T \mu$, subject to the constraint

σ_P^2 = Var(R_p) = $\sum_{i=1}^{N}\sum_{j=1}^{N} h_i \sigma_{ij} h_j = h^T C h$ is equal to a specified level of risk,

and $\sum_{i=1}^{N} h_i = 1$

We can impose additional constraints in the above two forms such as:

$h_i \geq 0, \forall i$ (Short Selling is forbidden)

$h_i \leq s, \forall i$ (Each asset cannot have more than a fraction of the total investment)

(3) Minimize $U = \sigma_P^2 - \lambda_E \mu_p$, subject to $\sum_{i=1}^{N} h_i = 1$

The first two forms simply require that the "optimal" portfolio is efficient. The third problem introduces the notion of a utility function where U is the *utility function* and the parameter λ_E is a measure of the *risk aversion* for the investor (the reciprocal of the *risk tolerance*). We may add additional constraints of the form:

- $Ah = b$
- $Ah \geq b$
- $h_i \geq 0, \forall i$ (short selling is forbidden)

The minimum-variance portfolio weights in the implementation are computed by using quadratic programming for the following problem:

Minimize: $\sigma_P^2 = Var(R_P) = \sum_{i=1}^{10}\sum_{j=1}^{10} h_i \sigma_{ij} h_j = h^T Ch$, subject to the constraint

$\sum_{i=1}^{10} h_i = 1$ and $h_i \geq 0, \forall i$ (short selling is not allowed).

Matlab's **quadprog** uses quadratic programming to solve for portfolio weights for a minimum variance portfolio with specified returns along the efficient frontier and it has been used for this project. In this project, we consider minimum variance portfolios on the efficient frontier. A sample of efficient frontiers and portfolio allocation bar charts are plotted for a couple of different time intervals (see Appendix B). The minimum variance portfolio lies at the bottom of the efficient frontier curve. An investor should only consider the portfolios that are on the efficient frontier. The portfolio allocation bar charts depict the different portfolio allocations over time. Portfolios during certain time intervals are fully diversified compared to the other time intervals where the portfolios do not include all the ten stocks.

4. Results and Analysis

4.1 Implementation of the Delta-Normal Method

Our data consists of 500 daily prices of ten stocks dated from 2nd Jan, 2001 (01/02/2001) to 27th May, 2003 (05/27/2003). The portfolio consisting of ten stocks is listed:

HITK	**Hi-Tech Pharmacal Inc**
WTSLA	**Wet Seal Inc**
MSEX	**Middlesex Water Co**
NWPX	**Northwest Pipe Co**
CMGI	**CMGI Inc**
NITE	**Knight Trading Group Inc**
BOKF	**BOK Financial Corp**
SMSC	**Standard Microsystems Corp**
GBND	**General Binding Corp**
ASIA	**Asia Info Holdings, Inc.**

The daily returns are computed as follows:

$$R_i(t) = (P_i(t+1) - P_i(t))/P_i(t) \text{ , for t=1, ……, 499 and i=1,…….10.}$$

The 500 days are divided into disjoint time periods of 100 daily returns each. The mean return, the standard deviation and the covariance matrix of the daily returns for the first 100 days are computed as follows:

$$\mu_i(100) = (1/100)\sum_{t=1}^{100} R_i(t) \text{, for i = 1,.......,10}$$

$$\sigma_i^2(100) = \sigma_i(100) = (1/99)\sum_{t=1}^{100}(R_i(t) - \mu_i(100))^2 \text{, for i = 1,.......,10}$$

$$\sigma_{ij}(100) = (1/99)\sum_{t=1}^{100}(R_i(t) - \mu_i(100))(R_j(t) - \mu_j(100)) \text{, for i,j=1,……,10}$$

The minimum-variance portfolio weights $\vec{h}$ are computed by using quadratic programming for the problem stated as follows:

Minimize : $\sigma_P^2 = Var(R_P) = \sum_{i=1}^{10}\sum_{j=1}^{10} h_i \sigma_{ij} h_j = h^T Ch$, subject to the constraint

$\sum_{i=1}^{10} h_i = 1$ and $h_i \geq 0, \forall i$ (short selling is forbidden)

The current return p is computed by multiplying the minimum-variance portfolio weights times the daily returns of the ten stocks for the 100th day.

$$p = \sum_{i=1}^{10} h_i R_i(100) = \vec{h}^T \vec{R}(100)$$

The risk measurement time horizon in the computation of Value-at-Risk is 100days. To compute the forecasted return P, we consider a moving window size of 100 days. So, P is the forecasted return of the portfolio for the 101st day :

$$P = \vec{h}^T \vec{\mu}(100)$$

The standard deviation of the portfolio P, denoted as σ_P , is given by.

$$\sigma_P = \left(\vec{h} C \vec{h}\right)^{1/2}$$

where $\bar{h}$ is the minimum-variance portfolio weights vector and σ_{ij} is the covariance matrix using the first 100 days. To compute the Value-at-Risk, we compute the standard deviation through:

$$\sigma_{P\,100\,days} = \sqrt{100} * \sigma_{P\,daily}$$

The Value-at-Risk at the 101st for the portfolio at a target probability of 95% is given by:

$$VaR(95\%) = 1.645\sigma_P + (p - \mu_P)$$

Now, the next time interval is considered by taking the next 100 daily returns (i.e, from the 2nd day – 101st day's returns) of the ten stocks and the same procedure is repeated for computing Value-at-Risk at the 102nd Day. The minimum-variance portfolio weights $\vec{h}$ are recomputed for each time interval by using quadratic programming.

The same procedure is repeated for 399 more time intervals and the corresponding VaR is computed for each time interval. Therefore, we compute Value-at-Risk for 400 time intervals. Finally, for a \$1000 investment in the portfolio of ten stocks we calculate the Value-at-Risk in dollars.

4.2 Implementation of the Historical Simulation Method

The same procedure is repeated for computing the mean return, the standard deviation, the covariance matrix of the daily returns and minimum-variance portfolio weights for the first 100 days as described in the implementation of the Delta-Normal Method (Section 4.1). The portfolio weights are applied to the daily returns from day 1 to day 100 to obtain a vector of 100 daily portfolio returns.

$$R_P(t) = \left(\sum_{i=1}^{10} h_i(t) R_i \right)$$

$R_P(t)$ is a vector of 100 daily portfolio returns and it is computed for each time interval. The distribution of daily portfolio returns is sorted and the 5% quantile of the distribution is taken since we consider a target probability of 95% i.e., p^* =0.95. We compute the 1-day VaR of the portfolio returns and we convert it into 100-day VaR by multiplying the 1-day VaR by square root of 100. The same procedure is repeated for 399 more time intervals and the corresponding VaR is computed in dollars for an investment of \$1000.

4.3 Results

VaR has been computed for the Delta-Normal method and the Historical Simulation method. The values of a 100-day VaR for an investment of $1000 for the 400 time intervals are obtained. Figure 6 shows the plot of the VaR in dollars for Historical Simulation method versus VaR in dollars for Delta Normal method against the corresponding date on which VaR was computed. Delta-Normal method curve oscillates more than the Historical Simulation method curve.

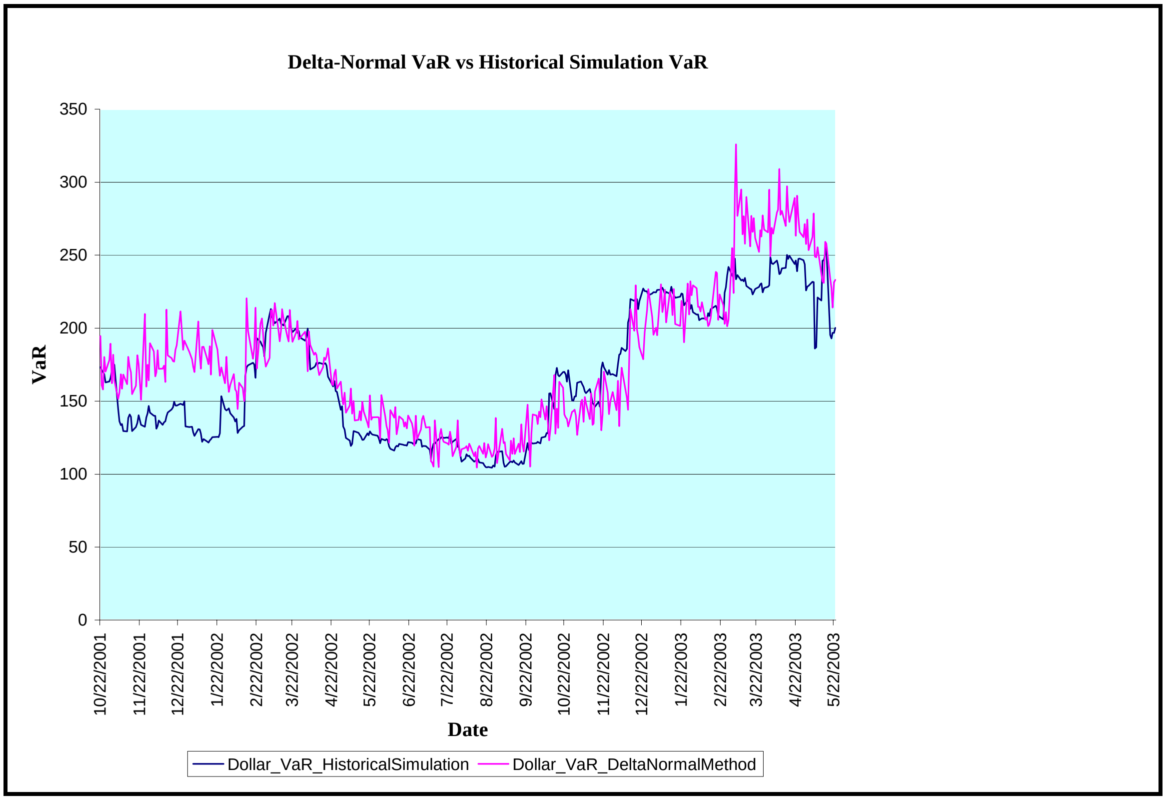

Figure 6: Delta-Normal Method VaR vs Historical Simulation Method VaR

The Delta-Normal method predicts a maximum loss of $92.38 more than the Historical Simulation method on March 6th 2003. The Historical Simulation method predicts a maximum loss of $59.87 more than the Delta-Normal method on December 11th 2002.

The distribution of the VaR computed by the Delta-Normal Method for the 400 time intervals is shown in Figure 7 and it is non-normal from the normality test.

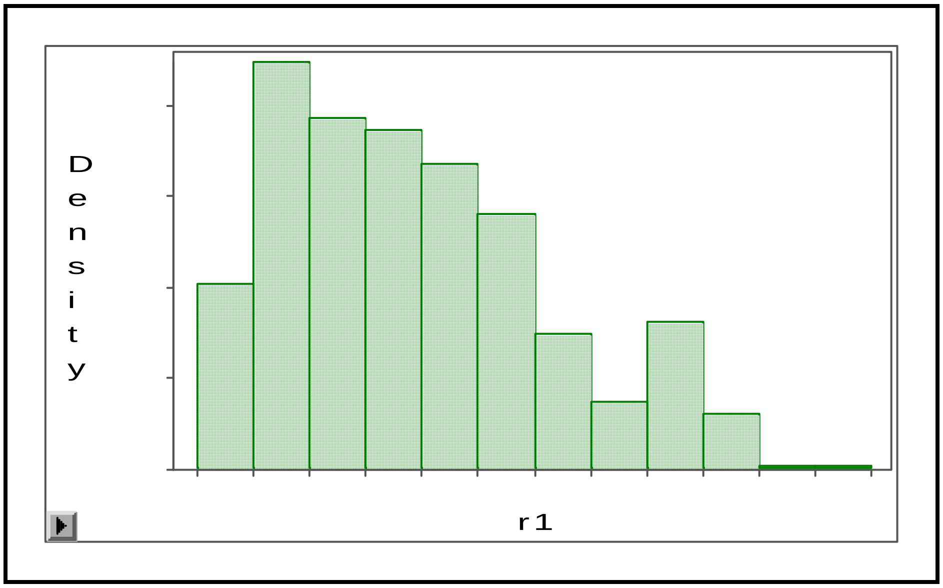

Figure 7: Distribution of VaR for Delta-Normal Method

The distribution of the VaR computed by the Historical Simulation method for the 400 time intervals is shown in Figure 8 and it is non-normal from the normality test.

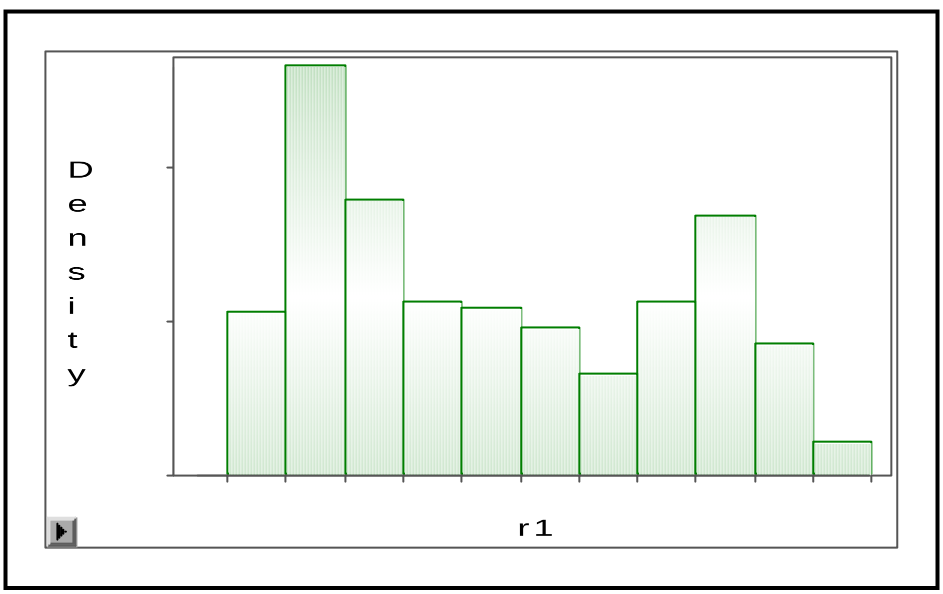

Figure 8: Distribution of VaR for Historical Simulation Method

4.4 Analysis:

The difference between the values of VaR obtained by the two methods is computed by the Delta-Normal VaR minus the Historical Simulation VaR in dollars. The Figure 9 below shows the plot of the difference in dollar VaR for Historical Simulation method versus the Delta-Normal Method against the corresponding date on which VaR was computed.

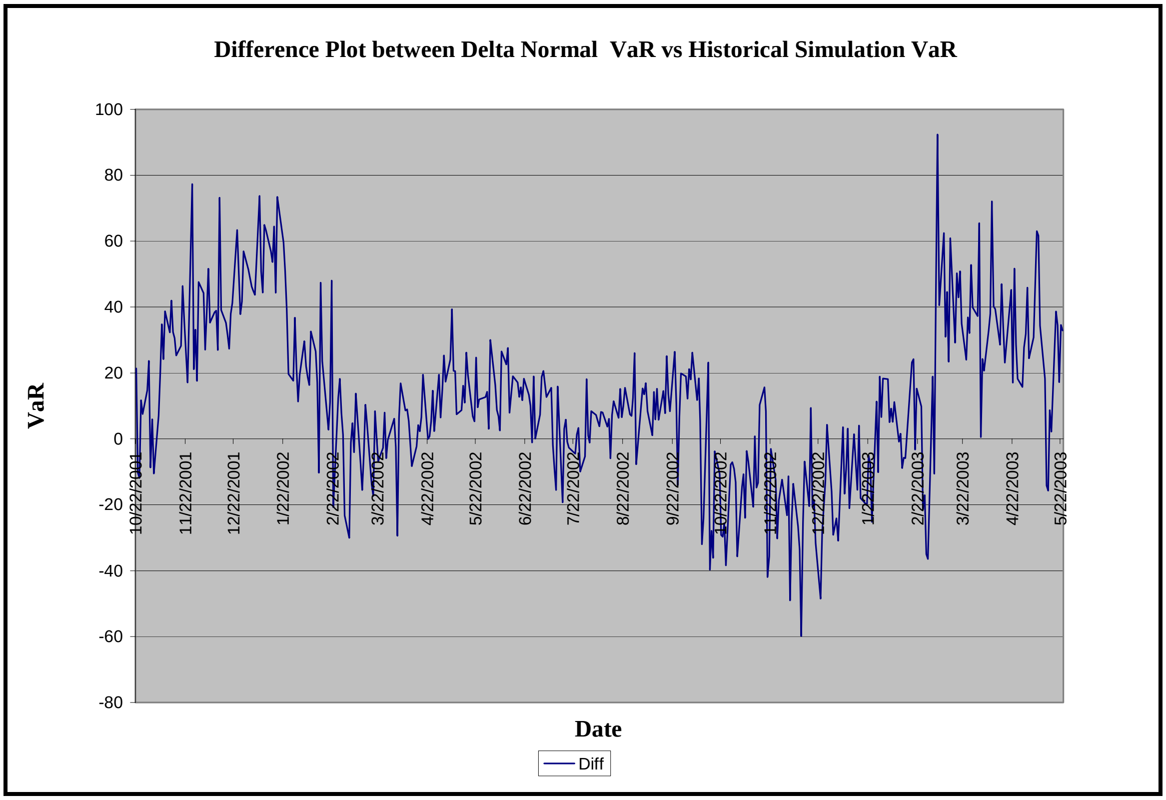

Figure 9: Plot of the difference between Delta-Normal VaR vs Historical Simulation VaR

The distribution of the difference in VaR between the Delta-Normal method and the Historical Simulation method is shown in the Figure 10. The distribution of the difference is normally distributed from the normality test (see Appendix C).

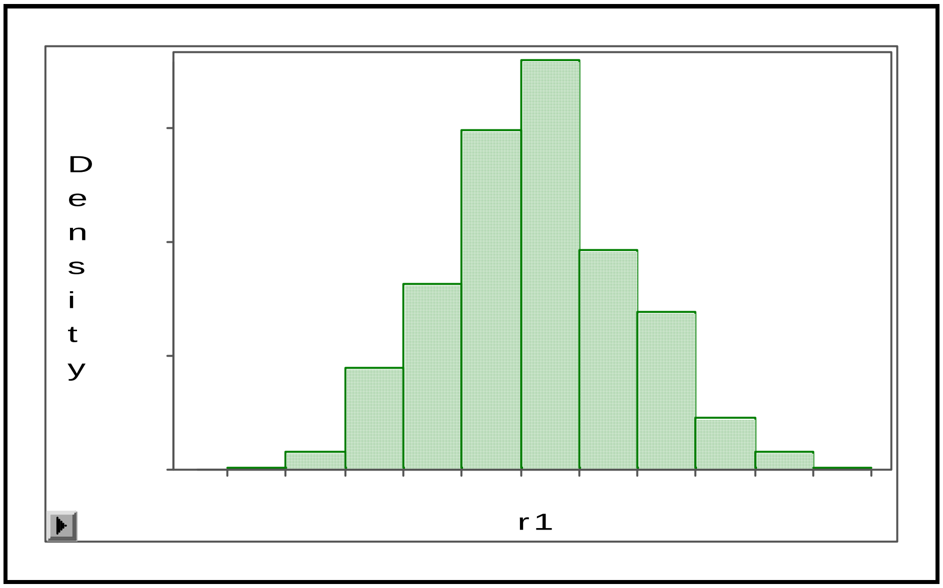

Figure 10: Plot of the distribution of the difference between Delta-Normal VaR and Historical Simulation VaR

On an average, an investor loses about $220 from Figure 6 with an initial investment of $1000. The maximum amount that an investor loses is $330 using the Delta-Normal method. Similarly, the maximum amount an investor loses is $255 using the Historical Simulation method. From Figure 9, we observe that a difference between Delta-Normal and Historical Simulation VaR is noticed during the beginning and the ending time intervals. For the time intervals in between, the Delta-Normal and Historical Simulation VaR appear to be quite close from Figure 6 but the largest percentage difference occurs on December 11th , 2002 when the Historical Simulation method exceeds the Delta method by 41.57% from Figure 11.

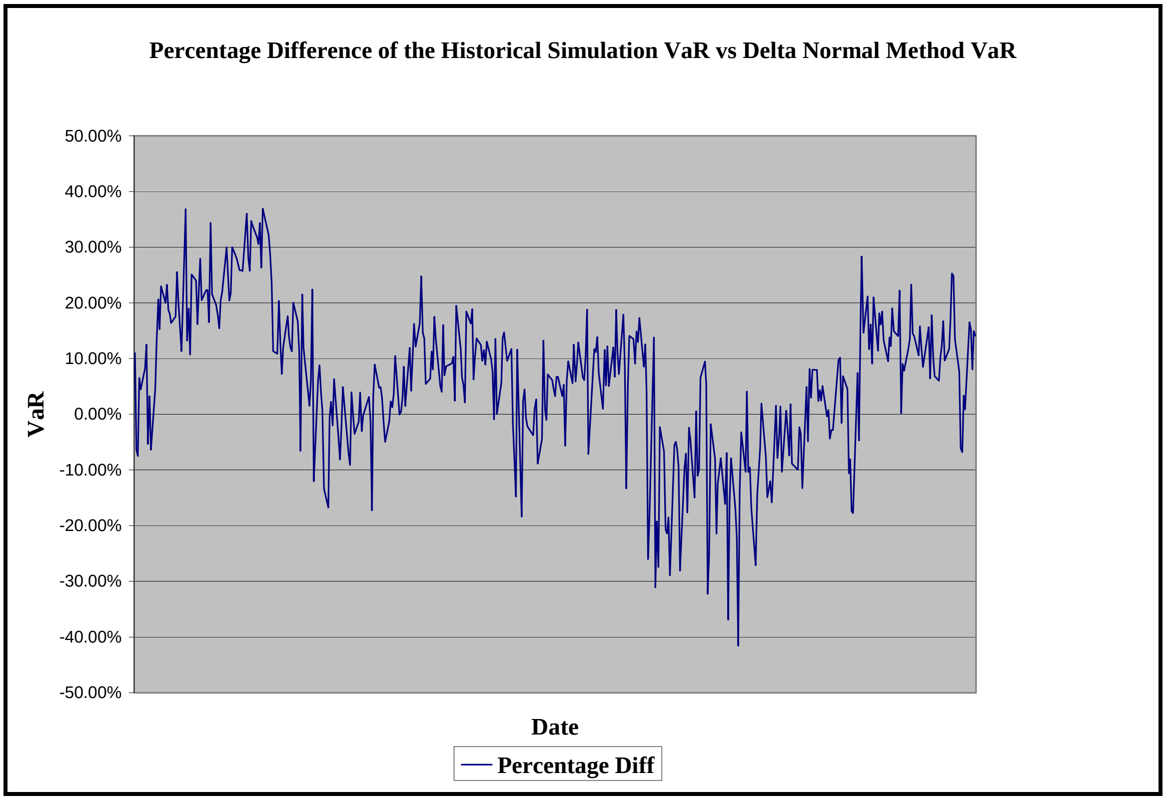

Figure 11: Plot of the Percentage difference between Delta-Normal VaR vs Historical Simulation VaR

The plot in Figure 11 oscillates much more on the positive phase (above 0%) than on the negative phase. The VaR computed by the Delta-Normal method exceeds Historical Simulation by a maximum value of 36.93%. The VaR computed by the Historical Simulation exceeds the Delta-Normal method by a maximum value of 41.57%.

The normality of the returns of the stocks is tested. The Shapiro-Wilk p-value is an indicator of the normality of the returns being considered. According to the Shapiro-Wilk normality test, we reject normality if p-value is less than 0.05 and we accept normality if the p-value is greater than 0.05. The results for the normality test for the different Phases can be seen in Appendix C. The results for the normality test for the whole data of returns of the ten stocks can be seen in Appendix D.

We divide the plot in Figure 6 into three phases. During phase I i.e., between the time interval November 5th, 2001 and February 12th, 2002 the Delta-Normal method predicts higher losses in VaR than the Historical Simulation method.

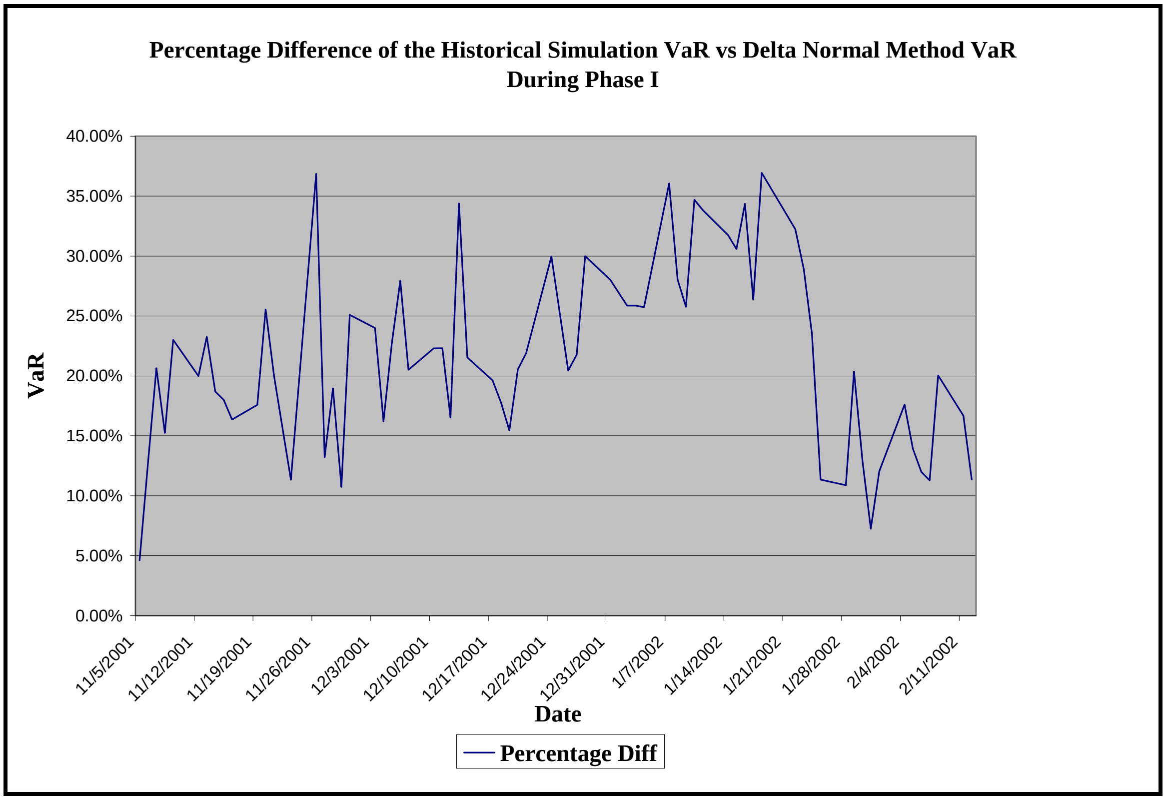

Figure 12: Plot of the Percentage difference between Delta-Normal VaR vs Historical Simulation VaR during Phase I

In the Phase I, we observe that the curve is only oscillating in the positive side. A difference in the estimates of VaR using the two methods is observed in Phase I from Figure 6. The results of the normality test in the Phase I show that the returns of the data are normally distributed (see Appendix C). Since the returns of the data are normally distributed during this phase, the Delta-Normal method would be a better approach to calculate VaR over Historical Simulation method. The VaR computed by the Delta-Normal method exceeds Historical Simulation method by a maximum value of 37%.

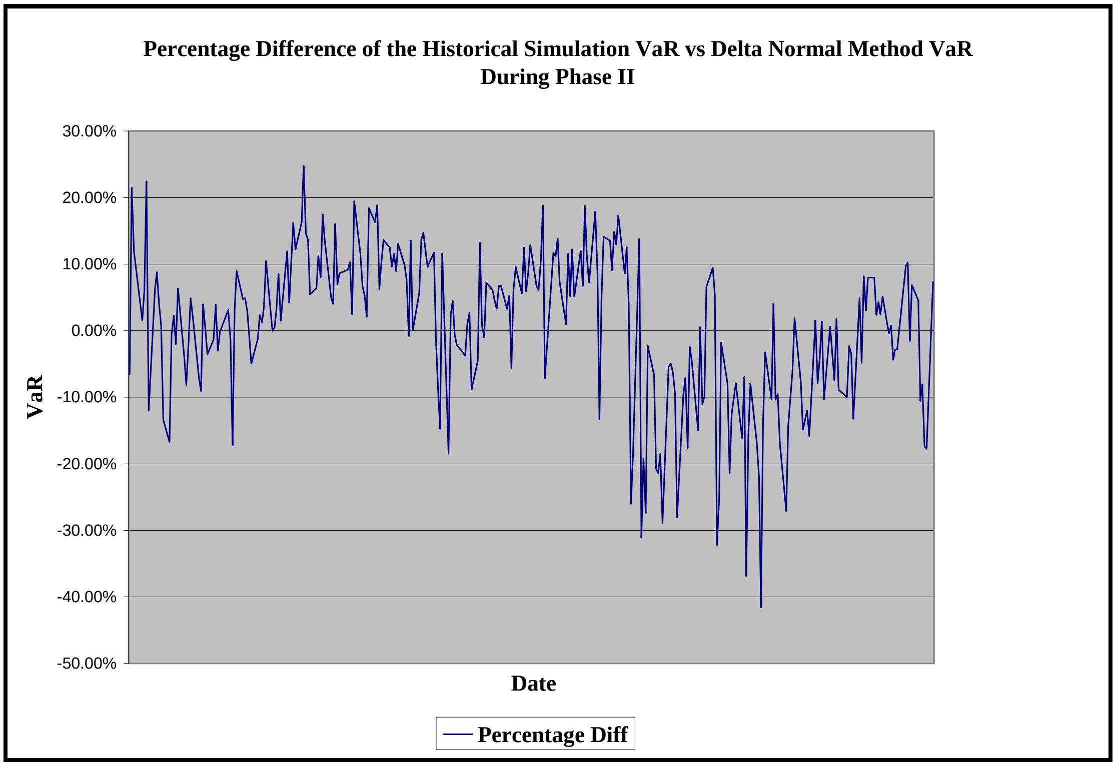

Figure 13: Plot of the Percentage difference between Delta-Normal VaR vs Historical Simulation VaR during Phase II

During Phase II i.e, between February 13th, 2002 and March 3rd, 2003, the results of the normality test show that the returns of the data are non-normal. The returns are positively skewed. With regards to kurtosis, half of the returns of the data are peaked with $kurt(X) > 3$ and half of the returns are flat with $kurt(X) < 3$. More oscillations in the plot are observed in the positive phase. Since the returns of the data are not normally distributed during this phase, the Historical-Simulation approach might be having an advantage over the Delta-Normal approach in calculating VaR. The VaR computed by the Historical Simulation exceeds the Delta-Normal method by a maximum value of 41.57%. The VaR computed by the Delta-Normal method exceeds Historical Simulation by a maximum value of 36.86%.

During Phase III i.e, between March 4th, 2003 and May 8th, 2003 the returns of the data are normally distributed (as shown in the Appendix C)

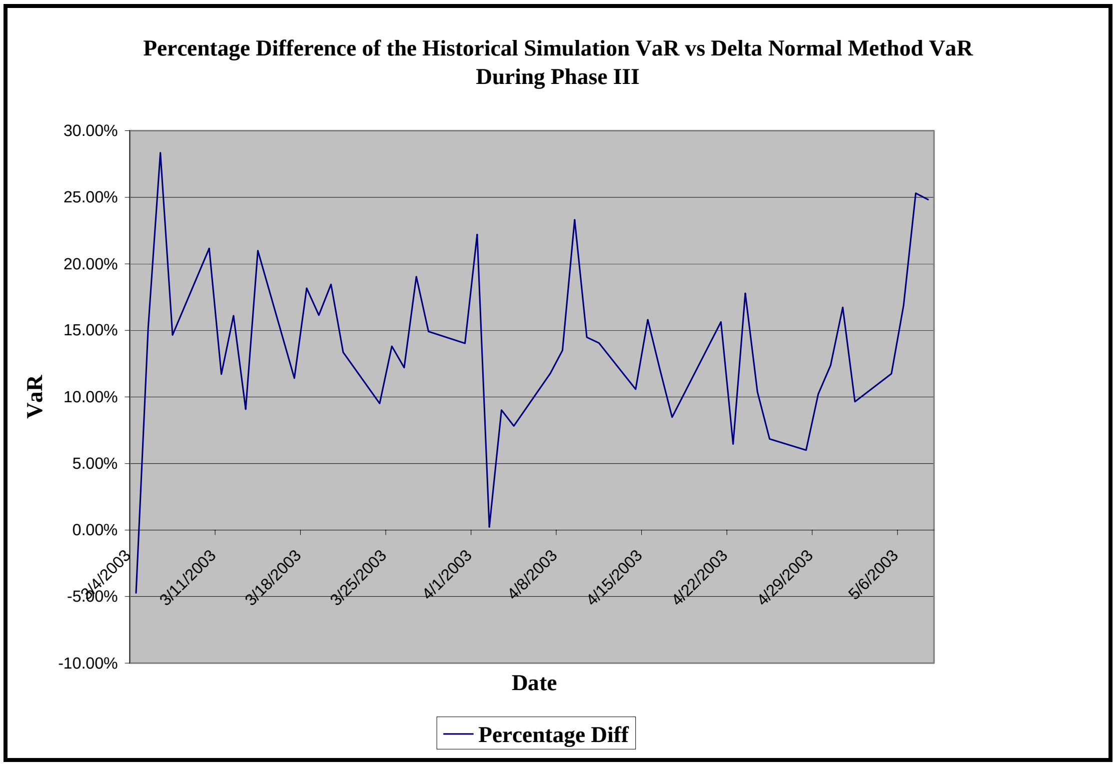

Figure 14: Plot of the Percentage difference between Delta-Normal VaR vs Historical Simulation VaR during Phase III

Since the returns of the data are normally distributed during this phase, the Delta-Normal method would be a better approach to calculate VaR. The VaR computed by the Historical Simulation exceeds the Delta-Normal method by a maximum value of 5%. The VaR computed by the Delta-Normal method exceeds Historical Simulation by a maximum value of 28%. For an initial investment of $1000, Figures 15 and 16 depict that the actual portfolio value during a particular time interval versus the (Investment –VaR). An investor is $(1-p^*)\%$ certain that the actual return on his investment will be atleast

($1000 - VaR).

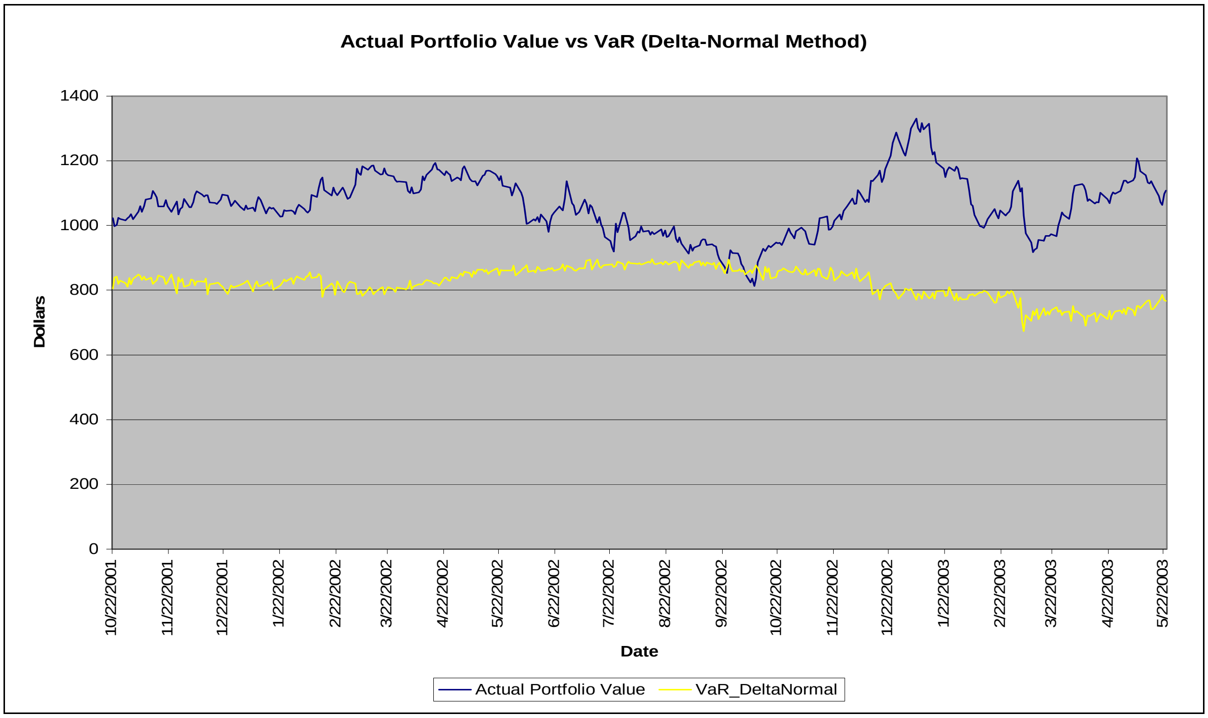

Figure 15: Plot of the Actual Portfolio Value vs (Investment – VaR) for Historical Simulation Method.

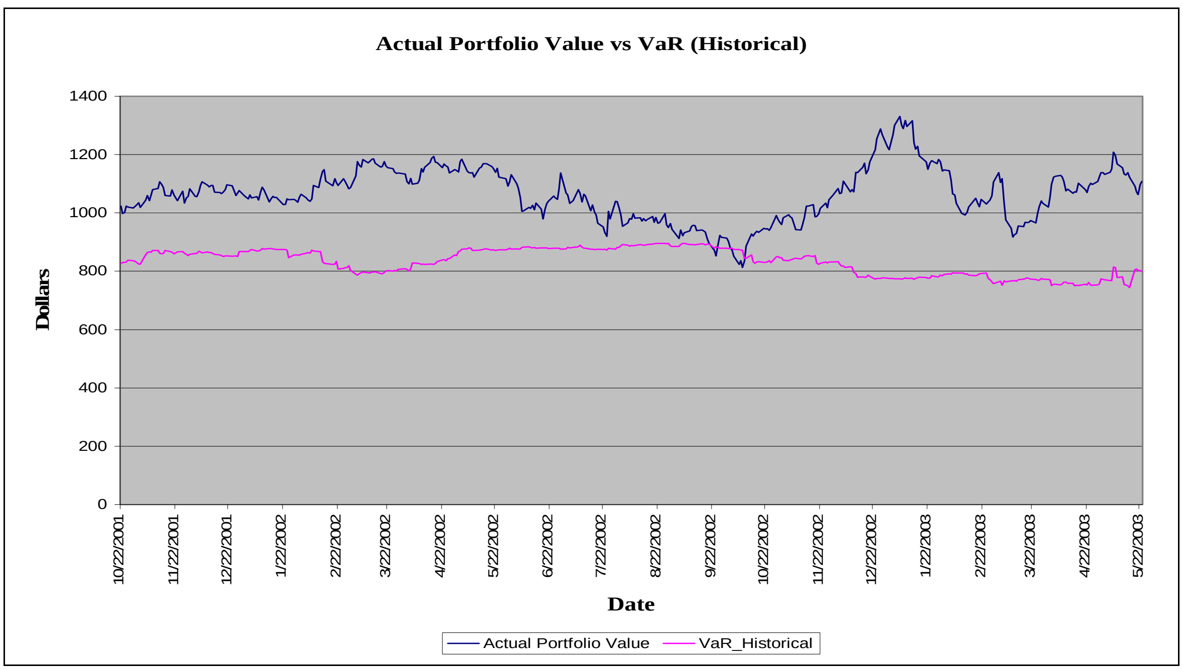

Figure 16: Plot of the Actual Portfolio Value vs (Investment – VaR) for Delta-Normal Method

5. Conclusion

The Delta-Normal Method is a suitable method to estimate VaR for linear portfolios and normally distributed returns. The Historical Simulation method is used to compute VaR both for linear and non-linear portfolios. The returns of the stock data considered in this project are normally distributed during Phase I and Phase III. So based on the normality of the returns of the data, the Delta-Normal Method is a better approach to calculate VaR compared to the Historical Simulation Method. Delta-Normal Method has an advantage of being easy to be implemented.

The actual return on the portfolio is greater than the Value-at-Risk for the Delta-Normal method for all the time intervals except from October 4th 2002 to October 11th 2002. The actual return on the portfolio is greater than the Value-at-Risk for the Historical Simulation method for all the time intervals except for the time intervals from September 20th 2002 to September 27th 2002 and from October 3rd 2002 to October 11th 2002 . Thus, any investor who would invest in this portfolio would go with the Delta-Normal Method of computing the Value-at-Risk.

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Appendix A: Matlab Code

Code for Calculating VaR using the Delta-Normal Method

```
% Matlab Code to Calculate VaR using Delta-Normal Method
% ReadTenStocks
Data = xlsread('Historical_Data.xls');
alpha=0.95;
m=100;

for i=1:400
mu = mean(Data(1+(i-1):100+(i-1),:))';
dim = size(mu,1);
C = cov(Data(1+(i-1):100+(i-1),:));

% Quadratic Programming to calculate the minimum-variance
% optimal portfolio weights.
% Start with small risk tolerance to approximate the min
variance portfolio

minrt = 0.001;

% You need an initial feasible point:  force the fully
% invested constraint.
x0 = zeros(dim,1);
slack = 1-sum(x0);
UB=[1 1 1 1 1 1 1 1 1 1];
LB=[0 0 0 0 0 0 0 0 0 0];

for j=1:dim
    x0(j) = min(slack,UB(j));
    slack = 1-sum(x0);
end

    rt = minrt;
                   x=gmqp(rt,mu,C,LB,UB,x0);
    muP = mu'*x;
    VarP = x'*C*x;
    PlotPoint = [muP, VarP];
    StackX = x;

% Now loop through the risk tolerances

numsteps = 100;
for rt= minrt : .005 : numsteps+minrt,
    x0 = x;
    x=gmqp(rt,mu,C,LB,UB,x0);
    muP = mu'*x;
    VarP = x'*C*x;
    PlotPoint = [PlotPoint; muP, VarP];
    StackX = [StackX,x];
end

% Plot the efficient frontier
figure(1);
last = size(PlotPoint,1);
```

```
xmax = 1.2*PlotPoint(last,1);
ymax = 1.2*PlotPoint(last,2);
xlim([0,xmax]);
ylim([0,ymax]);
plot(PlotPoint(:,2),PlotPoint(:,1),'-r','LineWidth',3);
grid on
title('Efficient Frontier')
xlabel('Risk = Portfolio Variance')
ylabel('Return')
%
% Plot the efficient portfolios in a bar chart
figure(2);
bar(StackX','stack')
grid on
xlim([0,numsteps])
ylim([0,1])
title('Efficient Portfolio Allocations')
xlabel('Different Portfolio Allocations')
Ylabel('Sum of Portfolio Allocations')

% Quadratic Programming using Optimization toolbox to calculate
% the minimum-variance Portfolio Weights.

c=[0 0 0 0 0 0 0 0 0 0];
Aeq =[1 1 1 1 1 1 1 1 1 1];
Beq =[1];
UB=[1 1 1 1 1 1 1 1 1 1];
LB=[0 0 0 0 0 0 0 0 0 0];
[x,fval,EXITFLAG,OUTPUT,lambda] =
quadprog(C,c,[],[],Aeq,Beq,LB,UB);
g2=Aeq*x;
fprintf('\nFinal Values\n')
fprintf('Optimum Design Variables\n')
fprintf('------------------------\n'),disp(x'),disp(sum(x'))
fprintf('Optimum function value\n')
fprintf('---------------------\n'),disp(fval)
fprintf('\nLagrange Multipliers for equality constraint\n')
fprintf('-----------------------------------------------\n')...
   ,disp(lambda.eqlin')
fprintf('\nEquality constraint\n')
fprintf('---------------------\n'),disp(g2)

% Multiply the minimum-variance portfolio weights by the
% next 100 daily returns
% p - Current Portfolio Return
% P - Forecasted Portfolio Return
p=Data(100+(i-1),:)*x;
P=x'*mu;
stdP=sqrt(x'*C*x);

% Compute the 100-Day VaR
VatHist(i,1)=(1.645*stdP*sqrt(100)+(p-P));
% Compute the Dollar Amount of VaR
Dollar_VatHist(i,1)=VatHist(i)*1000;

end
```

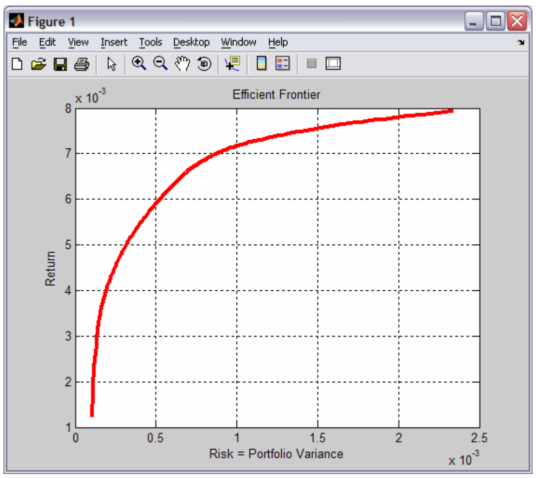
Code for Calculating VaR using the Historical-Simulation Method

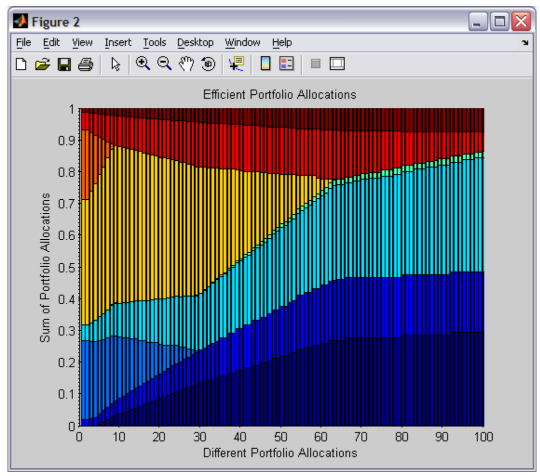
```
% Matlab Code to Compute the VaR using Historical Simulation Method.
% ReadTenStocks
Data = xlsread('Historical_Data.xls');
alpha=0.95;
m=100;

for i=1:400
mu = mean(Data(1+(i-1):100+(i-1),:))';
dim = size(mu,1);
C = cov(Data(1+(i-1):100+(i-1),:));

% Quadratic Programming to calculate the minimum-variance optimal
% portfolio weights.
% Start with small risk tolerance to approximate the min variance
portfolio
minrt = 0.001;

% You need an initial feasible point:  force the fully invested
% constraint.
x0 = zeros(dim,1);
slack = 1-sum(x0);
UB=[1 1 1 1 1 1 1 1 1 1];
LB=[0 0 0 0 0 0 0 0 0 0];

for j=1:dim
    x0(j) = min(slack,UB(j));
    slack = 1-sum(x0);
end
    rt = minrt;
    x=gmqp(rt,mu,C,LB,UB,x0);
    muP = mu'*x;
    VarP = x'*C*x;
    PlotPoint = [muP, VarP];
    StackX = x;

% Now loop through the risk tolerances
numsteps = 100;
for rt= minrt : .005 : numsteps+minrt,
    x0 = x;
    x=gmqp(rt,mu,C,LB,UB,x0);
    muP = mu'*x;
    VarP = x'*C*x;
    PlotPoint = [PlotPoint; muP, VarP];
    StackX = [StackX,x];
end

% Plot the effecient frontier
figure(1);
last = size(PlotPoint,1);
xmax = 1.2*PlotPoint(last,1);
ymax = 1.2*PlotPoint(last,2);
xlim([0,xmax]);
ylim([0,ymax]);
plot(PlotPoint(:,2),PlotPoint(:,1),'-r','LineWidth',3);
```

```
grid on
title('Efficient Frontier')
xlabel('Risk = Portfolio Variance')
ylabel('Return')

% Plot the efficient portfolios in a bar chart
figure(2);
bar(StackX','stack')
grid on
xlim([0,numsteps])
ylim([0,1])
title('Efficient Portfolio Allocations')
xlabel('Different Portfolio Allocations')
Ylabel('Sum of Portfolio Allocations')

% Quadratic Programming using Optimization toolbox to calculate the
% minimum
% variance Portfolio Weights.
c=[0 0 0 0 0 0 0 0 0 0];
Aeq =[1 1 1 1 1 1 1 1 1 1];
Beq =[1];
UB=[1 1 1 1 1 1 1 1 1 1];
LB=[0 0 0 0 0 0 0 0 0 0];
[x,fval,EXITFLAG,OUTPUT,lambda] =
quadprog(C,c,[],[],Aeq,Beq,LB,UB);
g2=Aeq*x;
fprintf('\nFinal Values\n')
fprintf('Optimum Design Variables\n')
fprintf('-------------------------\n'),disp(x'),disp(sum(x'))
fprintf('Optimum function value\n')
fprintf('---------------------\n'),disp(fval)
fprintf('\nLagrange Multipliers for equality constraint\n')
fprintf('-----------------------------------------------\n')...
   ,disp(lambda.eqlin')
fprintf('\nEquality constraint\n')
fprintf('---------------------\n'),disp(g2)

% Multiply the minimum-variance optimal portfolio weights by the
% next 100 daily returns
FutureData=Data(1+(i-1):100+(i-1),:);
Hist=FutureData*x;
% Sort the Portfolio Returns
sortHist=sort(Hist);
% Compute the Index for the specified quantile
index=round((1-alpha)*m);
% Compute the 100-day VaR
VatHist(i,1)=sortHist(index)*sqrt(100);
% Compute the Dollar Amount of VaR
Dollar_VatHist(i,1)=VatHist(i)*1000;

end
```

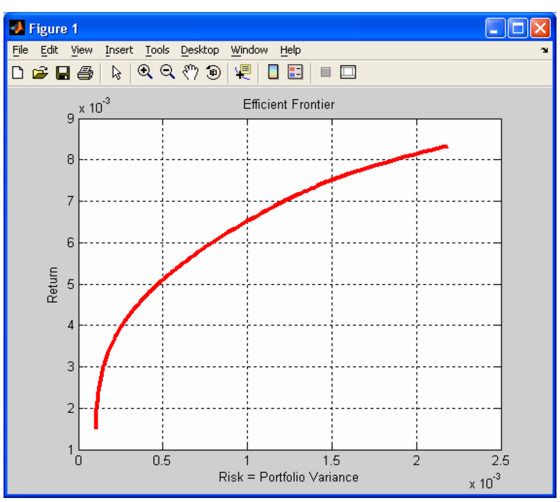
Appendix B: Efficient Frontiers and Portfolio Bar Allocations Charts for Different Time Intervals

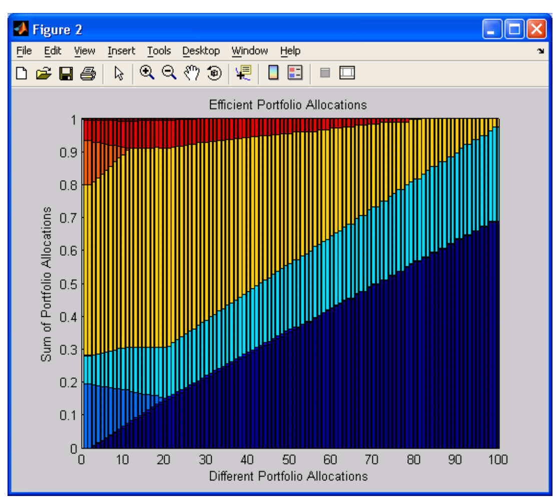
Efficient Frontier (Risk Vs Return) and Efficient Portfolio Allocations Bar Chart for the portfolio of ten stocks between 01/02/2001 and 05/25/2001



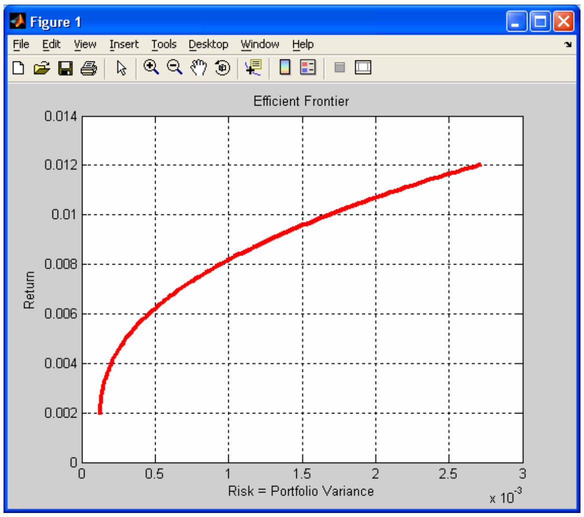


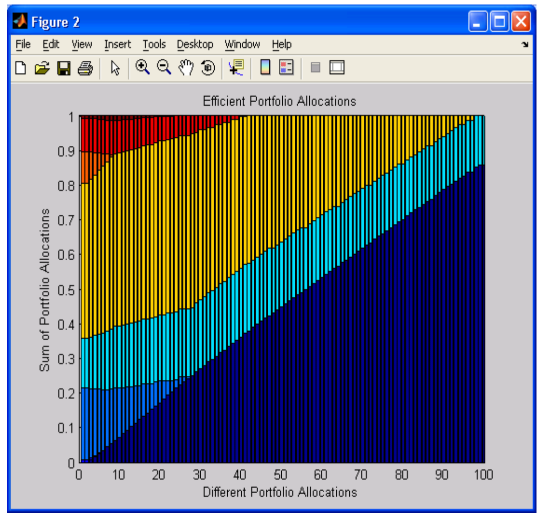
Efficient Frontier (Risk Vs Return) and Efficient Portfolio Allocations Bar Chart for the portfolio of ten stocks between 02/01/2001 and 06/26/2001



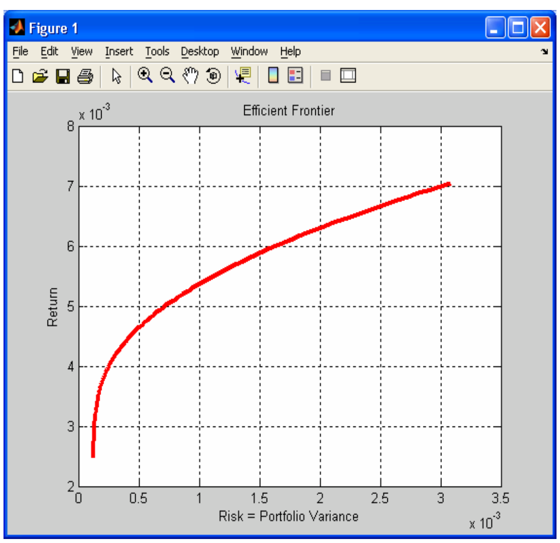


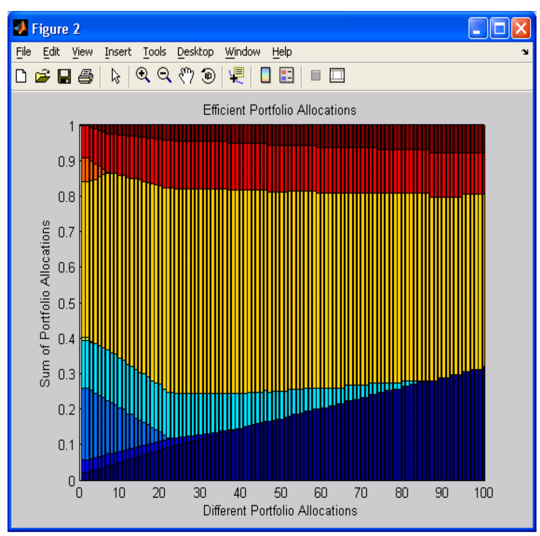
Efficient Frontier (Risk Vs Return) and Efficient Portfolio Allocations Bar Chart for the portfolio of ten stocks between 03/05/2001 and 07/26/2001



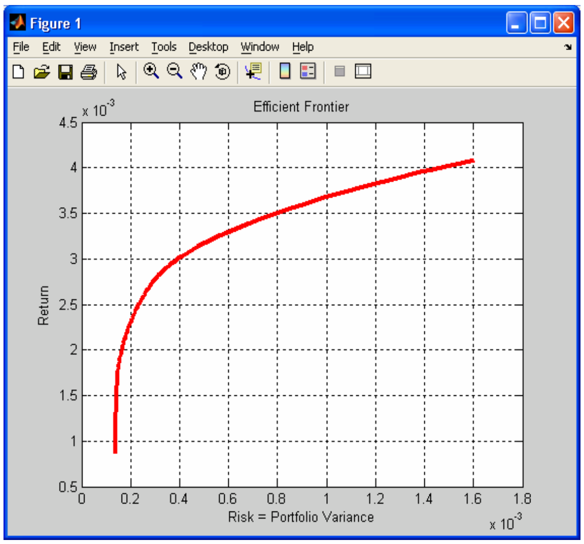


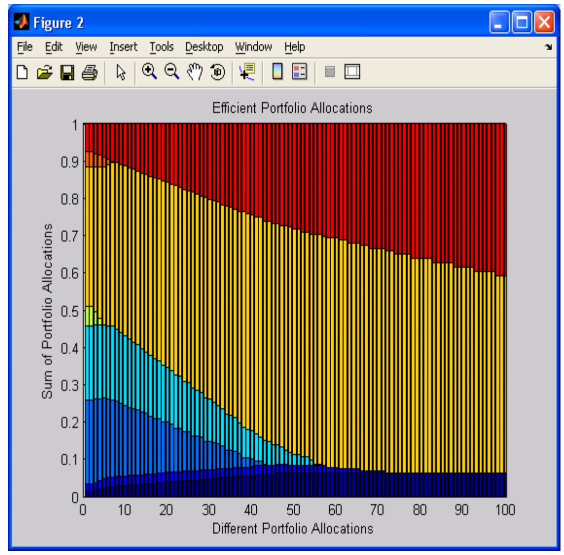
Efficient Frontier (Risk Vs Return) and Efficient Portfolio Allocations Bar Chart for the portfolio of ten stocks between 04/03/2001 and 08/24/2001



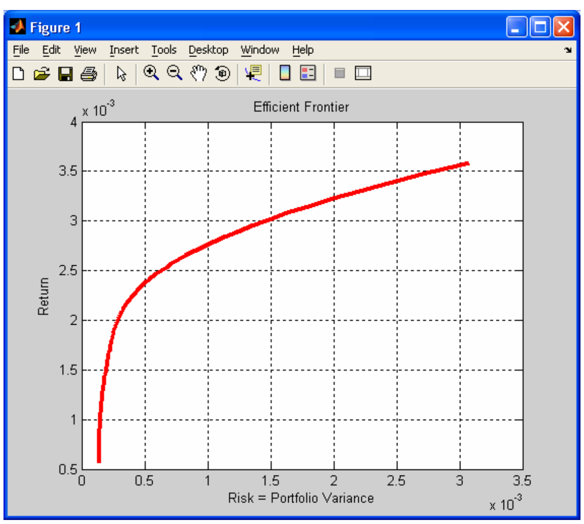


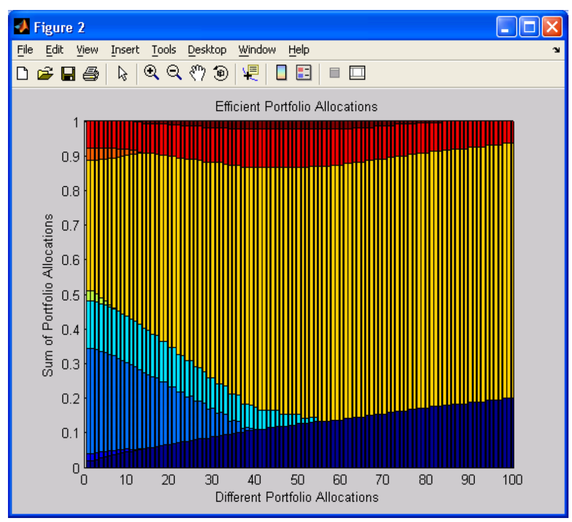
Efficient Frontier (Risk Vs Return) and Efficient Portfolio Allocations Bar Chart for the portfolio of ten stocks between 05/03/2001 and 10/01/2001





Efficient Frontier (Risk Vs Return) and Efficient Portfolio Allocations Bar Chart for the portfolio of ten stocks between 06/04/2001 and 10/30/2001

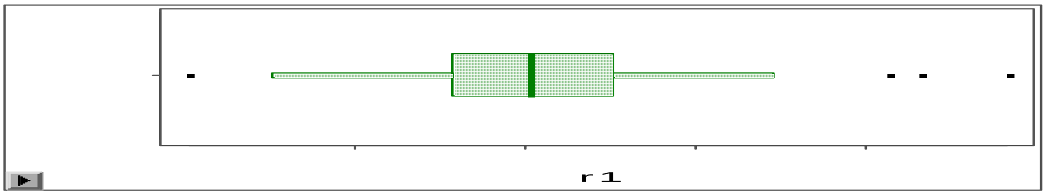


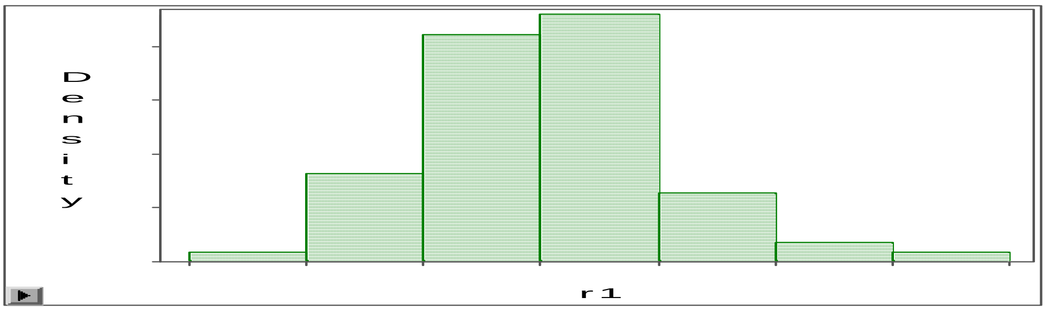


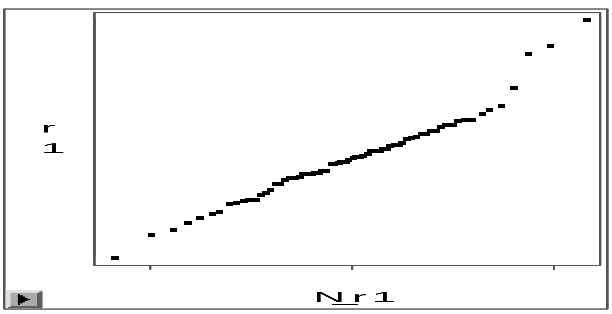
Appendix C: SAS Results I

Phase I: The normality test for the returns of the data from November 5th, 2001 to February 12th, 2002.

r1







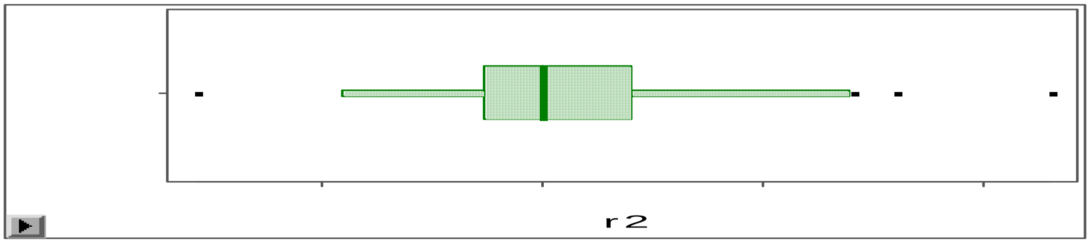
Moments			
N	68.0000	Sum Wgts	68.0000
Mean	0.0022	Sum	0.1488
Std Dev	0.0425	Variance	0.0018
Skewness	0.6279	Kurtosis	1.6520
USS	0.1216	CSS	0.1213
CV	1944.2537	Std Mean	0.0052

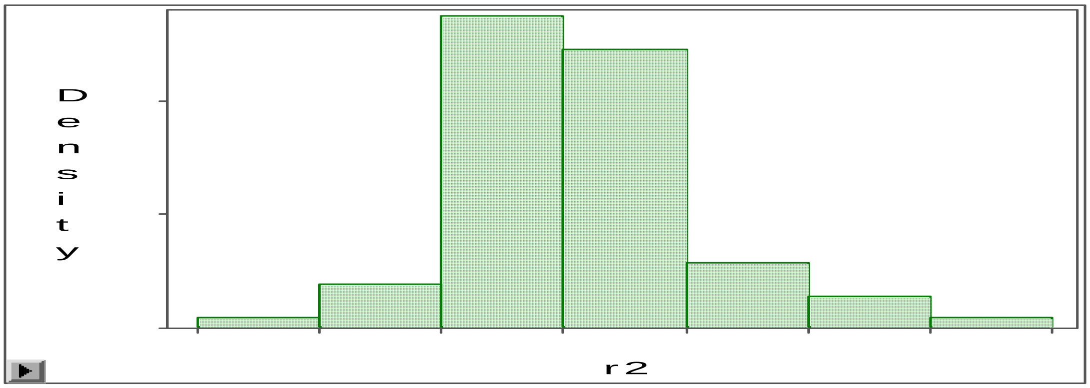
Quantiles			
100% Max	0.1419	99.0%	0.1419
75% Q3	0.0253	97.5%	0.1161
50% Med	0.0014	95.0%	0.0727
25% Q1	-0.0215	90.0%	0.0469
0% Min	-0.0986	10.0%	-0.0514
Range	0.2405	5.0%	-0.0630
Q3-Q1	0.0468	2.5%	-0.0743
Mode	.	1.0%	-0.0986

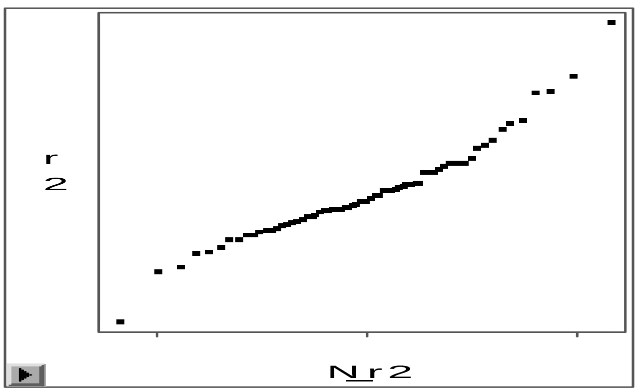
Tests for Normality

Test	Statistic	Value	p-value
Shapiro-Wilk		0.965062	0.0536
Kolmogorov-Smirnov		0.073749	>.1500
Cramer-von Mises		0.073865	0.2476
Anderson-Darling		0.572427	0.1371

r2







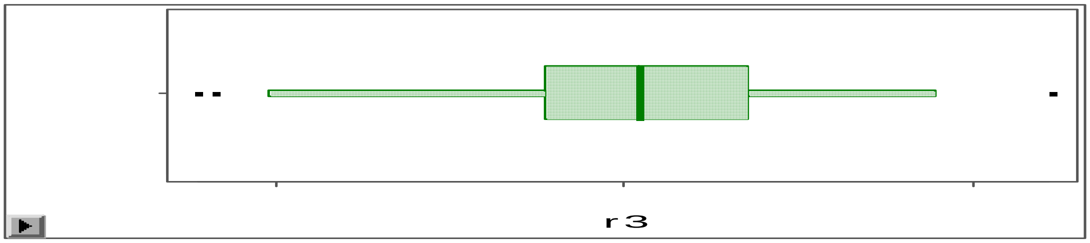
Moments			
N	68.0000	Sum Wgts	68.0000
Mean	0.0051	Sum	0.3460
Std Dev	0.0307	Variance	0.0009
Skewness	0.8355	Kurtosis	2.3925
USS	0.0649	CSS	0.0631
CV	603.1218	Std Mean	0.0037

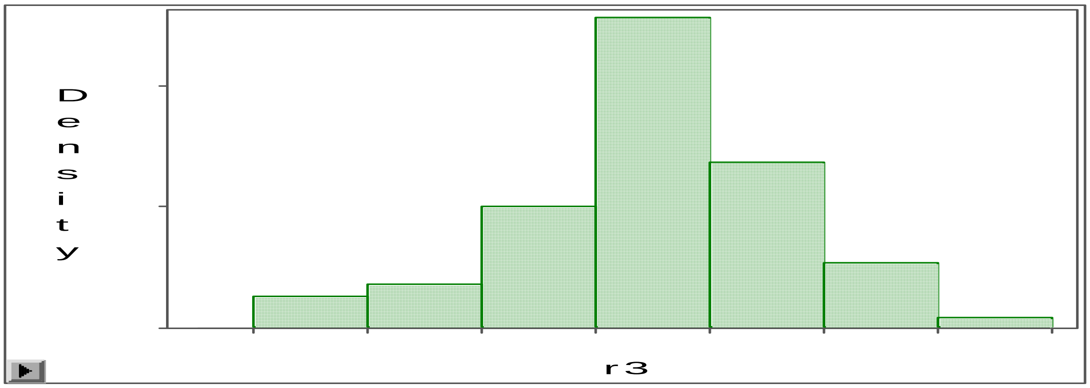
Quantiles			
100% Max	0.1154	99.0%	0.1154
75% Q3	0.0199	97.5%	0.0803
50% Med	0	95.0%	0.0694
25% Q1	-0.0131	90.0%	0.0461
0% Min	-0.0781	10.0%	-0.0248
Range	0.1935	5.0%	-0.0331
Q3-Q1	0.0331	2.5%	-0.0454
Mode	0	1.0%	-0.0781

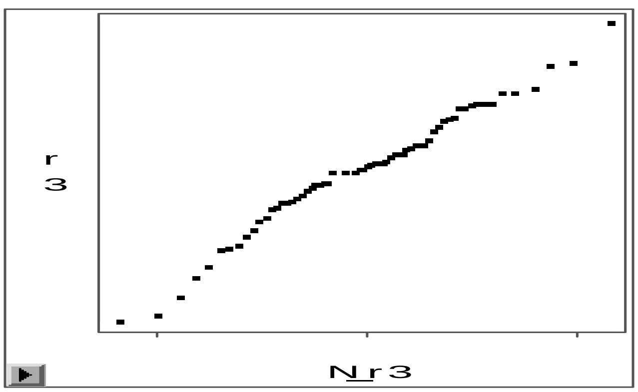
Tests for Normality

Test Statistic	Value	p-value
Shapiro-Wilk	0.946095	0.0053
Kolmogorov-Smirnov	0.114516	0.0250
Cramer-von Mises	0.187583	0.0076
Anderson-Darling	1.129527	0.0057

r3







Moments

N	68.0000	Sum Wgts	68.0000
Mean	0.0006	Sum	0.0428
Std Dev	0.0096	Variance	9.206E-05
Skewness	-0.3614	Kurtosis	0.5339
USS	0.0062	CSS	0.0062
CV	1524.2441	Std Mean	0.0012

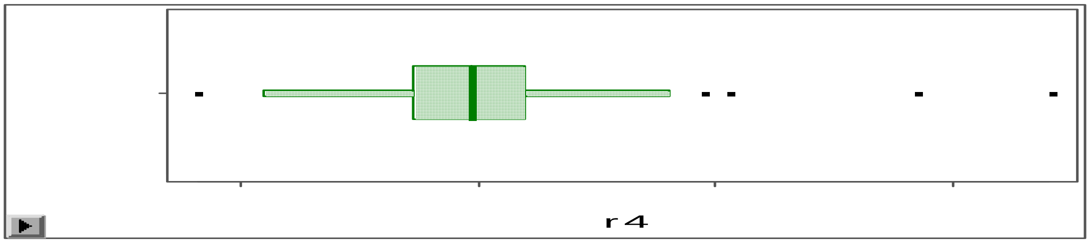
Quantiles

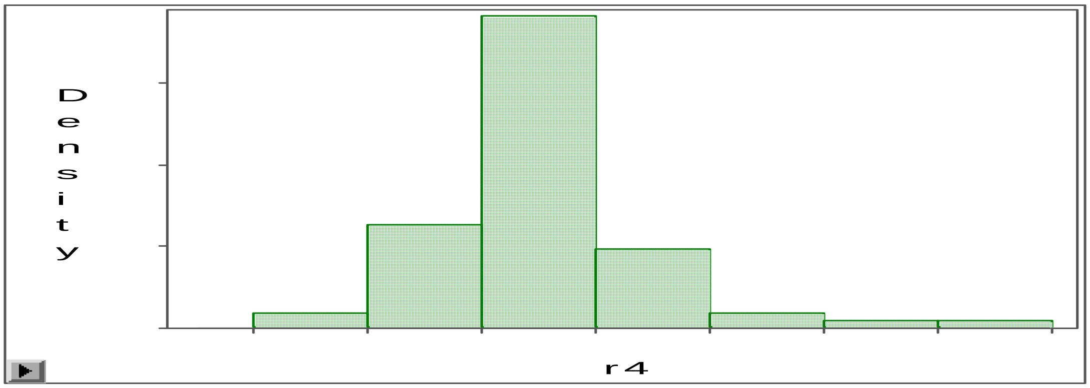
100% Max	0.0245	99.0%	0.0245
75% Q3	0.0070	97.5%	0.0179
50% Med	-0.0007	95.0%	0.0136
25% Q1	-0.0046	90.0%	0.0129
0% Min	-0.0245	10.0%	-0.0126
Range	0.0490	5.0%	-0.0173
Q3-Q1	0.0116	2.5%	-0.0235
Mode	0	1.0%	-0.0245

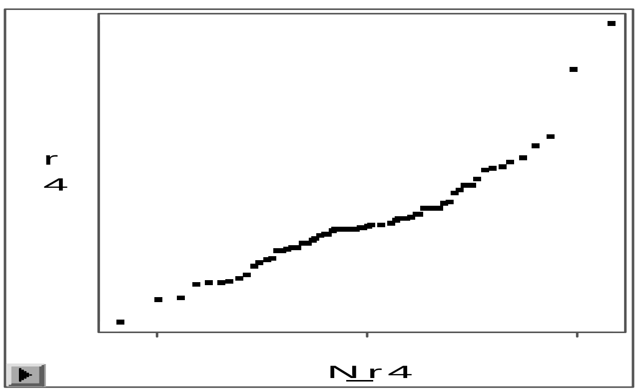
Tests for Normality

Test	Statistic	Value	p-value
Shapiro-Wilk		0.978457	0.2872
Kolmogorov-Smirnov		0.101250	0.0833
Cramer-von Mises		0.097814	0.1210
Anderson-Darling		0.559790	0.1463

r4







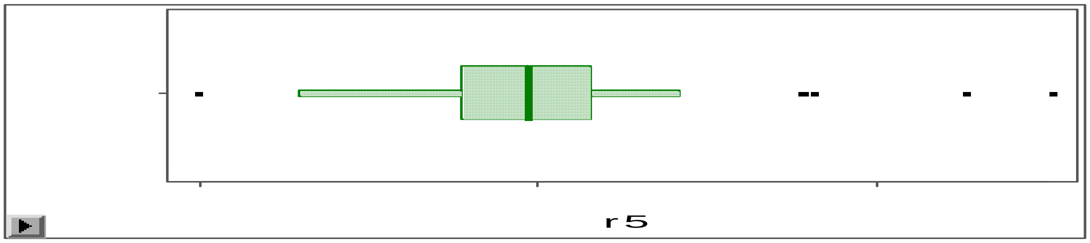
Moments			
N	68.0000	Sum Wgts	68.0000
Mean	0.0014	Sum	0.0957
Std Dev	0.0290	Variance	0.0008
Skewness	1.3839	Kurtosis	4.4404
USS	0.0563	CSS	0.0562
CV	2058.4513	Std Mean	0.0035

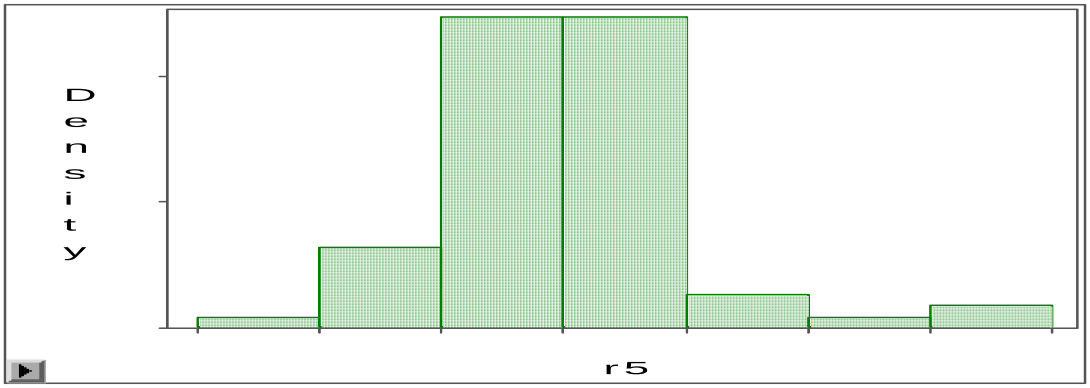
Quantiles			
100% Max	0.1208	99.0%	0.1208
75% Q3	0.0097	97.5%	0.0926
50% Med	-0.0016	95.0%	0.0476
25% Q1	-0.0139	90.0%	0.0342
0% Min	-0.0592	10.0%	-0.0341
Range	0.1801	5.0%	-0.0364
Q3-Q1	0.0236	2.5%	-0.0452
Mode	0	1.0%	-0.0592

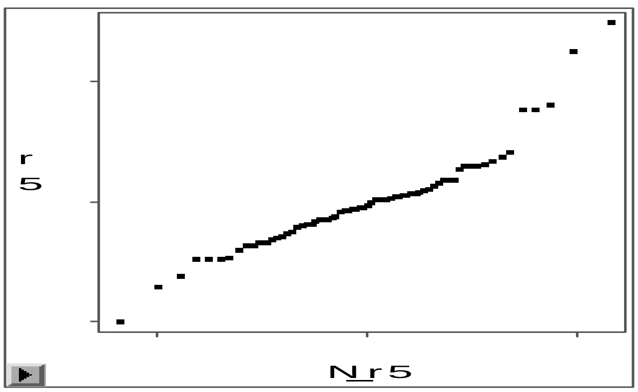
Tests for Normality

Test	Statistic	Value	p-value
Shapiro-Wilk		0.902218	0.0001
Kolmogorov-Smirnov		0.149775	<.0100
Cramer-von Mises		0.313700	<.0050
Anderson-Darling		1.687083	<.0050

r5







Moments

N	68.0000	Sum Wgts	68.0000
Mean	0.0010	Sum	0.0690
Std Dev	0.0811	Variance	0.0066
Skewness	1.1247	Kurtosis	3.4432
USS	0.4407	CSS	0.4406
CV	7993.4217	Std Mean	0.0098

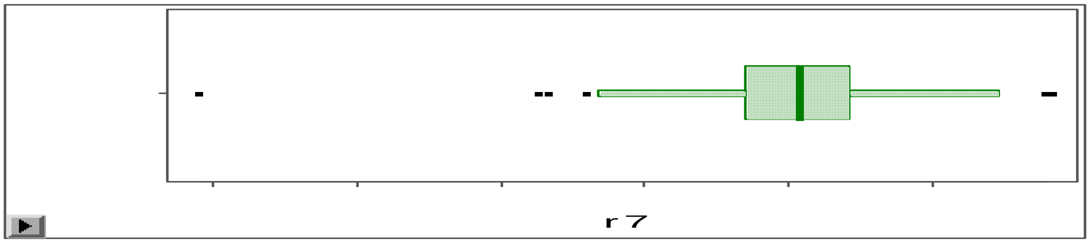
Quantiles

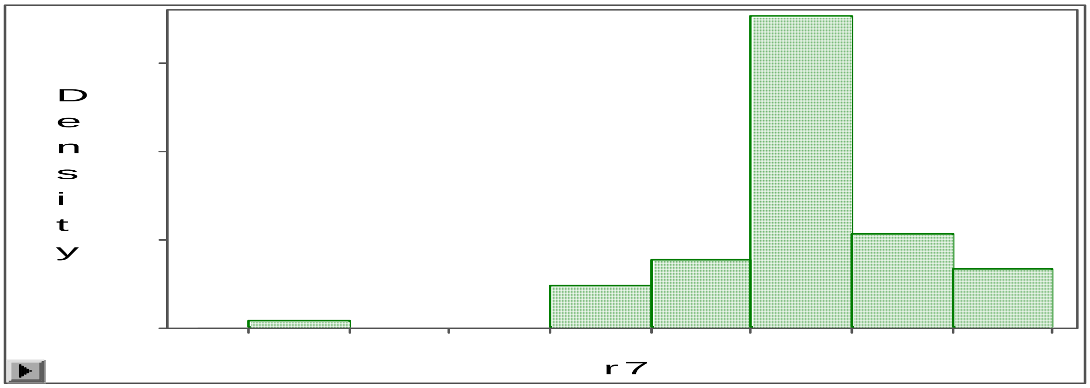
100% Max	0.3043	99.0%	0.3043
75% Q3	0.0311	97.5%	0.2538
50% Med	-0.0058	95.0%	0.1573
25% Q1	-0.0448	90.0%	0.0765
0% Min	-0.2009	10.0%	-0.0909
Range	0.5052	5.0%	-0.0957
Q3-Q1	0.0760	2.5%	-0.1410
Mode	.	1.0%	-0.2009

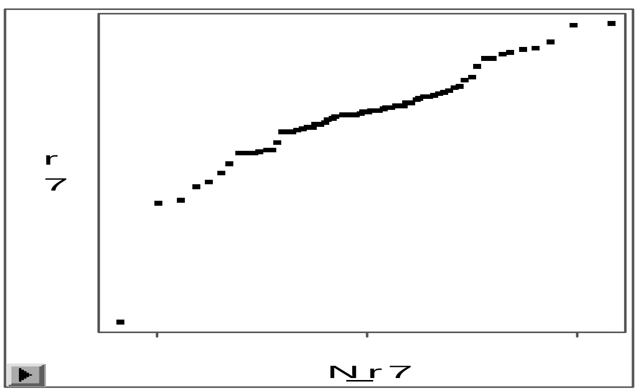
Tests for Normality

Test Statistic	Value	p-value
Shapiro-Wilk	0.917076	0.0002
Kolmogorov-Smirnov	0.129856	< 0.0100
Cramer-von Mises	0.246902	< 0.0050
Anderson-Darling	1.553019	< 0.0050

r7







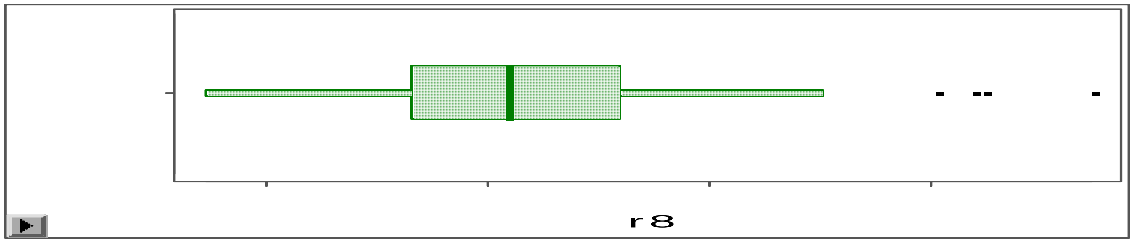
Moments			
N	68.0000	Sum Wgts	68.0000
Mean	0.0003	Sum	0.0197
Std Dev	0.0180	Variance	0.0003
Skewness	-1.3433	Kurtosis	5.7264
USS	0.0217	CSS	0.0217
CV	6222.4908	Std Mean	0.0022

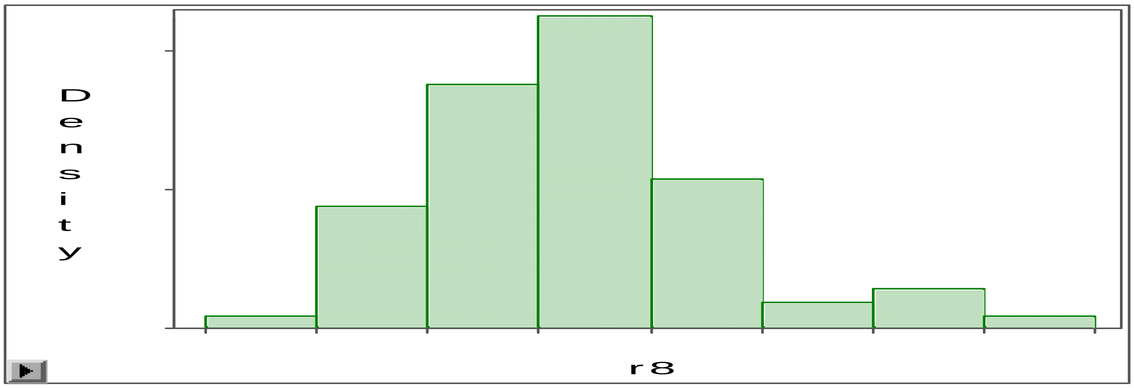
Quantiles			
100% Max	0.0365	99.0%	0.0365
75% Q3	0.0083	97.5%	0.0355
50% Med	0.0014	95.0%	0.0262
25% Q1	-0.0063	90.0%	0.0239
0% Min	-0.0822	10.0%	-0.0192
Range	0.1186	5.0%	-0.0284
Q3-Q1	0.0146	2.5%	-0.0348
Mode	0	1.0%	-0.0822

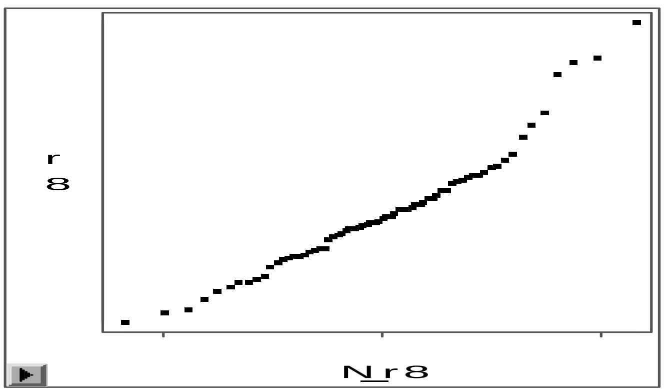
Tests for Normality

Test Statistic	Value	p-value
Shapiro-Wilk	0.901789	0.0001
Kolmogorov-Smirnov	0.145857	<.0100
Cramer-von Mises	0.275569	<.0050
Anderson-Darling	1.461733	<.0050

r8







Moments

N	68.0000	Sum Wgts	68.0000
Mean	0.0093	Sum	0.6309
Std Dev	0.0399	Variance	0.0016
Skewness	0.9386	Kurtosis	1.4774
USS	0.1124	CSS	0.1066
CV	429.8851	Std Mean	0.0048

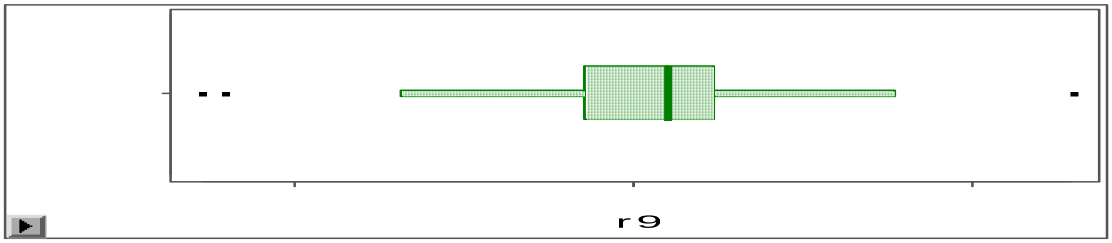
Quantiles

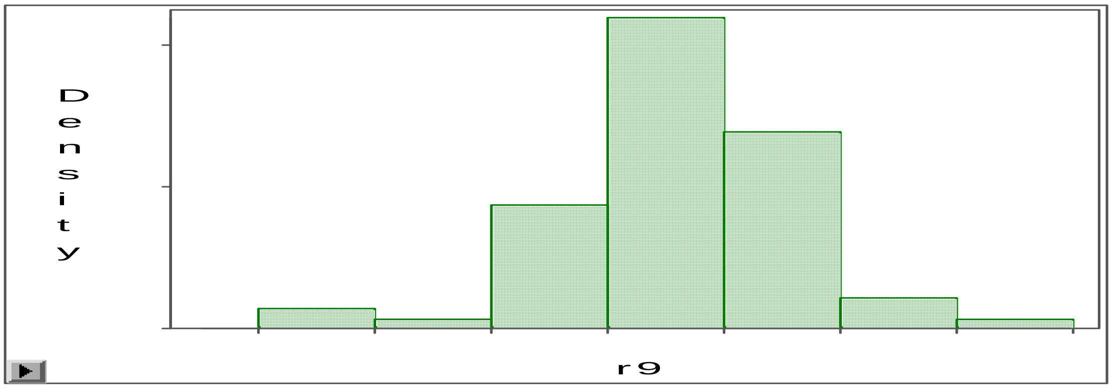
100% Max	0.1368	99.0%	0.1368
75% Q3	0.0299	97.5%	0.1126
50% Med	0.0048	95.0%	0.1018
25% Q1	-0.0175	90.0%	0.0599
0% Min	-0.0637	10.0%	-0.0371
Range	0.2005	5.0%	-0.0482
Q3-Q1	0.0474	2.5%	-0.0571
Mode	.	1.0%	-0.0637

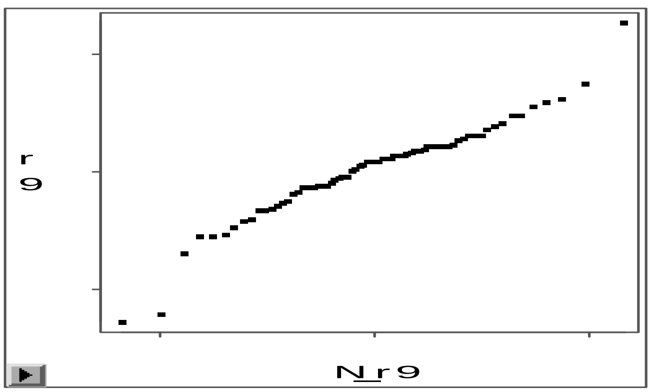
Tests for Normality

Test Statistic	Value	p-value
Shapiro-Wilk	0.943650	0.0040
Kolmogorov-Smirnov	0.093203	0.1474
Cramer-von Mises	0.140357	0.0325
Anderson-Darling	0.979630	0.0141

r9







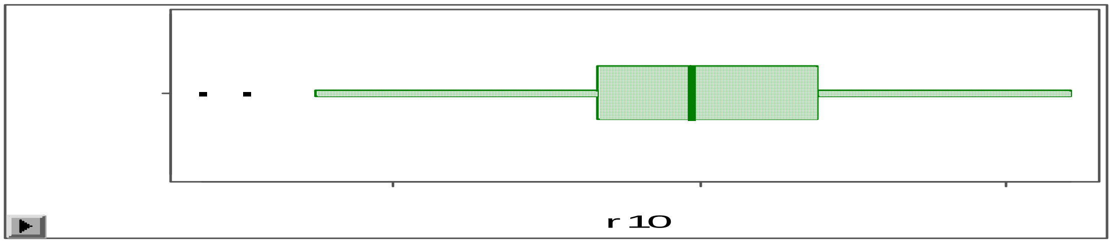
Moments			
N	68.0000	Sum Wgts	68.0000
Mean	0.0038	Sum	0.2574
Std Dev	0.0404	Variance	0.0016
Skewness	-0.5024	Kurtosis	2.6285
USS	0.1104	CSS	0.1094
CV	1067.2390	Std Mean	0.0049

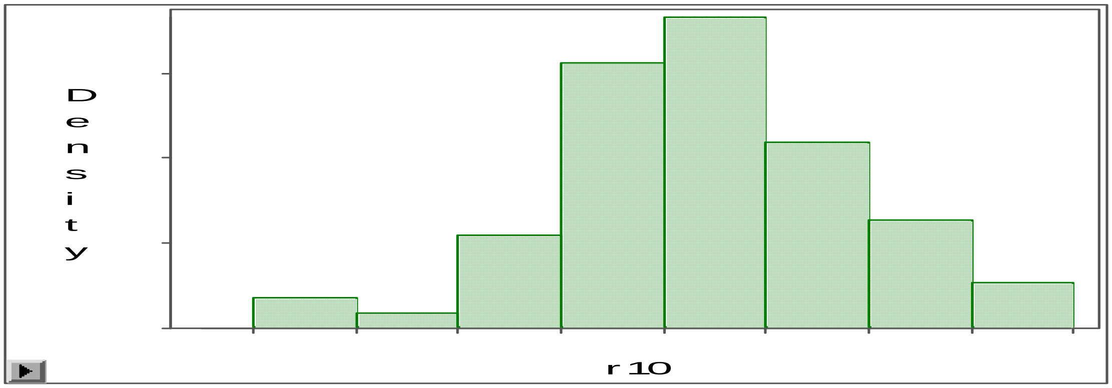
Quantiles			
100% Max	0.1297	99.0%	0.1297
75% Q3	0.0239	97.5%	0.0772
50% Med	0.0102	95.0%	0.0616
25% Q1	-0.0143	90.0%	0.0489
0% Min	-0.1272	10.0%	-0.0452
Range	0.2569	5.0%	-0.0535
Q3-Q1	0.0382	2.5%	-0.1205
Mode	.	1.0%	-0.1272

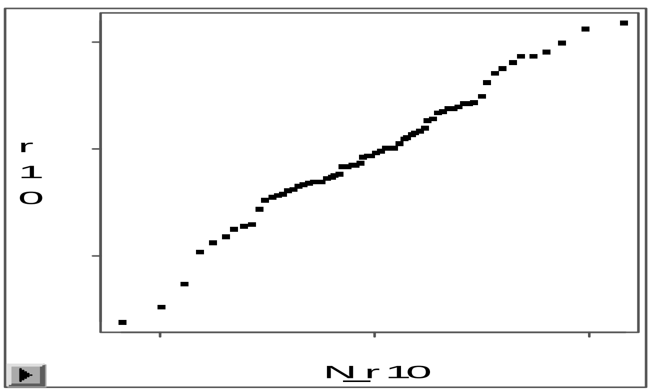
Tests for Normality

Test Statistic	Value	p-value
Shapiro-Wilk	0.947254	0.0061
Kolmogorov-Smirnov	0.094858	0.1318
Cramer-von Mises	0.161865	0.0175
Anderson-Darling	1.007328	0.0115

r10







Moments			
N	68.0000	Sum Wgts	68.0000
Mean	-0.0015	Sum	-0.1036
Std Dev	0.0585	Variance	0.0034
Skewness	-0.2394	Kurtosis	0.4216
USS	0.2294	CSS	0.2293
CV	-3839.7879	Std Mean	0.0071

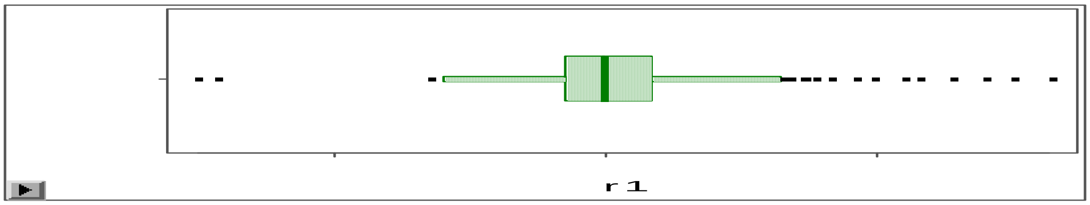
Quantiles			
100% Max	0.1218	99.0%	0.1218
75% Q3	0.0387	97.5%	0.1158
50% Med	-0.0026	95.0%	0.0932
25% Q1	-0.0329	90.0%	0.0831
0% Min	-0.1621	10.0%	-0.0728
Range	0.2839	5.0%	-0.0946
Q3-Q1	0.0715	2.5%	-0.1476
Mode	0	1.0%	-0.1621

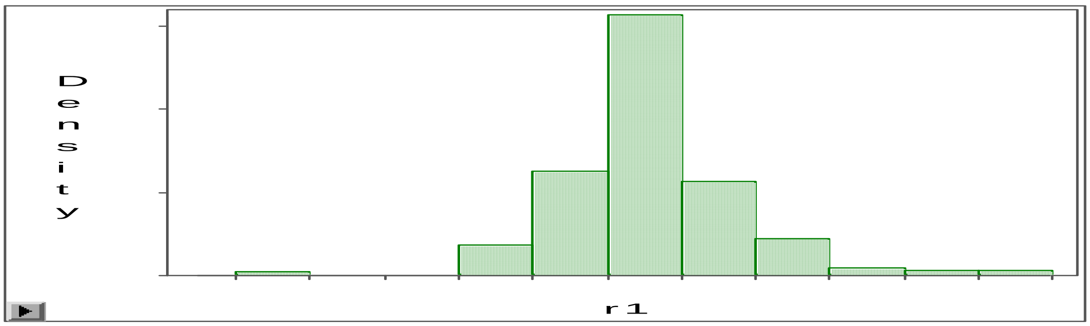
Tests for Normality

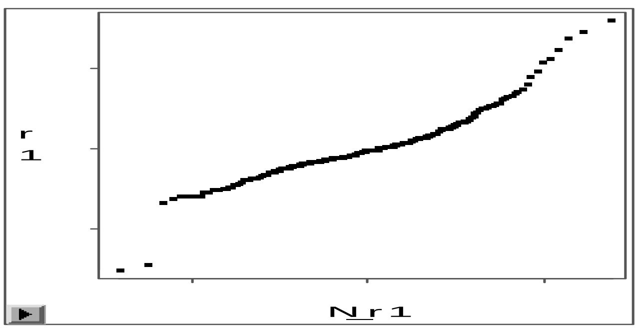
Test Statistic	Value	p-value
Shapiro-Wilk	0.982241	0.4434
Kolmogorov-Smirnov	0.078841	>.1500
Cramer-von Mises	0.053257	>.2500
Anderson-Darling	0.359895	>.2500

Phase II: The normality test for the returns of the data from November Feb13th, 2002 to Mar 3rd, 2003.

r1







Moments

N	264.0000	Sum Wgts	264.0000
Mean	0.0030	Sum	0.7863
Std Dev	0.0375	Variance	0.0014
Skewness	0.7336	Kurtosis	4.2281
USS	0.3714	CSS	0.3690
CV	1257.6326	Std Mean	0.0023

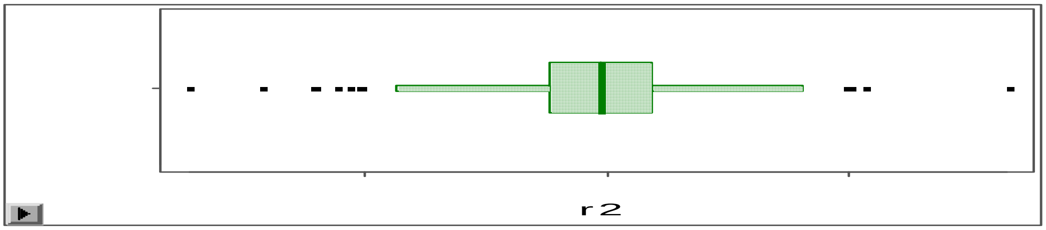
Quantiles

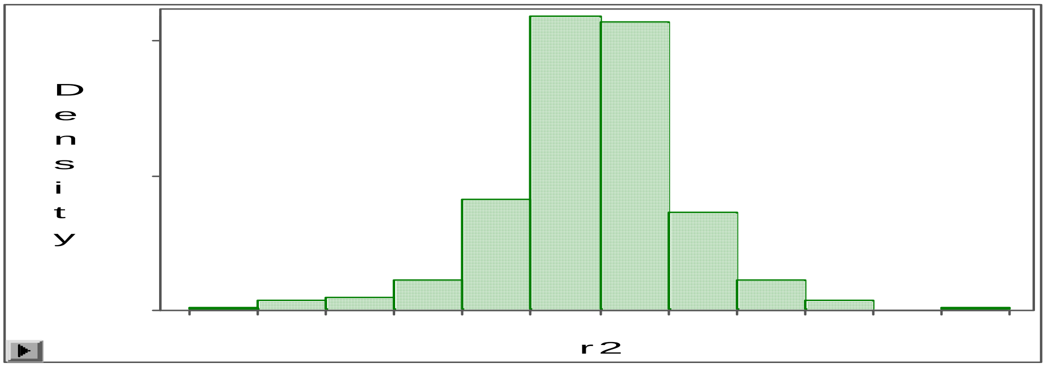
100% Max	0.1640	99.0%	0.1401
75% Q3	0.0166	97.5%	0.0991
50% Med	-0.0009	95.0%	0.0677
25% Q1	-0.0153	90.0%	0.0481
0% Min	-0.1504	10.0%	-0.0345
Range	0.3144	5.0%	-0.0476
Q3-Q1	0.0319	2.5%	-0.0575
Mode	0	1.0%	-0.0644

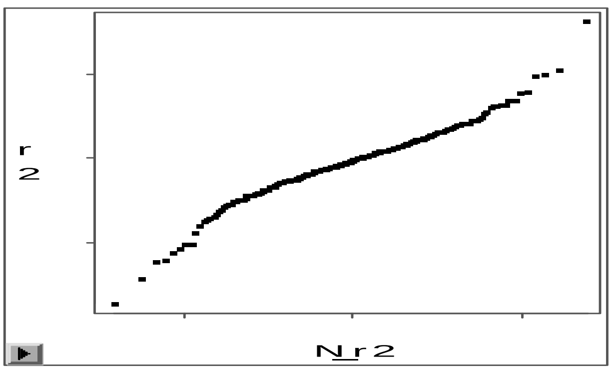
Tests for Normality

Test	Statistic	Value	p-value
Shapiro-Wilk		0.913339	0.0000
Kolmogorov-Smirnov		0.114482	<.0100
Cramer-von Mises		1.027240	<.0050
Anderson-Darling		5.830583	<.0050

r2







Moments

N	264.0000	Sum Wgts	264.0000
Mean	-0.0034	Sum	-0.8891
Std Dev	0.0399	Variance	0.0016
Skewness	-0.3201	Kurtosis	2.9217
USS	0.4223	CSS	0.4193
CV	-1185.5829	Std Mean	0.0025

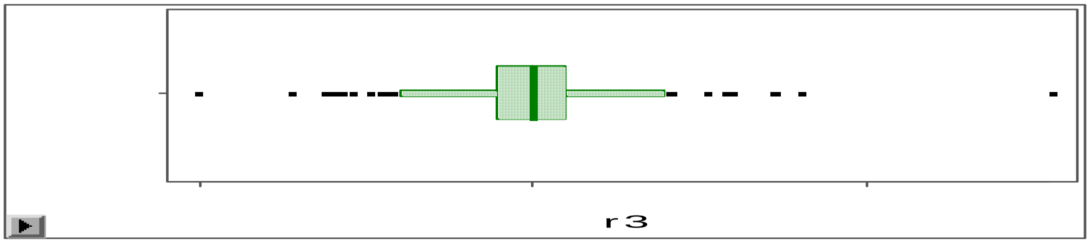
Quantiles

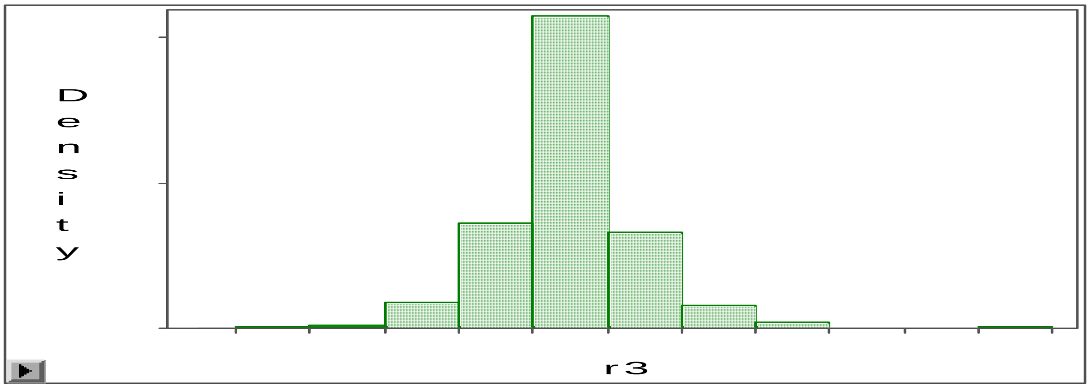
100% Max	0.1657	99.0%	0.1008
75% Q3	0.0185	97.5%	0.0710
50% Med	-0.0031	95.0%	0.0559
25% Q1	-0.0240	90.0%	0.0407
0% Min	-0.1725	10.0%	-0.0478
Range	0.3382	5.0%	-0.0685
Q3-Q1	0.0425	2.5%	-0.1020
Mode	0	1.0%	-0.1213

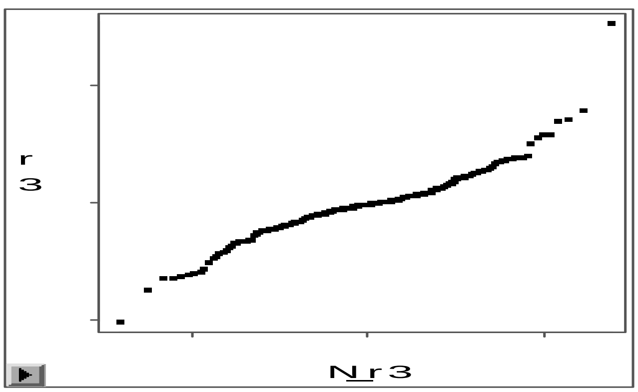
Tests for Normality

Test Statistic	Value	p-value
Shapiro-Wilk	0.959154	0.0000
Kolmogorov-Smirnov	0.070372	<.0100
Cramer-von Mises	0.370503	<.0050
Anderson-Darling	2.412135	<.0050

r3







Moments

N	264.0000	Sum Wgts	264.0000
Mean	0.0002	Sum	0.0406
Std Dev	0.0256	Variance	0.0007
Skewness	0.6579	Kurtosis	6.2874
USS	0.1722	CSS	0.1721
CV	16631.4519	Std Mean	0.0016

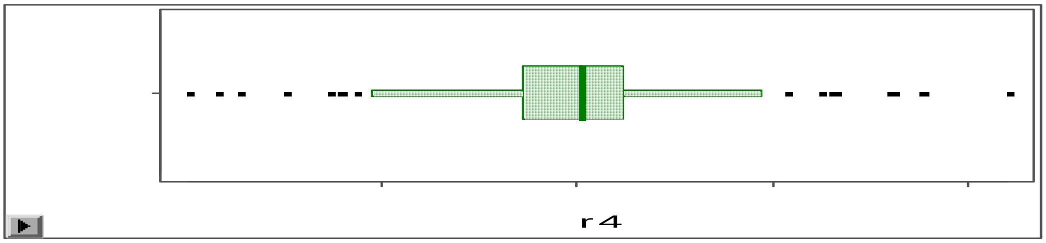
Quantiles

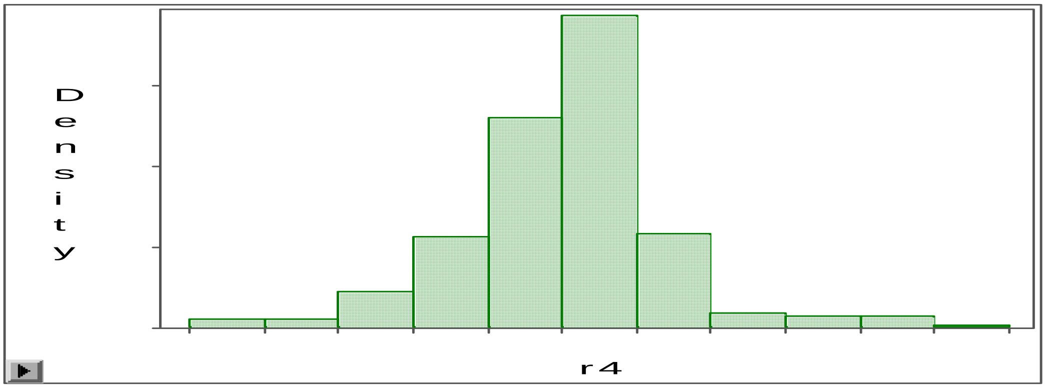
100% Max	0.1559	99.0%	0.0727
75% Q3	0.0099	97.5%	0.0575
50% Med	0	95.0%	0.0393
25% Q1	-0.0107	90.0%	0.0271
0% Min	-0.1009	10.0%	-0.0264
Range	0.2567	5.0%	-0.0403
Q3-Q1	0.0206	2.5%	-0.0586
Mode	0	1.0%	-0.0628

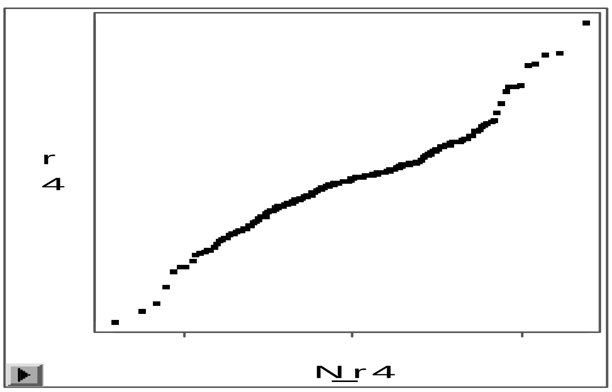
Tests for Normality

Test Statistic	Value	p-value
Shapiro-Wilk	0.920442	0.0000
Kolmogorov-Smirnov	0.112552	<.0100
Cramer-von Mises	0.940599	<.0050
Anderson-Darling	4.991225	<.0050

r4







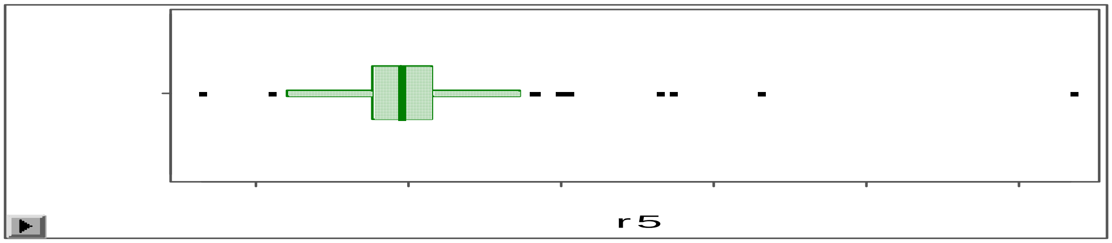
Moments			
N	264.0000	Sum Wgts	264.0000
Mean	-0.0005	Sum	-0.1305
Std Dev	0.0276	Variance	0.0008
Skewness	0.1609	Kurtosis	2.6753
USS	0.2007	CSS	0.2006
CV	-5589.1318	Std Mean	0.0017

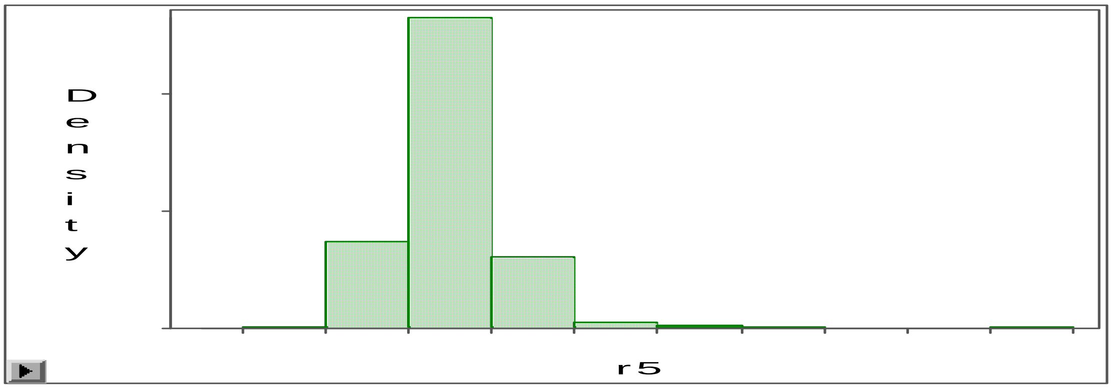
Quantiles			
100% Max	0.1108	99.0%	0.0883
75% Q3	0.0116	97.5%	0.0658
50% Med	0.0011	95.0%	0.0400
25% Q1	-0.0139	90.0%	0.0273
0% Min	-0.0989	10.0%	-0.0335
Range	0.2097	5.0%	-0.0463
Q3-Q1	0.0255	2.5%	-0.0598
Mode	0	1.0%	-0.0860

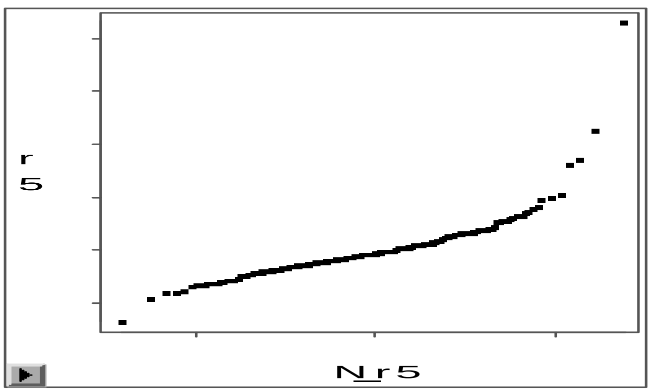
Tests for Normality

Test Statistic	Value	p-value
Shapiro-Wilk	0.949599	0.0000
Kolmogorov-Smirnov	0.103886	<.0100
Cramer-von Mises	0.731882	<.0050
Anderson-Darling	4.076214	<.0050

r5







Moments

N	264.0000	Sum Wgts	264.0000
Mean	0.0015	Sum	0.4012
Std Dev	0.0964	Variance	0.0093
Skewness	3.5226	Kurtosis	26.9238
USS	2.4446	CSS	2.4440
CV	6343.1705	Std Mean	0.0059

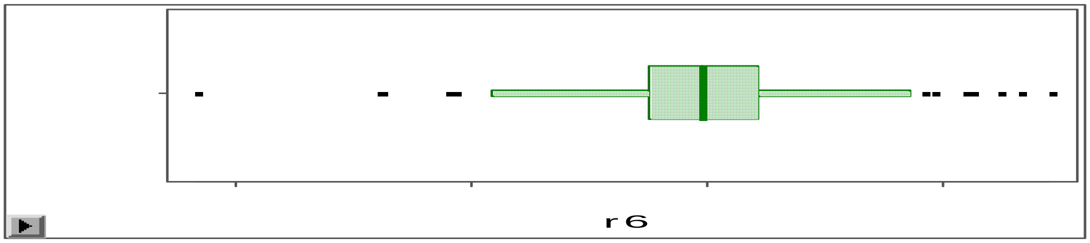
Quantiles

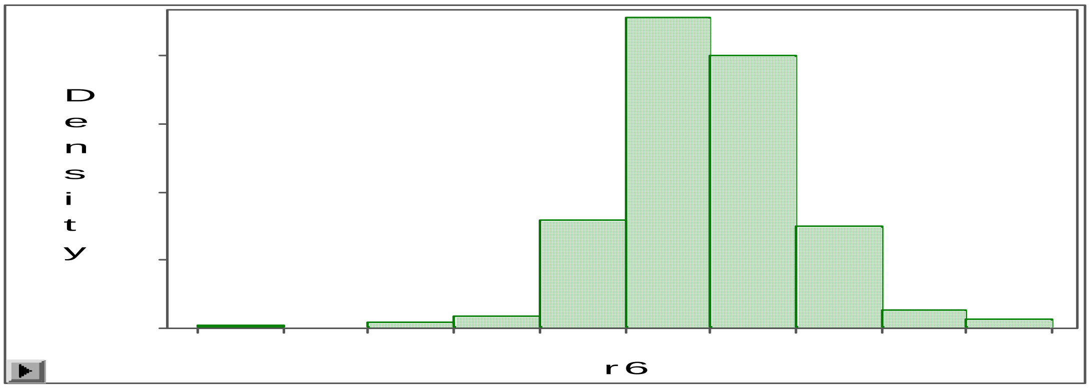
100% Max	0.8710	99.0%	0.3462
75% Q3	0.0319	97.5%	0.2000
50% Med	-0.0116	95.0%	0.1300
25% Q1	-0.0476	90.0%	0.0820
0% Min	-0.2710	10.0%	-0.0806
Range	1.1420	5.0%	-0.1136
Q3-Q1	0.0796	2.5%	-0.1280
Mode	0	1.0%	-0.1586

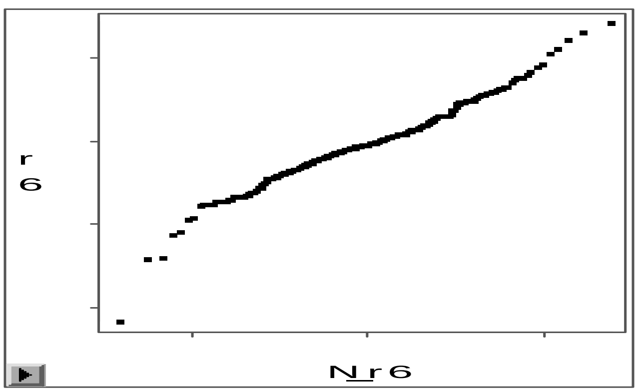
Tests for Normality

Test	Statistic	Value	p-value
Shapiro-Wilk		0.769678	0.0000
Kolmogorov-Smirnov		0.132268	<.0100
Cramer-von Mises		1.526938	<.0050
Anderson-Darling		9.056801	<.0050

r6







Moments			
N	264.0000	Sum Wgts	264.0000
Mean	-0.0015	Sum	-0.4027
Std Dev	0.0452	Variance	0.0020
Skewness	-0.2252	Kurtosis	2.3983
USS	0.5376	CSS	0.5370
CV	-2962.1083	Std Mean	0.0028

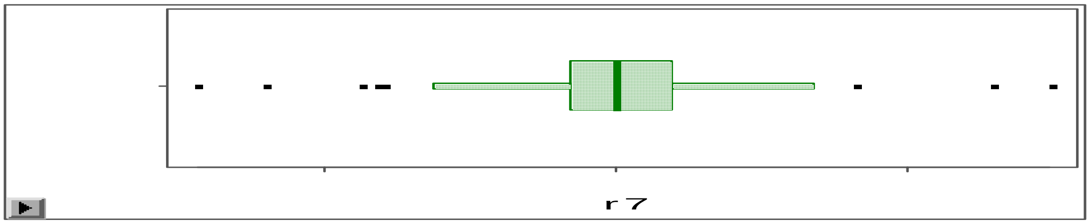
Quantiles			
100% Max	0.1456	99.0%	0.1242
75% Q3	0.0213	97.5%	0.0916
50% Med	-0.0022	95.0%	0.0673
25% Q1	-0.0254	90.0%	0.0542
0% Min	-0.2162	10.0%	-0.0585
Range	0.3618	5.0%	-0.0695
Q3-Q1	0.0467	2.5%	-0.0897
Mode	0	1.0%	-0.1382

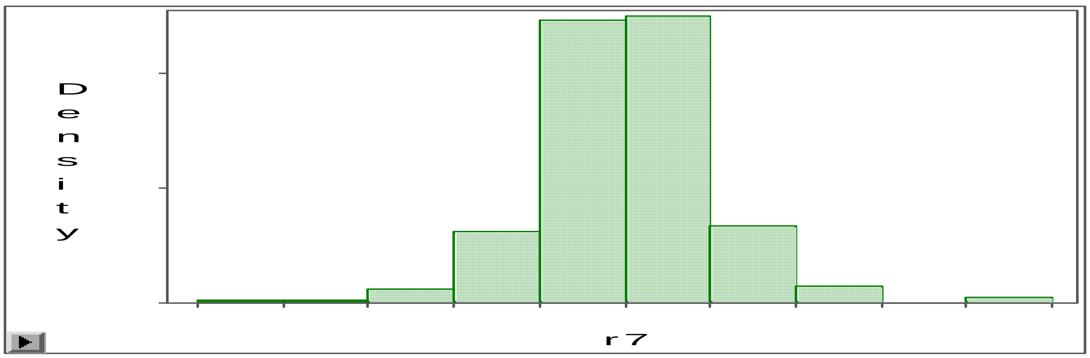
Tests for Normality

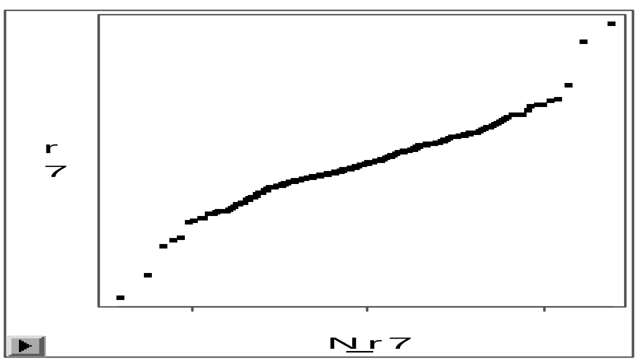
Test Statistic	Value	p-value
Shapiro-Wilk	0.971241	0.0000
Kolmogorov-Smirnov	0.072551	< .0100
Cramer-von Mises	0.325473	< .0050
Anderson-Darling	1.752683	< .0050

Tests for Normality		
Test Statistic	Value	p-value
Shapiro-Wilk	0.971241	0.0000
Kolmogorov-Smirnov	0.072551	<.0100
Cramer-von Mises	0.325473	<.0050
Anderson-Darling	1.752683	<.0050

r7





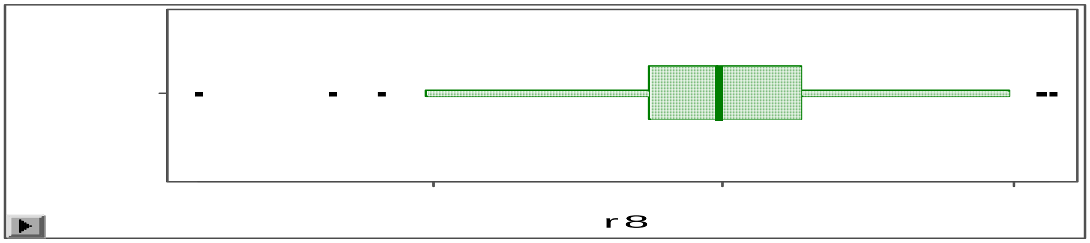


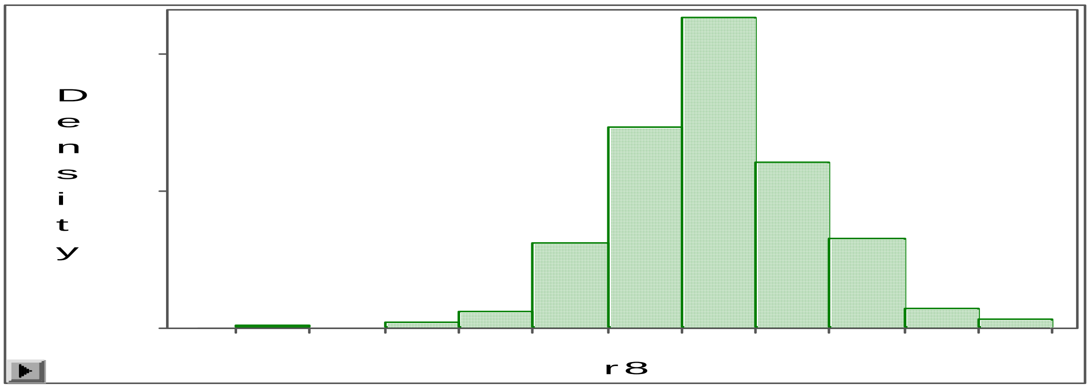
Moments			
N	264.0000	Sum Wgts	264.0000
Mean	0.0003	Sum	0.0753
Std Dev	0.0161	Variance	0.0003
Skewness	0.0002	Kurtosis	4.0222
USS	0.0681	CSS	0.0680
CV	5638.1436	Std Mean	0.0010

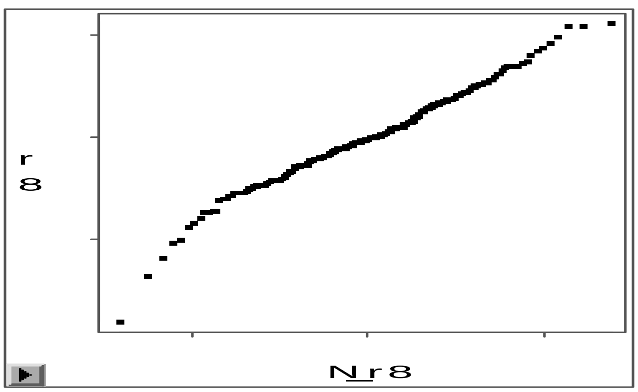
Quantiles			
100% Max	0.0749	99.0%	0.0414
75% Q3	0.0096	97.5%	0.0312
50% Med	0	95.0%	0.0248
25% Q1	-0.0077	90.0%	0.0167
0% Min	-0.0718	10.0%	-0.0178
Range	0.1467	5.0%	-0.0251
Q3-Q1	0.0172	2.5%	-0.0307
Mode	0	1.0%	-0.0437

Tests for Normality		
Test Statistic	Value	p-value
Shapiro-Wilk	0.950404	0.0000
Kolmogorov-Smirnov	0.074254	<.0100
Cramer-von Mises	0.334442	<.0050
Anderson-Darling	2.198921	<.0050

r8







Moments

N	264.0000	Sum Wgts	264.0000
Mean	-0.0003	Sum	-0.0794
Std Dev	0.0418	Variance	0.0017
Skewness	-0.2223	Kurtosis	1.4223
USS	0.4591	CSS	0.4590
CV	-13897.575	Std Mean	0.0026

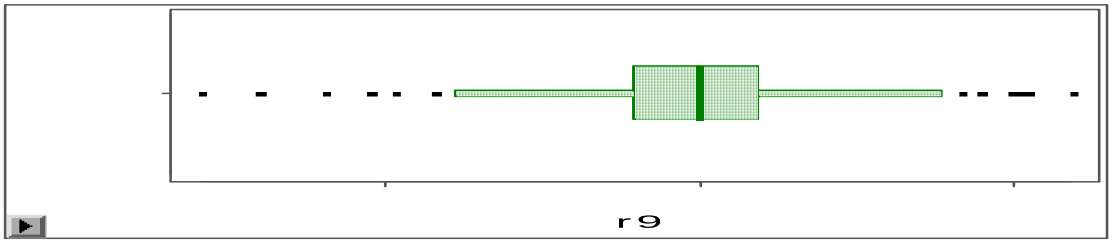
Quantiles

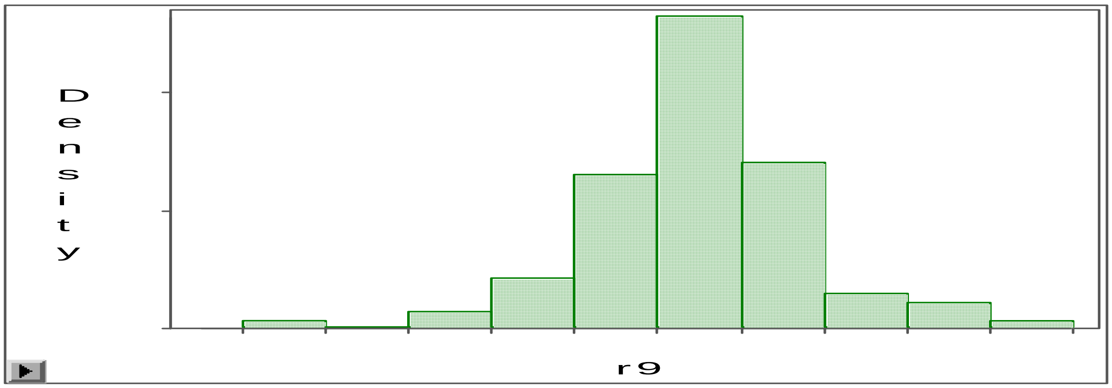
100% Max	0.1137	99.0%	0.1097
75% Q3	0.0275	97.5%	0.0858
50% Med	-0.0015	95.0%	0.0704
25% Q1	-0.0256	90.0%	0.0518
0% Min	-0.1811	10.0%	-0.0474
Range	0.2948	5.0%	-0.0590
Q3-Q1	0.0531	2.5%	-0.0832
Mode	0	1.0%	-0.1181

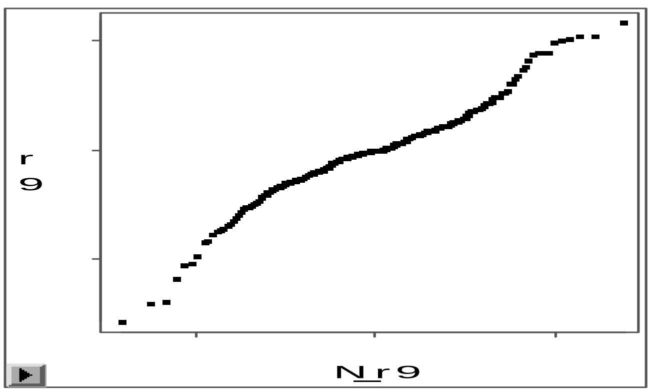
Tests for Normality

Test Statistic	Value	p-value
Shapiro-Wilk	0.983378	0.0036
Kolmogorov-Smirnov	0.054229	0.0576
Cramer-von Mises	0.143009	0.0304
Anderson-Darling	0.844311	0.0304

r9







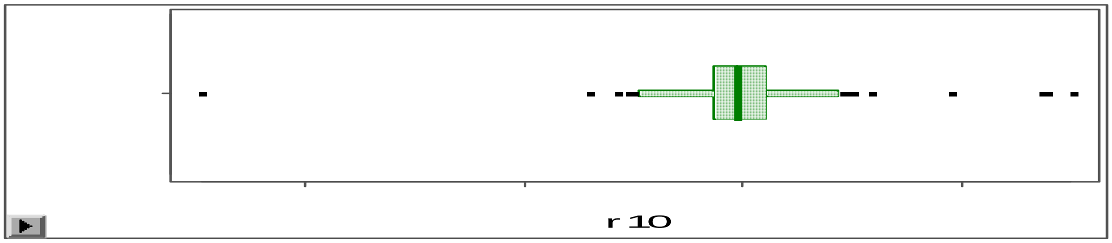
Moments			
N	264.0000	Sum Wgts	264.0000
Mean	-0.0007	Sum	-0.1833
Std Dev	0.0405	Variance	0.0016
Skewness	-0.2881	Kurtosis	2.1616
USS	0.4322	CSS	0.4321
CV	-5838.1076	Std Mean	0.0025

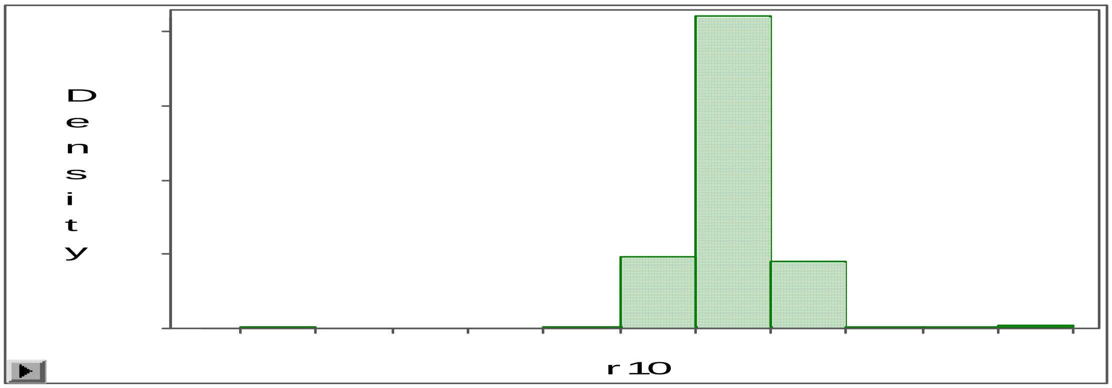
Quantiles			
100% Max	0.1190	99.0%	0.1050
75% Q3	0.0191	97.5%	0.0905
50% Med	0	95.0%	0.0690
25% Q1	-0.0207	90.0%	0.0436
0% Min	-0.1579	10.0%	-0.0464
Range	0.2769	5.0%	-0.0694
Q3-Q1	0.0397	2.5%	-0.0962
Mode	0	1.0%	-0.1394

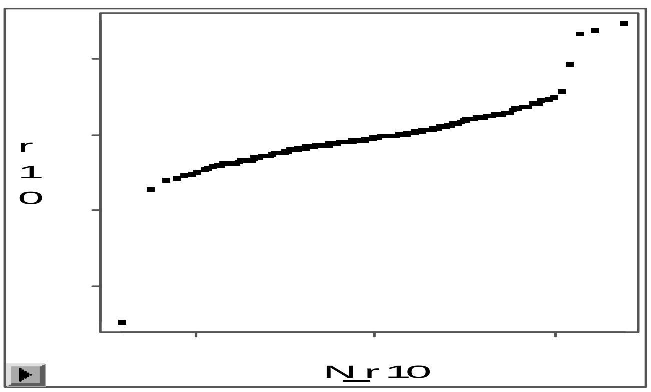
Tests for Normality

Test Statistic	Value	p-value
Shapiro-Wilk	0.955806	0.0000
Kolmogorov-Smirnov	0.082538	<.0100
Cramer-von Mises	0.632481	<.0050
Anderson-Darling	3.644373	<.0050

r10







Moments			
N	264.0000	Sum Wgts	264.0000
Mean	-0.0014	Sum	-0.3577
Std Dev	0.0604	Variance	0.0037
Skewness	-0.7546	Kurtosis	21.2881
USS	0.9609	CSS	0.9604
CV	-4459.5150	Std Mean	0.0037

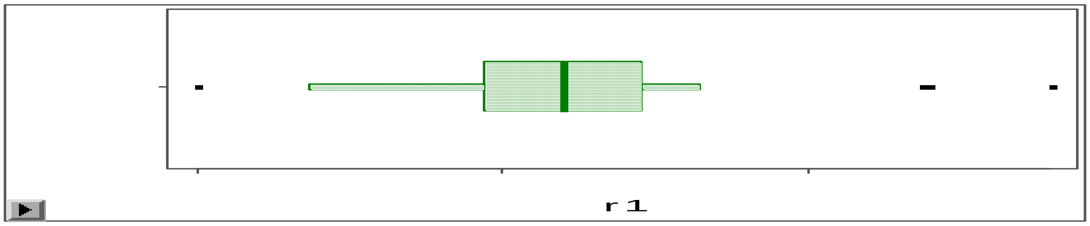
Quantiles			
100% Max	0.3032	99.0%	0.2739
75% Q3	0.0217	97.5%	0.0974
50% Med	-0.0046	95.0%	0.0750
25% Q1	-0.0253	90.0%	0.0539
0% Min	-0.4937	10.0%	-0.0559
Range	0.7969	5.0%	-0.0713
Q3-Q1	0.0470	2.5%	-0.0932
Mode	0	1.0%	-0.1137

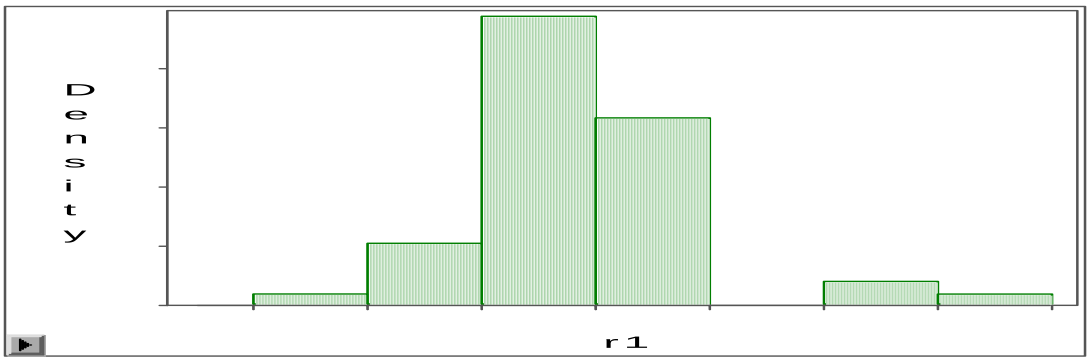
Tests for Normality

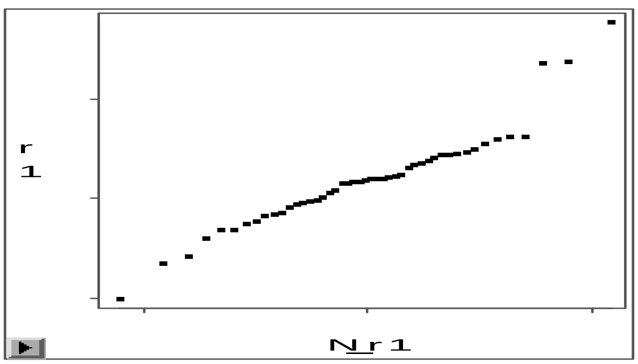
Test	Statistic	Value	p-value
Shapiro-Wilk		0.780821	0.0000
Kolmogorov-Smirnov		0.112125	<.0100
Cramer-von Mises		1.488802	<.0050
Anderson-Darling		9.087392	<.0050

Phase III: The normality test for the returns of the data from November Mar 4th, 2003 to Mar 8th, 2002.

r1







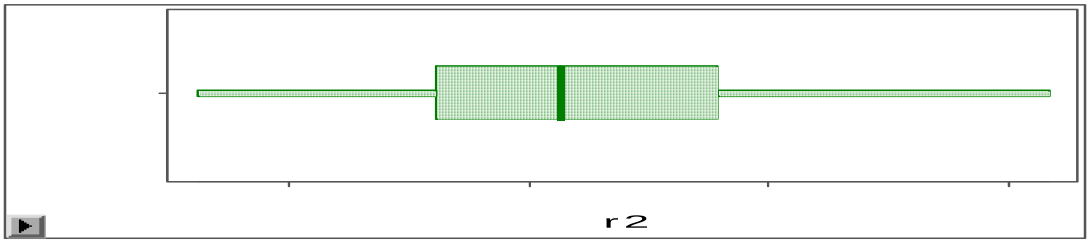
Moments			
N	47.0000	Sum Wgts	47.0000
Mean	0.0201	Sum	0.9426
Std Dev	0.0493	Variance	0.0024
Skewness	0.7961	Kurtosis	2.5975
USS	0.1309	CSS	0.1120
CV	246.0573	Std Mean	0.0072

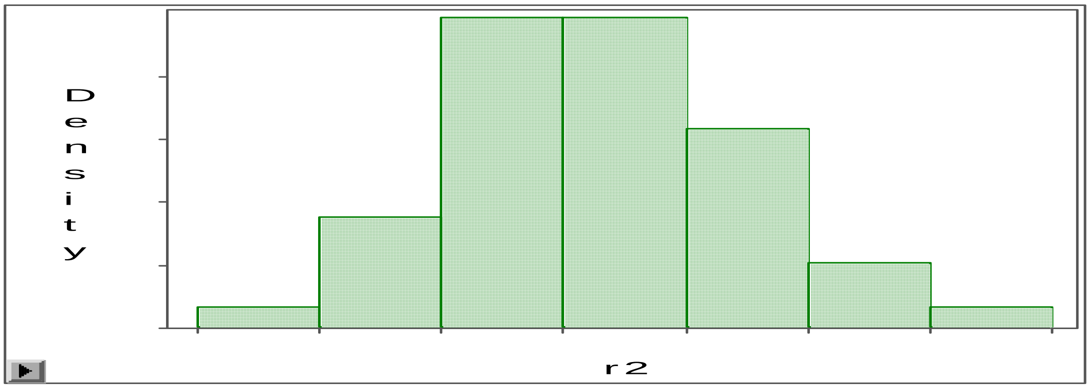
Quantiles			
100% Max	0.1797	99.0%	0.1797
75% Q3	0.0451	97.5%	0.1395
50% Med	0.0192	95.0%	0.1374
25% Q1	-0.0068	90.0%	0.0634
0% Min	-0.1004	10.0%	-0.0305
Range	0.2801	5.0%	-0.0569
Q3-Q1	0.0519	2.5%	-0.0635
Mode	.	1.0%	-0.1004

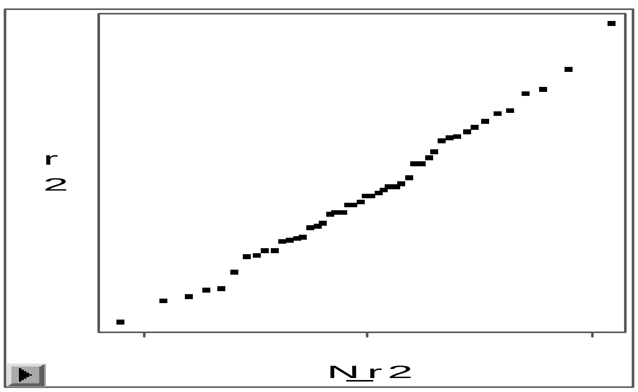
Tests for Normality

Test Statistic	Value	p-value
Shapiro-Wilk	0.928885	0.0069
Kolmogorov-Smirnov	0.123031	0.0747
Cramer-von Mises	0.149104	0.0237
Anderson-Darling	1.047707	0.0088

r2







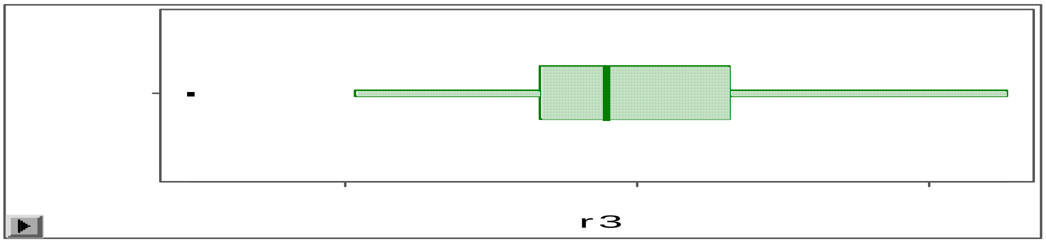
Moments			
N	47.0000	Sum Wgts	47.0000
Mean	0.0079	Sum	0.3693
Std Dev	0.0389	Variance	0.0015
Skewness	0.3045	Kurtosis	-0.1174
USS	0.0724	CSS	0.0695
CV	494.6081	Std Mean	0.0057

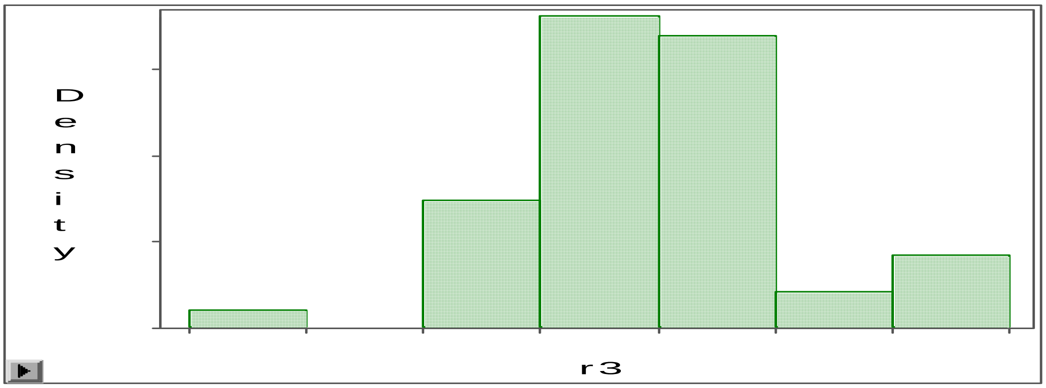
Quantiles			
100% Max	0.1092	99.0%	0.1092
75% Q3	0.0396	97.5%	0.0817
50% Med	-0.0068	95.0%	0.0700
25% Q1	-0.0193	90.0%	0.0573
0% Min	-0.0689	10.0%	-0.0485
Range	0.1781	5.0%	-0.0526
Q3-Q1	0.0589	2.5%	-0.0557
Mode	.	1.0%	-0.0689

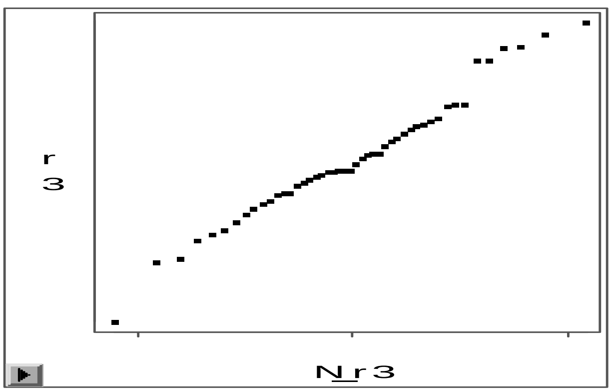
Tests for Normality

Test Statistic	Value	p-value
Shapiro-Wilk	0.988303	0.9151
Kolmogorov-Smirnov	0.077942	>.1500
Cramer-von Mises	0.029883	>.2500
Anderson-Darling	0.178227	>.2500

r3







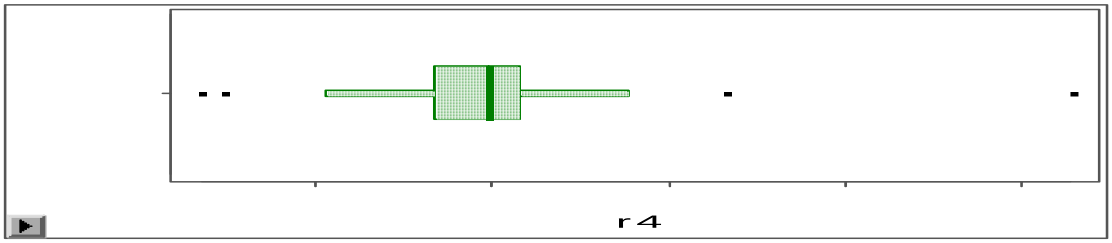
Moments			
N	47.0000	Sum Wgts	47.0000
Mean	-0.0003	Sum	-0.0134
Std Dev	0.0117	Variance	0.0001
Skewness	0.1265	Kurtosis	0.3575
USS	0.0063	CSS	0.0063
CV	-4081.0230	Std Mean	0.0017

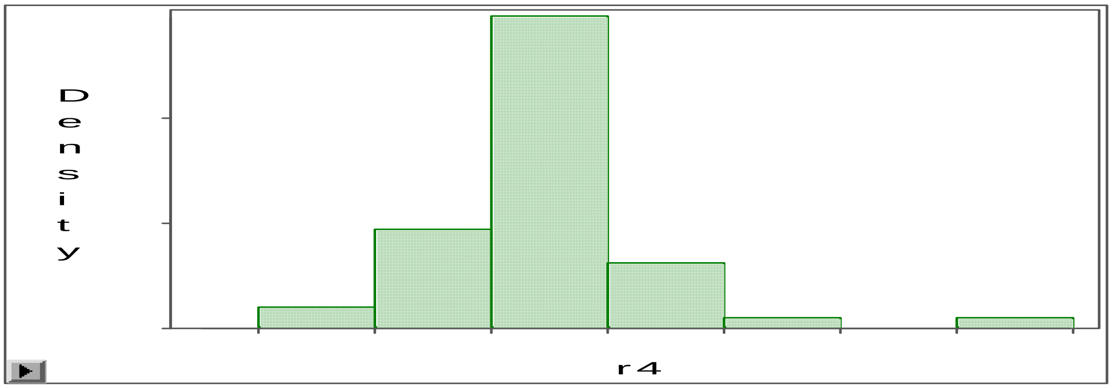
Quantiles			
100% Max	0.0254	99.0%	0.0254
75% Q3	0.0062	97.5%	0.0229
50% Med	-0.0023	95.0%	0.0207
25% Q1	-0.0068	90.0%	0.0181
0% Min	-0.0307	10.0%	-0.0143
Range	0.0561	5.0%	-0.0189
Q3-Q1	0.0130	2.5%	-0.0194
Mode	.	1.0%	-0.0307

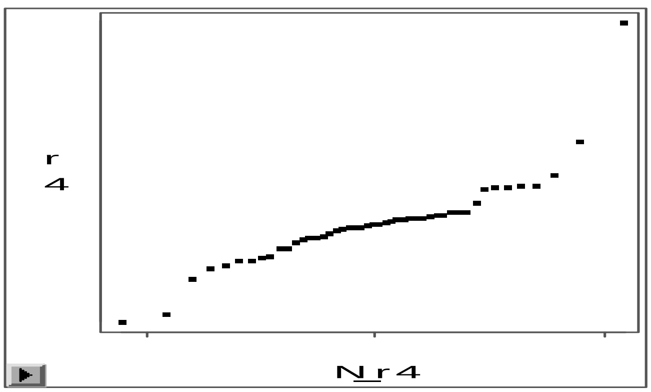
Tests for Normality

Test Statistic	Value	p-value
Shapiro-Wilk	0.979179	0.5588
Kolmogorov-Smirnov	0.077947	>.1500
Cramer-von Mises	0.057325	>.2500
Anderson-Darling	0.389317	>.2500

r4







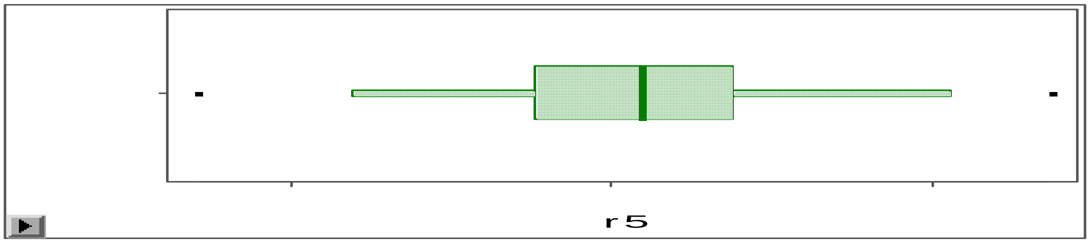
Moments			
N	47.0000	Sum Wgts	47.0000
Mean	-0.0011	Sum	-0.0506
Std Dev	0.0363	Variance	0.0013
Skewness	1.8062	Kurtosis	9.1172
USS	0.0607	CSS	0.0607
CV	-3372.3369	Std Mean	0.0053

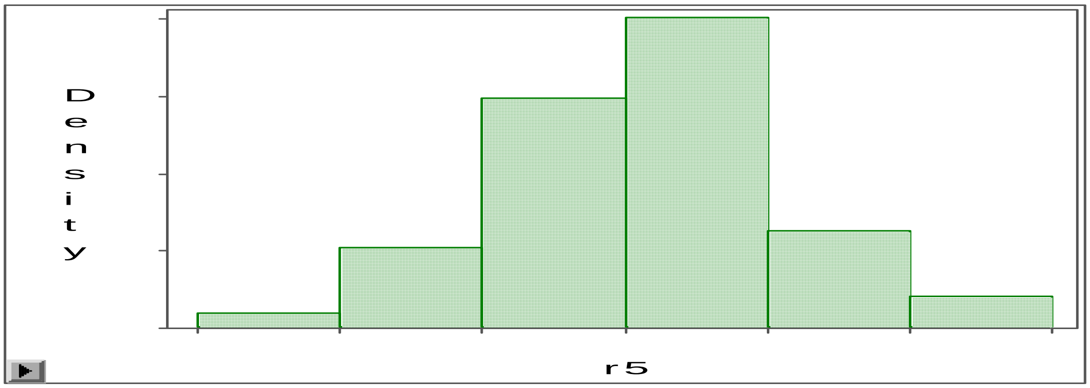
Quantiles			
100% Max	0.1646	99.0%	0.1646
75% Q3	0.0081	97.5%	0.0668
50% Med	-0.0008	95.0%	0.0387
25% Q1	-0.0163	90.0%	0.0294
0% Min	-0.0821	10.0%	-0.0353
Range	0.2467	5.0%	-0.0470
Q3-Q1	0.0244	2.5%	-0.0757
Mode	.	1.0%	-0.0821

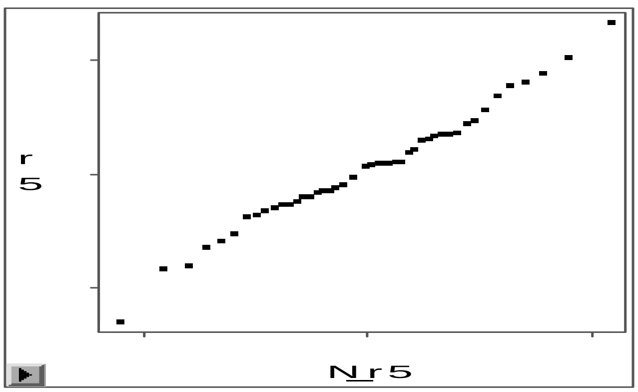
Tests for Normality

Test Statistic	Value	p-value
Shapiro-Wilk	0.827526	0.0000
Kolmogorov-Smirnov	0.204972	<.0100
Cramer-von Mises	0.353628	<.0050
Anderson-Darling	2.026491	<.0050

r5







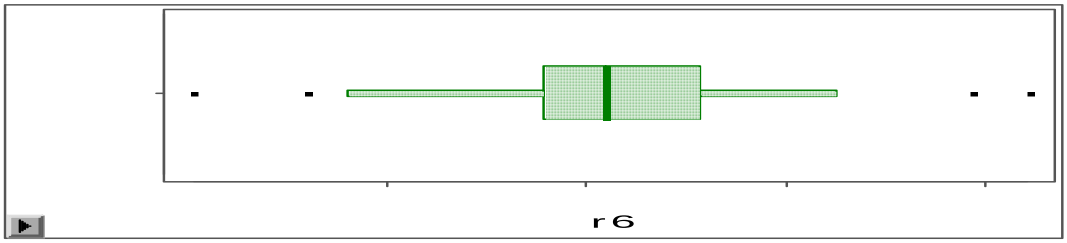
Moments			
N	47.0000	Sum Wgts	47.0000
Mean	0.0073	Sum	0.3446
Std Dev	0.0509	Variance	0.0026
Skewness	0.0607	Kurtosis	0.7065
USS	0.1216	CSS	0.1191
CV	694.0398	Std Mean	0.0074

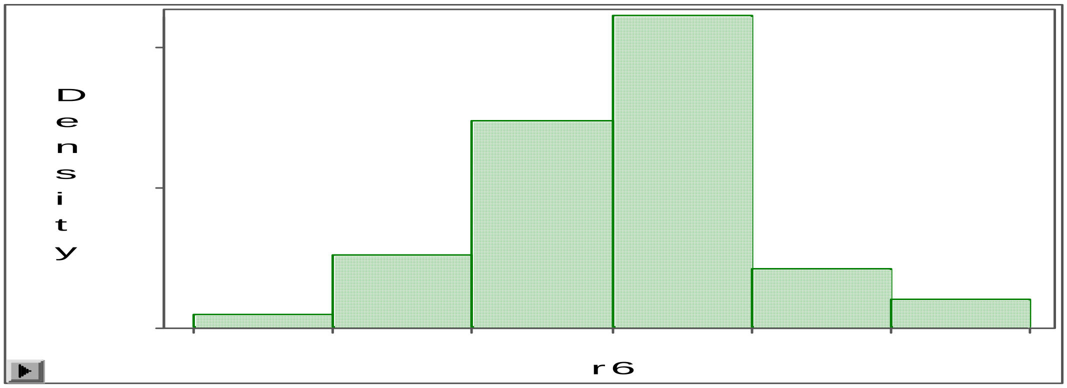
Quantiles			
100% Max	0.1376	99.0%	0.1376
75% Q3	0.0383	97.5%	0.1064
50% Med	0.0093	95.0%	0.0920
25% Q1	-0.0235	90.0%	0.0811
0% Min	-0.1290	10.0%	-0.0562
Range	0.2666	5.0%	-0.0789
Q3-Q1	0.0618	2.5%	-0.0806
Mode	0	1.0%	-0.1290

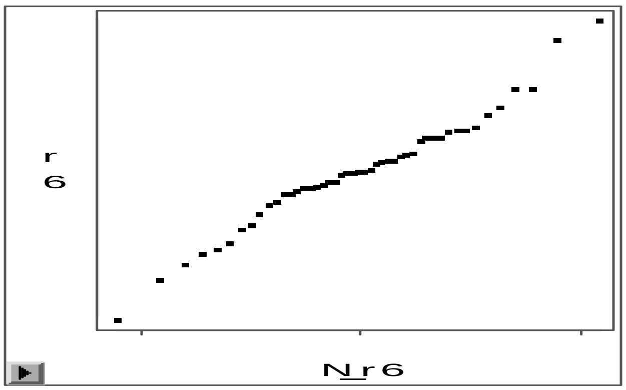
Tests for Normality

Test Statistic	Value	p-value
Shapiro-Wilk	0.987718	0.8979
Kolmogorov-Smirnov	0.087038	>.1500
Cramer-von Mises	0.047568	>.2500
Anderson-Darling	0.283878	>.2500

r6







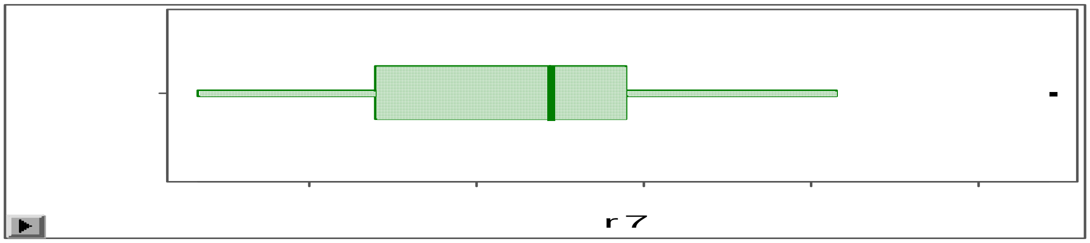
Moments			
N	47.0000	Sum Wgts	47.0000
Mean	0.0050	Sum	0.2346
Std Dev	0.0393	Variance	0.0015
Skewness	0.0566	Kurtosis	1.1848
USS	0.0723	CSS	0.0711
CV	787.9661	Std Mean	0.0057

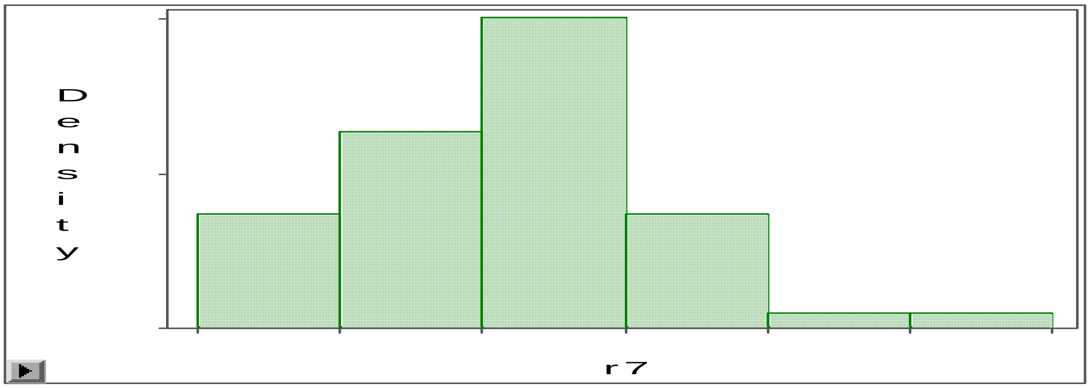
Quantiles			
100% Max	0.1108	99.0%	0.1108
75% Q3	0.0283	97.5%	0.0963
50% Med	0.0047	95.0%	0.0623
25% Q1	-0.0108	90.0%	0.0495
0% Min	-0.0987	10.0%	-0.0490
Range	0.2095	5.0%	-0.0601
Q3-Q1	0.0391	2.5%	-0.0703
Mode	.	1.0%	-0.0987

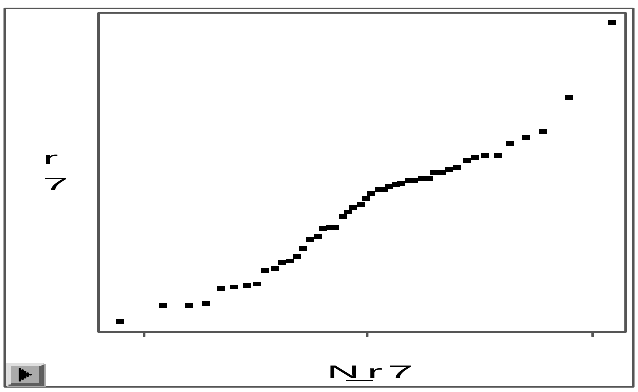
Tests for Normality

Test Statistic	Value	p-value
Shapiro-Wilk	0.974621	0.3925
Kolmogorov-Smirnov	0.110193	>.1500
Cramer-von Mises	0.091511	0.1443
Anderson-Darling	0.524577	0.1802

r7







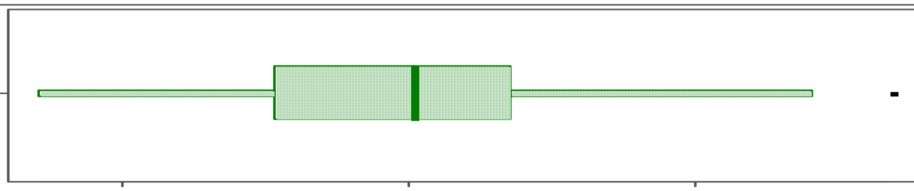
Moments			
N	47.0000	Sum Wgts	47.0000
Mean	0.0026	Sum	0.1218
Std Dev	0.0103	Variance	0.0001
Skewness	0.3251	Kurtosis	0.6333
USS	0.0052	CSS	0.0049
CV	397.6993	Std Mean	0.0015

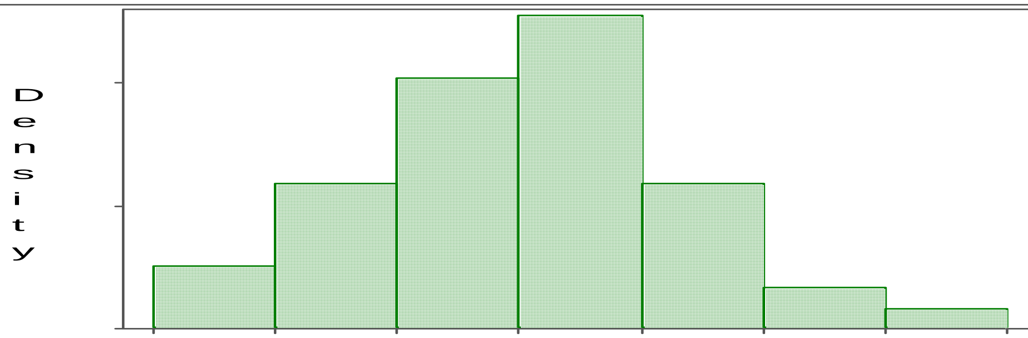
Quantiles			
100% Max	0.0343	99.0%	0.0343
75% Q3	0.0089	97.5%	0.0214
50% Med	0.0043	95.0%	0.0158
25% Q1	-0.0061	90.0%	0.0139
0% Min	-0.0167	10.0%	-0.0110
Range	0.0510	5.0%	-0.0136
Q3-Q1	0.0150	2.5%	-0.0137
Mode	.	1.0%	-0.0167

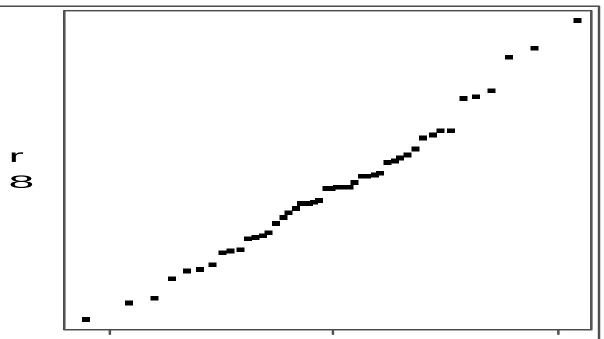
Tests for Normality

Test Statistic	Value	p-value
Shapiro-Wilk	0.964238	0.1583
Kolmogorov-Smirnov	0.094373	>.1500
Cramer-von Mises	0.074981	0.2395
Anderson-Darling	0.495097	0.2132

r8







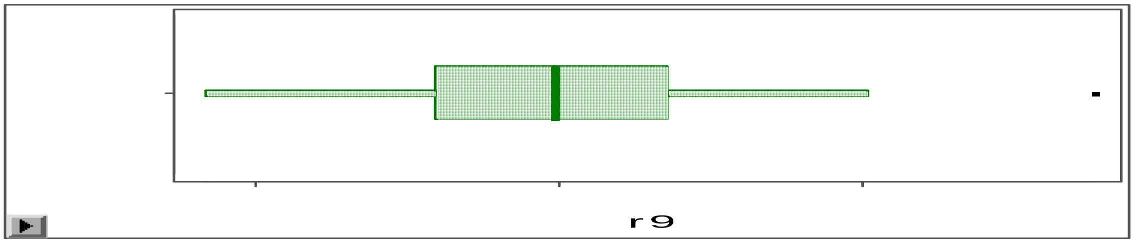
Moments			
N	47.0000	Sum Wgts	47.0000
Mean	0.0001	Sum	0.0065
Std Dev	0.0330	Variance	0.0011
Skewness	0.4113	Kurtosis	0.1581
USS	0.0501	CSS	0.0501
CV	23701.3160	Std Mean	0.0048

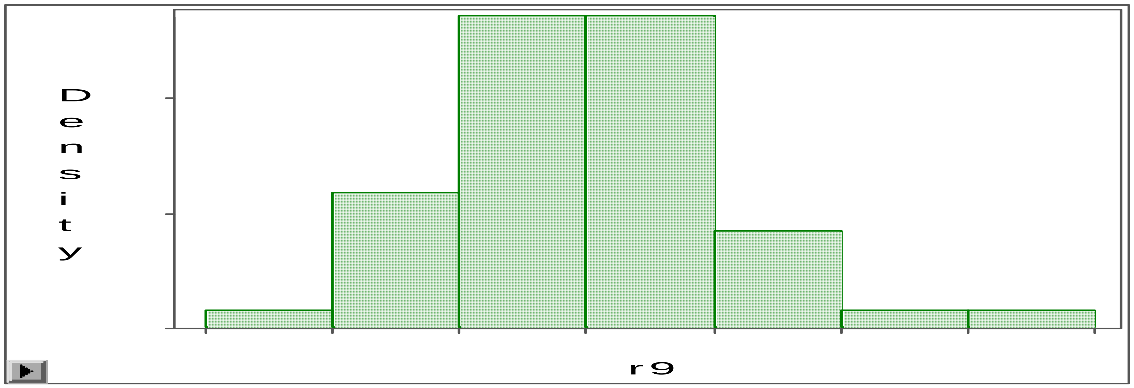
Quantiles			
100% Max	0.0845	99.0%	0.0845
75% Q3	0.0176	97.5%	0.0704
50% Med	0.0007	95.0%	0.0656
25% Q1	-0.0235	90.0%	0.0465
0% Min	-0.0645	10.0%	-0.0402
Range	0.1490	5.0%	-0.0537
Q3-Q1	0.0411	2.5%	-0.0556
Mode	.	1.0%	-0.0645

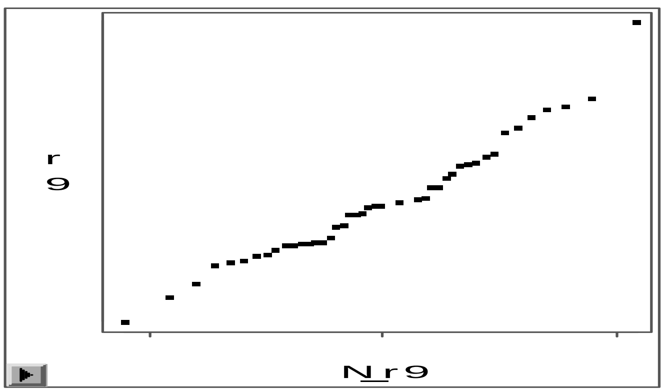
Tests for Normality

Test	Statistic	Value	p-value
Shapiro-Wilk		0.981665	0.6620
Kolmogorov-Smirnov		0.084278	>.1500
Cramer-von Mises		0.039325	>.2500
Anderson-Darling		0.264965	>.2500

r9







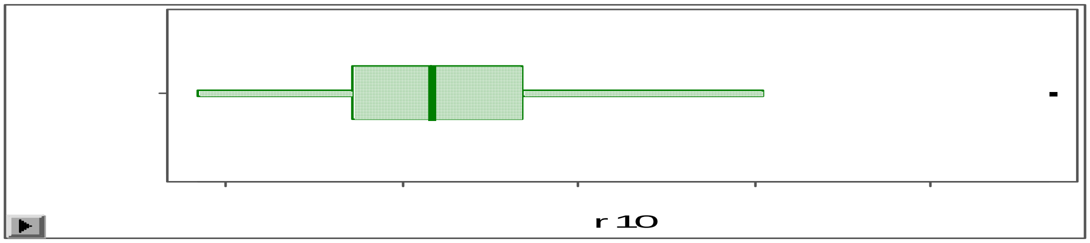
Moments			
N	47.0000	Sum Wgts	47.0000
Mean	-3.129E-06	Sum	-0.0001
Std Dev	0.0280	Variance	0.0008
Skewness	0.7097	Kurtosis	1.0845
USS	0.0361	CSS	0.0361
CV	-894858.91	Std Mean	0.0041

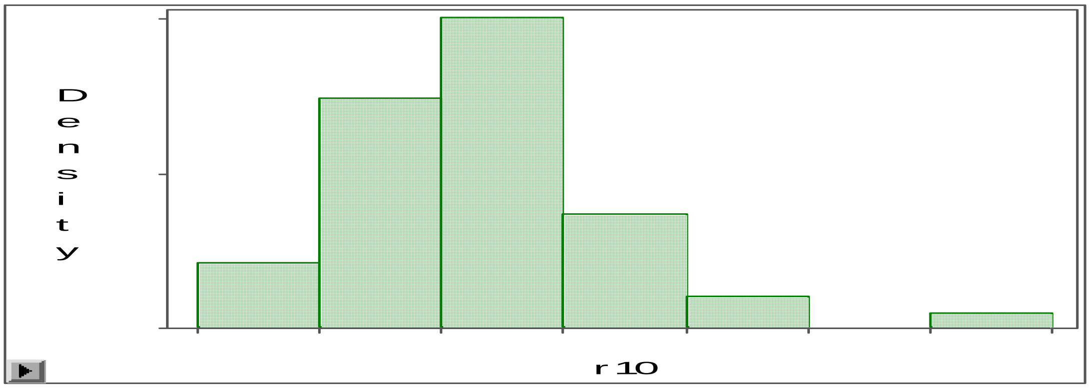
Quantiles			
100% Max	0.0882	99.0%	0.0882
75% Q3	0.0175	97.5%	0.0507
50% Med	-0.0012	95.0%	0.0469
25% Q1	-0.0205	90.0%	0.0417
0% Min	-0.0585	10.0%	-0.0291
Range	0.1466	5.0%	-0.0396
Q3-Q1	0.0380	2.5%	-0.0459
Mode	0	1.0%	-0.0585

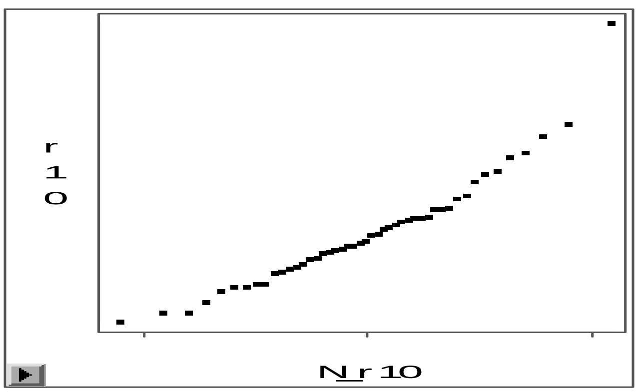
Tests for Normality

Test	Statistic	Value	p-value
Shapiro-Wilk		0.964025	0.1553
Kolmogorov-Smirnov		0.124990	0.0657
Cramer-von Mises		0.101355	0.1060
Anderson-Darling		0.579608	0.1297

r10







Moments			
N	47.0000	Sum Wgts	47.0000
Mean	0.0149	Sum	0.7008
Std Dev	0.0450	Variance	0.0020
Skewness	1.2650	Kurtosis	3.1747
USS	0.1035	CSS	0.0931
CV	301.7155	Std Mean	0.0066

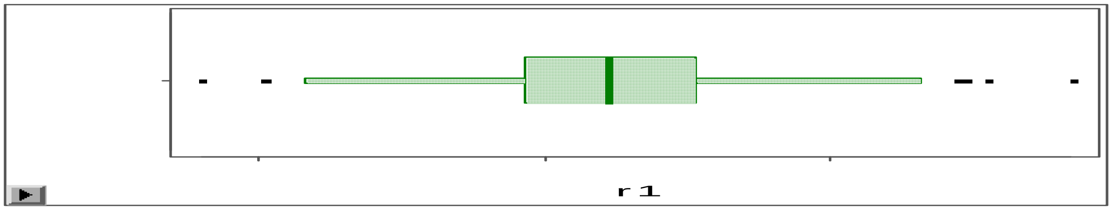
Quantiles			
100% Max	0.1840	99.0%	0.1840
75% Q3	0.0338	97.5%	0.1019
50% Med	0.0077	95.0%	0.0923
25% Q1	-0.0143	90.0%	0.0750
0% Min	-0.0583	10.0%	-0.0336
Range	0.2423	5.0%	-0.0502
Q3-Q1	0.0481	2.5%	-0.0502
Mode	.	1.0%	-0.0583

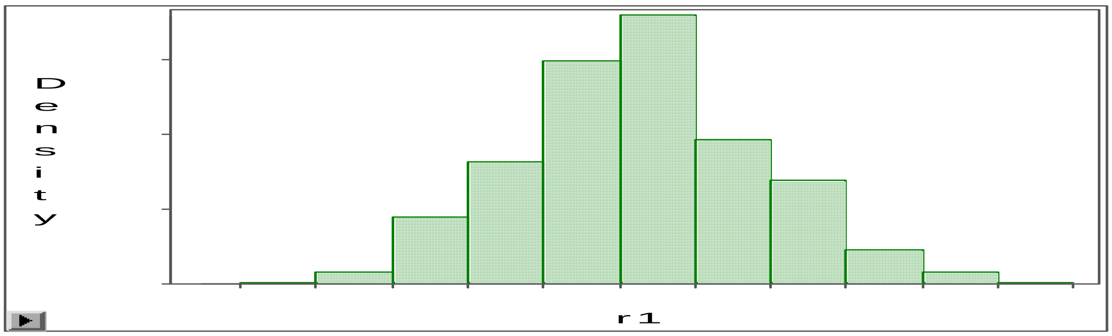
Tests for Normality

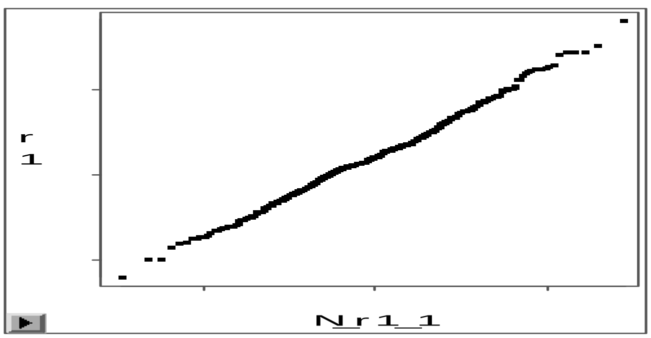
Test Statistic	Value	p-value
Shapiro-Wilk	0.926853	0.0058
Kolmogorov-Smirnov	0.115185	0.1184
Cramer-von Mises	0.104073	0.0973
Anderson-Darling	0.669338	0.0795

Distribution of the difference of the VaR between the Delta-Normal method and the Historical Simulation method.

r1







Moments

N	400.0000	Sum Wgts	400.0000
Mean	11.9768	Sum	4790.7353
Std Dev	24.2997	Variance	590.4743
Skewness	0.1576	Kurtosis	0.1834
USS	292977.121	CSS	235599.259
CV	202.8889	Std Mean	1.2150

Quantiles

100% Max	92.3762	99.0%	73.2942
75% Q3	26.3045	97.5%	63.3556
50% Med	11.1979	95.0%	52.2329
25% Q1	-3.4456	90.0%	44.3482
0% Min	-59.8667	10.0%	-19.6814
Range	152.2429	5.0%	-29.1408
Q3-Q1	29.7501	2.5%	-35.3006
Mode	.	1.0%	-40.8248

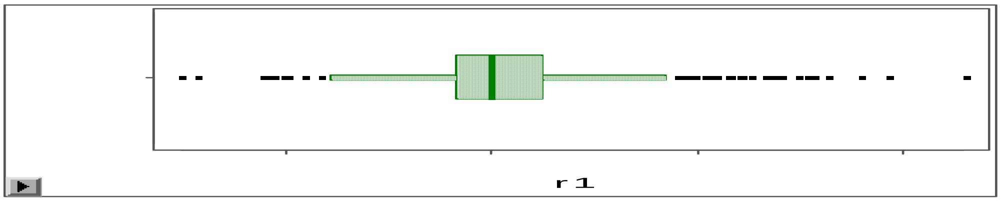
Tests for Normality

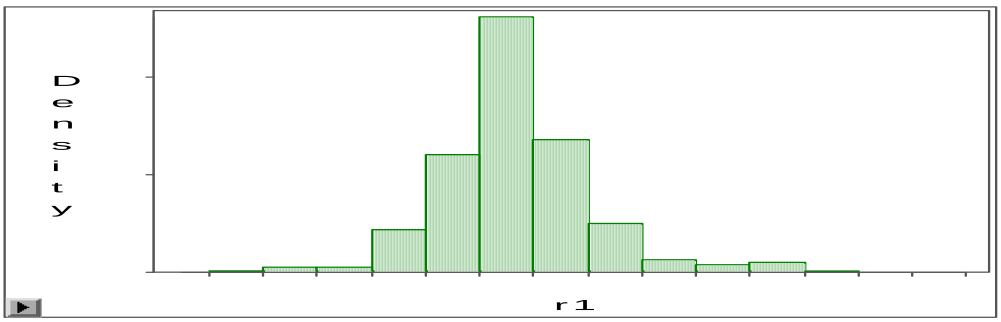
Test	Statistic	Value	p-value
Shapiro-Wilk		0.995499	0.3050
Kolmogorov-Smirnov		0.050205	0.0155
Cramer-von Mises		0.136841	0.0375
Anderson-Darling		0.699356	0.0712

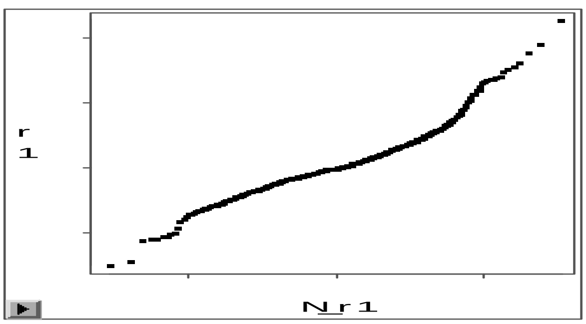
Appendix D: SAS Results II

Test for normality for the whole historical data of the portfolio of ten stocks from 01/02/2001 to 05/27/2003

r1







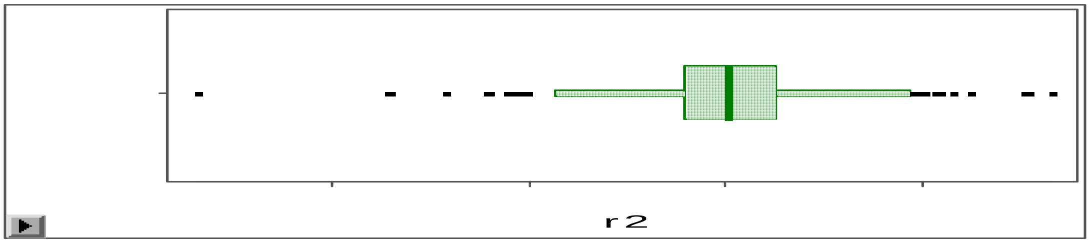
Moments			
N	599.0000	Sum Wgts	599.0000
Mean	0.0054	Sum	3.2399
Std Dev	0.0441	Variance	0.0019
Skewness	0.8878	Kurtosis	3.3834
USS	1.1810	CSS	1.1635
CV	815.5090	Std Mean	0.0018

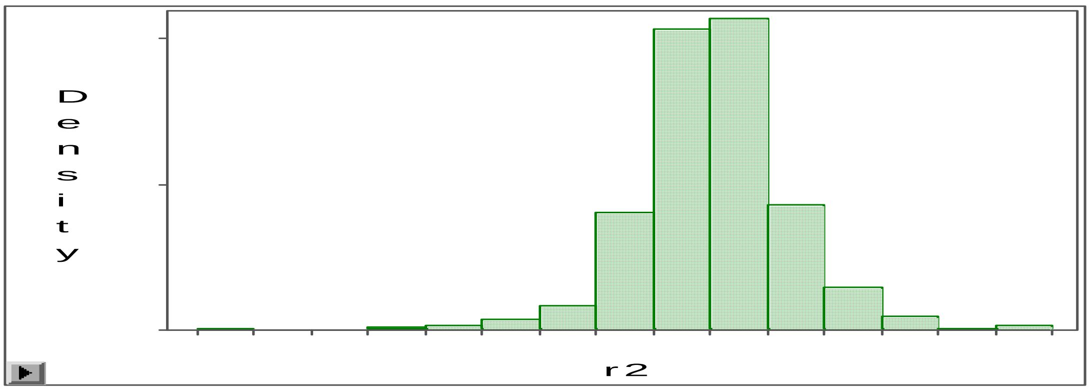
Quantiles			
100% Max	0.2308	99.0%	0.1538
75% Q3	0.0245	97.5%	0.1272
50% Med	0	95.0%	0.0806
25% Q1	-0.0169	90.0%	0.0545
0% Min	-0.1504	10.0%	-0.0411
Range	0.3812	5.0%	-0.0573
Q3-Q1	0.0414	2.5%	-0.0705
Mode	0	1.0%	-0.1058

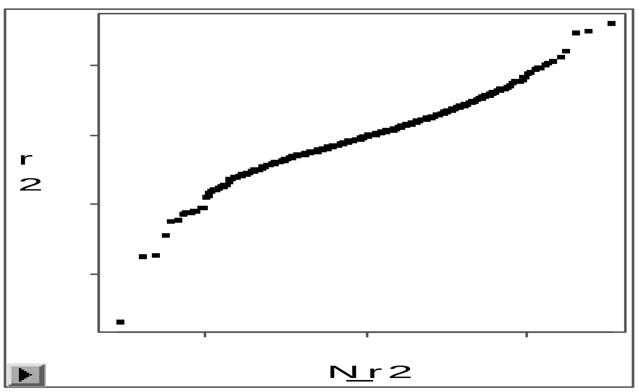
Tests for Normality

Test Statistic	Value	p-value
Shapiro-Wilk	0.935692	0.0000
Kolmogorov-Smirnov	0.091148	<.0100
Cramer-von Mises	1.675610	<.0050
Anderson-Darling	9.804214	<.0050

r2







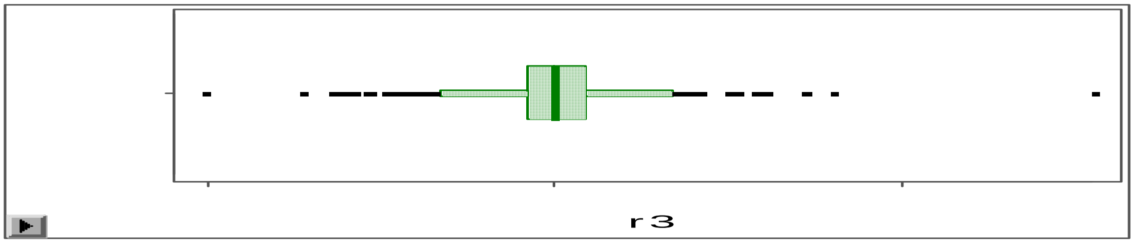
Moments			
N	599.0000	Sum Wgts	599.0000
Mean	0.0010	Sum	0.6289
Std Dev	0.0421	Variance	0.0018
Skewness	-0.4530	Kurtosis	4.4381
USS	1.0586	CSS	1.0580
CV	4006.2249	Std Mean	0.0017

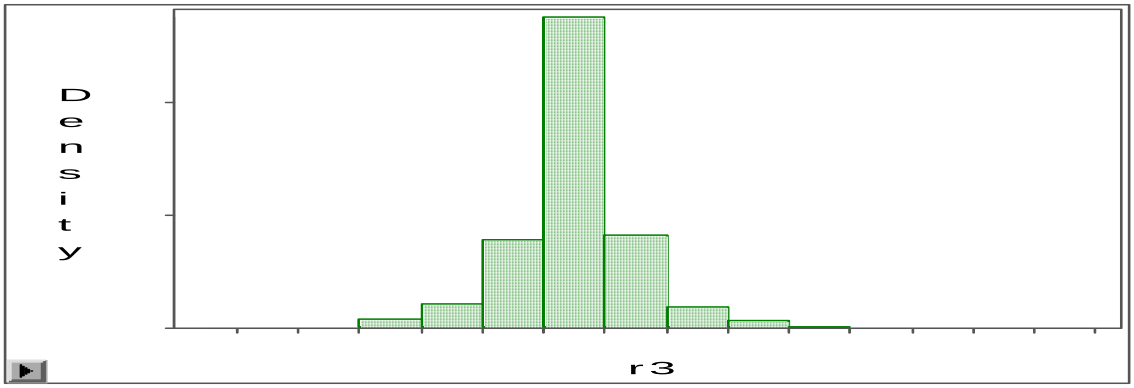
Quantiles			
100% Max	0.1657	99.0%	0.1092
75% Q3	0.0245	97.5%	0.0866
50% Med	0.0011	95.0%	0.0671
25% Q1	-0.0210	90.0%	0.0495
0% Min	-0.2689	10.0%	-0.0433
Range	0.4346	5.0%	-0.0592
Q3-Q1	0.0455	2.5%	-0.0846
Mode	0	1.0%	-0.1207

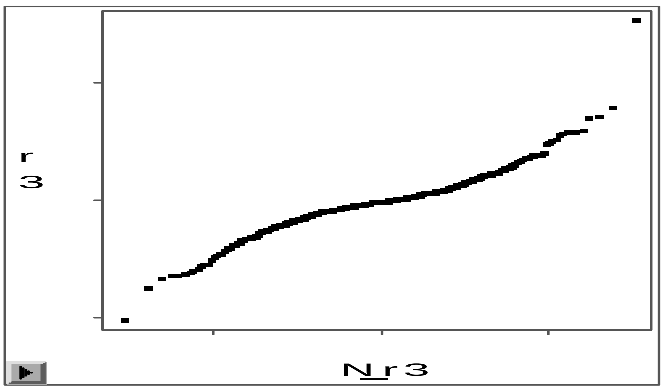
Tests for Normality

Test Statistic	Value	p-value
Shapiro-Wilk	0.953752	0.0000
Kolmogorov-Smirnov	0.062355	<.0100
Cramer-von Mises	0.695312	<.0050
Anderson-Darling	4.475988	<.0050

r3







Moments

N	599.0000	Sum Wgts	599.0000
Mean	0.0003	Sum	0.2007
Std Dev	0.0212	Variance	0.0004
Skewness	0.5279	Kurtosis	6.8512
USS	0.2682	CSS	0.2682
CV	6320.8848	Std Mean	0.0009

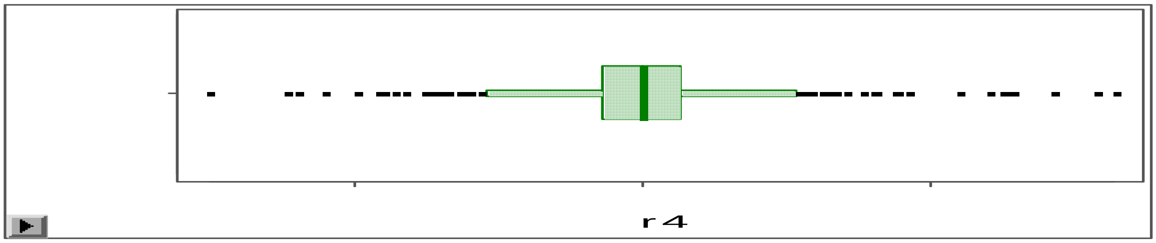
Quantiles

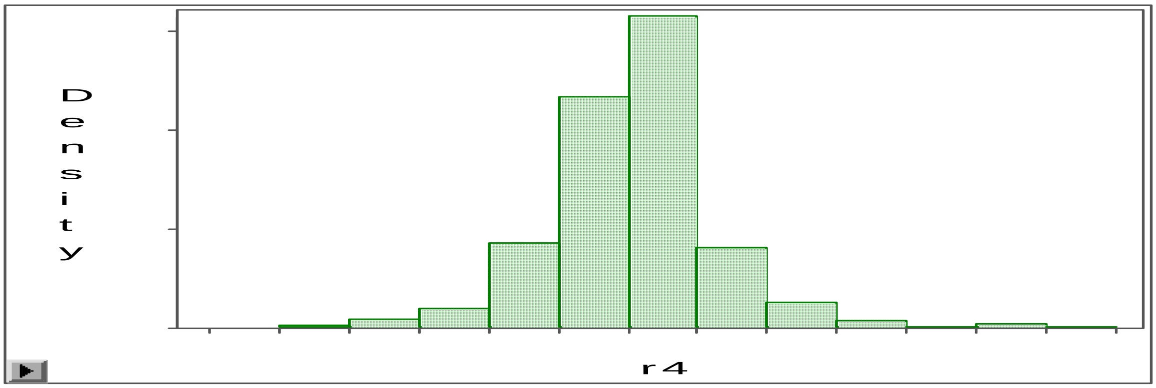
100% Max	0.1559	99.0%	0.0607
75% Q3	0.0089	97.5%	0.0421
50% Med	0	95.0%	0.0346
25% Q1	-0.0082	90.0%	0.0238
0% Min	-0.1009	10.0%	-0.0225
Range	0.2567	5.0%	-0.0326
Q3-Q1	0.0171	2.5%	-0.0458
Mode	0	1.0%	-0.0610

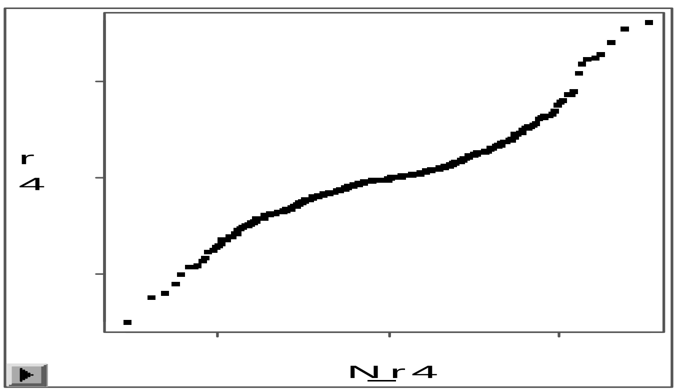
Tests for Normality

Test Statistic	Value	p-value
Shapiro-Wilk	0.920364	0.0000
Kolmogorov-Smirnov	0.103566	<.0100
Cramer-von Mises	2.225156	<.0050
Anderson-Darling	11.76144	<.0050

r4







Moments

N	599.0000	Sum Wgts	599.0000
Mean	0.0012	Sum	0.6922
Std Dev	0.0331	Variance	0.0011
Skewness	0.4496	Kurtosis	4.6753
USS	0.6567	CSS	0.6559
CV	2866.1144	Std Mean	0.0014

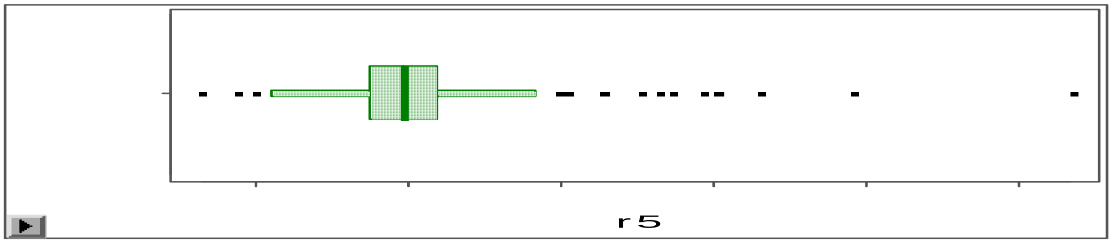
Quantiles

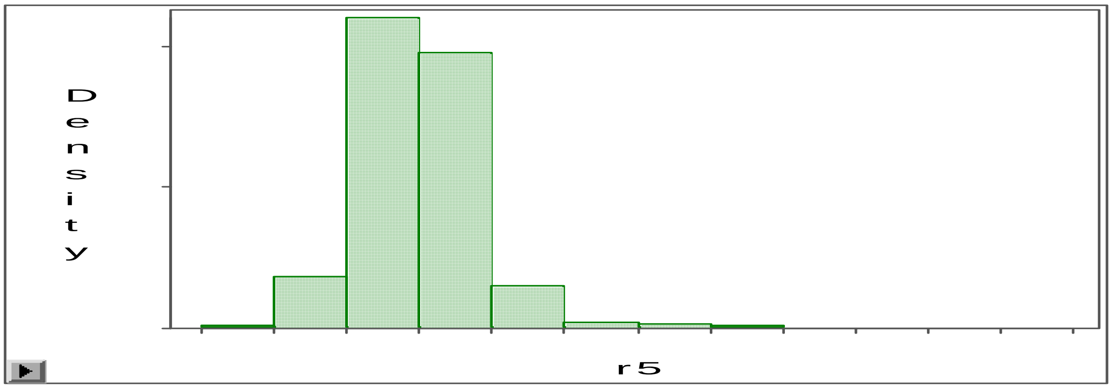
100% Max	0.1646	99.0%	0.1250
75% Q3	0.0132	97.5%	0.0714
50% Med	5.848E-05	95.0%	0.0547
25% Q1	-0.0139	90.0%	0.0348
0% Min	-0.1500	10.0%	-0.0346
Range	0.3146	5.0%	-0.0482
Q3-Q1	0.0271	2.5%	-0.0671
Mode	0	1.0%	-0.0915

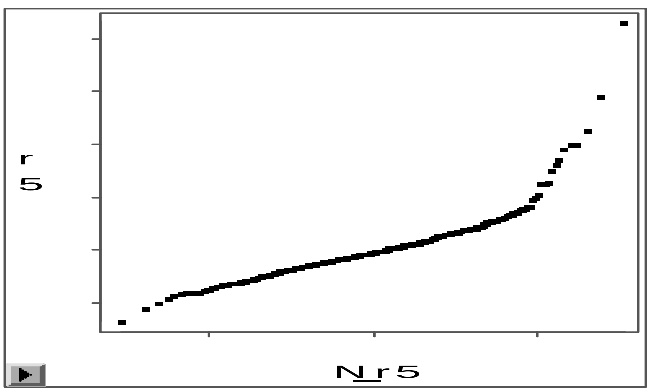
Tests for Normality

Test Statistic	Value	p-value
Shapiro-Wilk	0.919367	0.0000
Kolmogorov-Smirnov	0.114377	<.0100
Cramer-von Mises	2.411792	<.0050
Anderson-Darling	12.95930	<.0050

r5







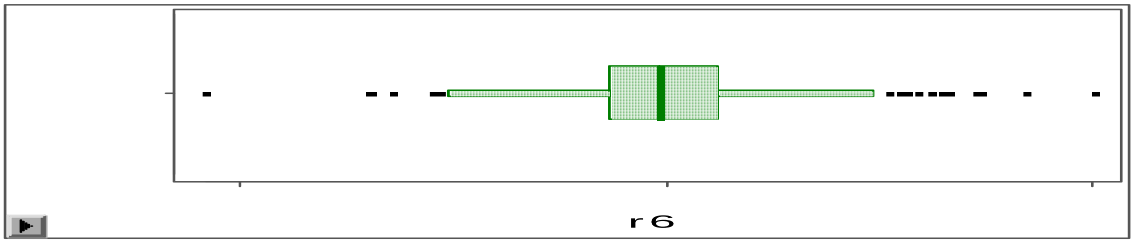
Moments			
N	599.0000	Sum Wgts	599.0000
Mean	0.0019	Sum	1.1499
Std Dev	0.0932	Variance	0.0087
Skewness	2.5477	Kurtosis	17.0512
USS	5.2019	CSS	5.1997
CV	4857.6172	Std Mean	0.0038

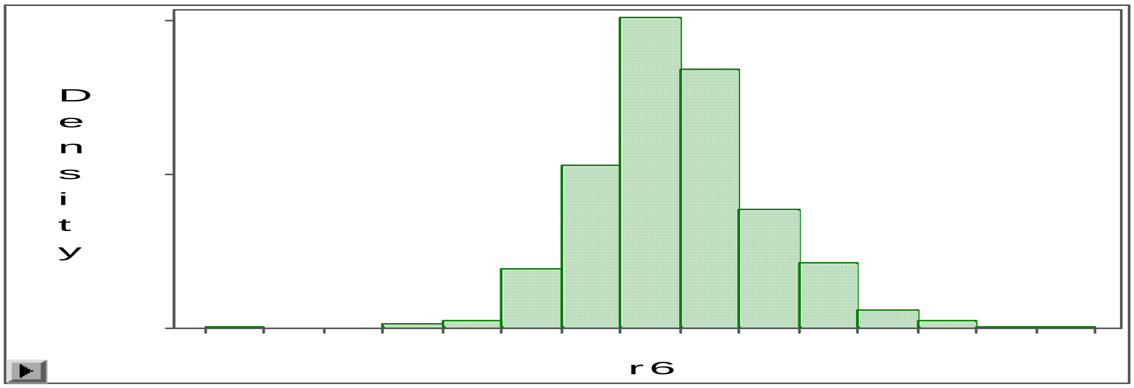
Quantiles			
100% Max	0.8710	99.0%	0.3880
75% Q3	0.0387	97.5%	0.2000
50% Med	-0.0072	95.0%	0.1321
25% Q1	-0.0505	90.0%	0.0909
0% Min	-0.2710	10.0%	-0.0950
Range	1.1420	5.0%	-0.1221
Q3-Q1	0.0892	2.5%	-0.1410
Mode	0	1.0%	-0.1630

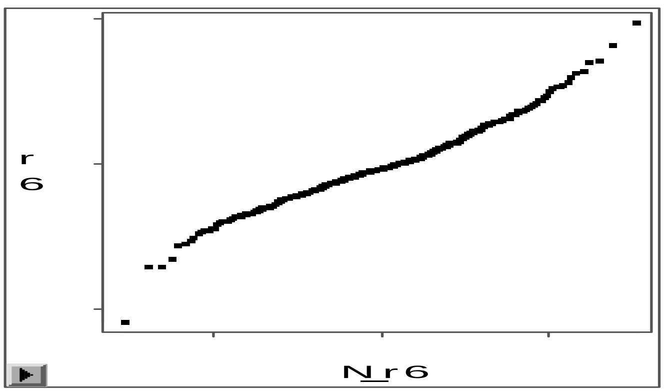
Tests for Normality

Test Statistic	Value	p-value
Shapiro-Wilk	0.843875	0.0000
Kolmogorov-Smirnov	0.100851	<.0100
Cramer-von Mises	1.886487	<.0050
Anderson-Darling	11.96463	<.0050

r6







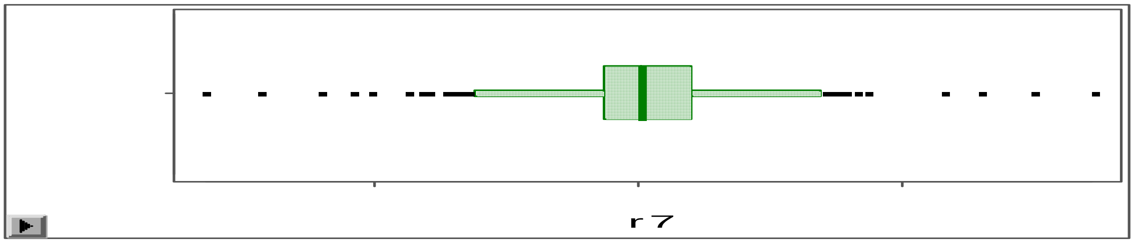
Moments			
N	599.0000	Sum Wgts	599.0000
Mean	-0.0003	Sum	-0.2050
Std Dev	0.0448	Variance	0.0020
Skewness	0.2925	Kurtosis	2.0289
USS	1.2002	CSS	1.2001
CV	-13090.764	Std Mean	0.0018

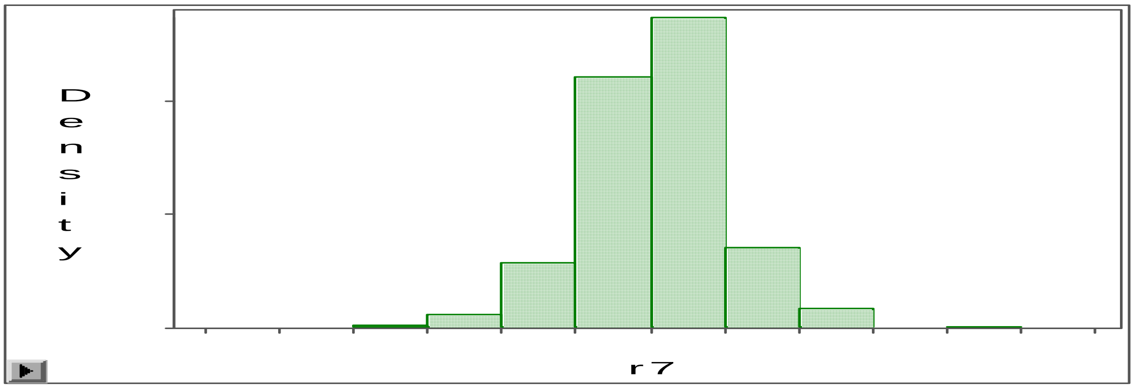
Quantiles			
100% Max	0.2009	99.0%	0.1290
75% Q3	0.0244	97.5%	0.0963
50% Med	-0.0029	95.0%	0.0763
25% Q1	-0.0269	90.0%	0.0590
0% Min	-0.2162	10.0%	-0.0532
Range	0.4172	5.0%	-0.0678
Q3-Q1	0.0513	2.5%	-0.0801
Mode	0	1.0%	-0.1065

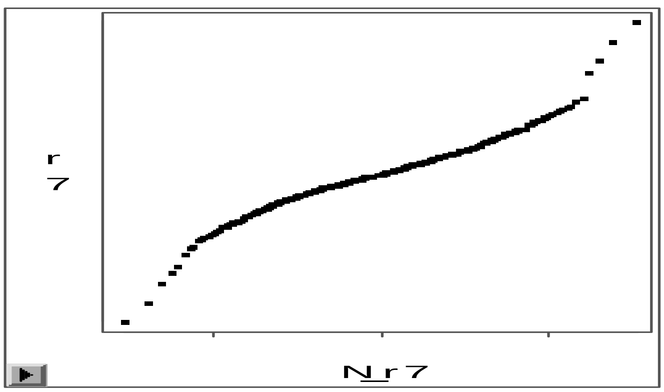
Tests for Normality

Test Statistic	Value	p-value
Shapiro-Wilk	0.976002	0.0000
Kolmogorov-Smirnov	0.072686	<.0100
Cramer-von Mises	0.652545	<.0050
Anderson-Darling	3.475965	<.0050

r7







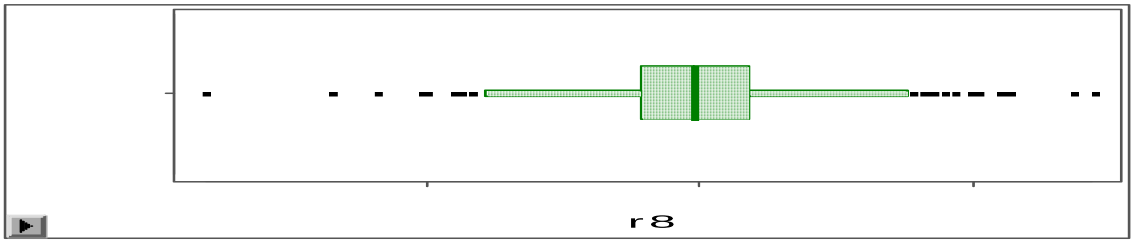
Moments			
N	599.0000	Sum Wgts	599.0000
Mean	0.0011	Sum	0.6764
Std Dev	0.0161	Variance	0.0003
Skewness	-0.0226	Kurtosis	4.4173
USS	0.1550	CSS	0.1542
CV	1422.1070	Std Mean	0.0007

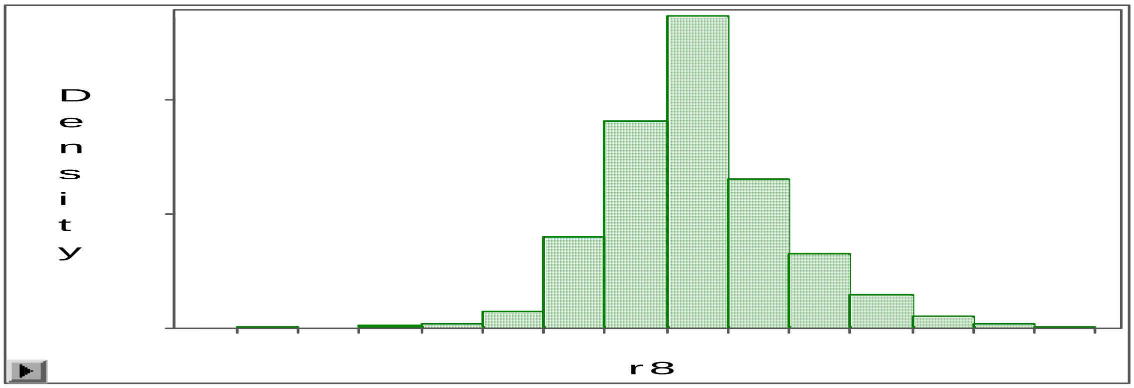
Quantiles			
100% Max	0.0865	99.0%	0.0414
75% Q3	0.0097	97.5%	0.0328
50% Med	0.0006	95.0%	0.0260
25% Q1	-0.0067	90.0%	0.0191
0% Min	-0.0822	10.0%	-0.0161
Range	0.1687	5.0%	-0.0245
Q3-Q1	0.0164	2.5%	-0.0313
Mode	0	1.0%	-0.0437

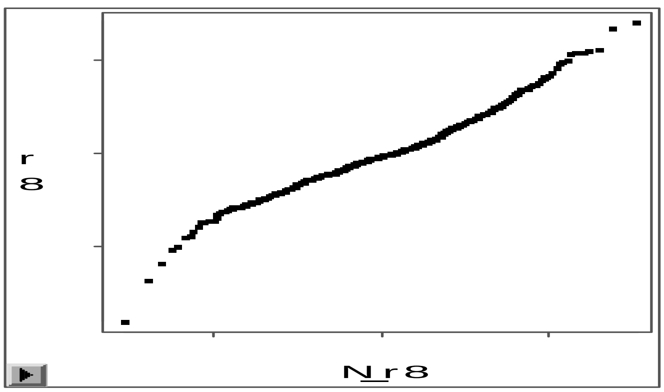
Tests for Normality

Test Statistic	Value	p-value
Shapiro-Wilk	0.949485	0.0000
Kolmogorov-Smirnov	0.065481	< .0100
Cramer-von Mises	0.863140	< .0050
Anderson-Darling	5.187065	< .0050

r8







Moments

N	599.0000	Sum Wgts	599.0000
Mean	3.257E-05	Sum	0.0195
Std Dev	0.0373	Variance	0.0014
Skewness	0.2204	Kurtosis	1.9380
USS	0.8302	CSS	0.8302
CV	114413.276	Std Mean	0.0015

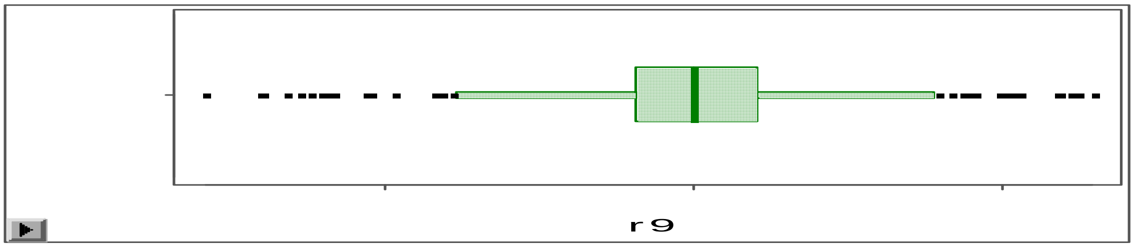
Quantiles

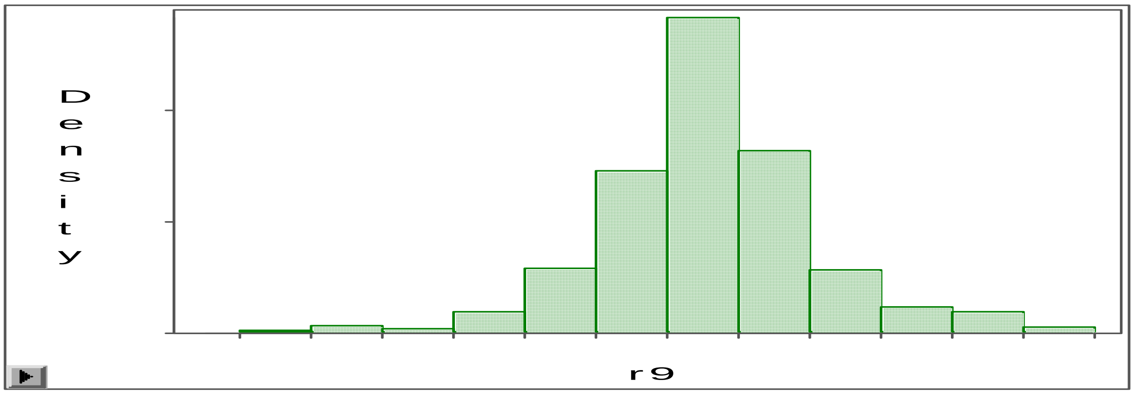
100% Max	0.1446	99.0%	0.1100
75% Q3	0.0176	97.5%	0.0840
50% Med	-0.0019	95.0%	0.0684
25% Q1	-0.0214	90.0%	0.0456
0% Min	-0.1811	10.0%	-0.0429
Range	0.3256	5.0%	-0.0548
Q3-Q1	0.0390	2.5%	-0.0684
Mode	0	1.0%	-0.0900

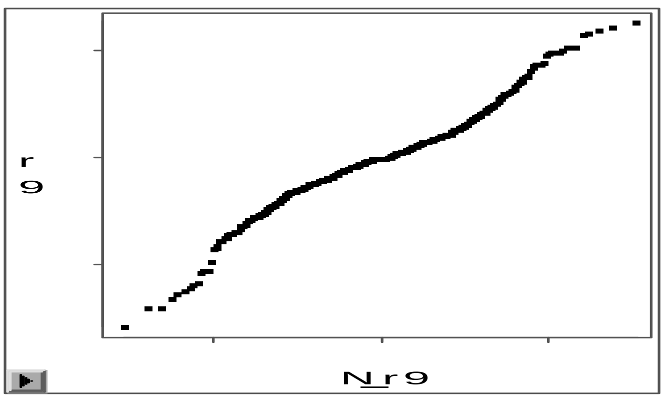
Tests for Normality

Test Statistic	Value	p-value
Shapiro-Wilk	0.974545	0.0000
Kolmogorov-Smirnov	0.072823	<.0100
Cramer-von Mises	0.704586	<.0050
Anderson-Darling	3.902639	<.0050

r9







Moments

N	599.0000	Sum Wgts	599.0000
Mean	0.0011	Sum	0.6869
Std Dev	0.0396	Variance	0.0016
Skewness	-0.1206	Kurtosis	2.0294
USS	0.9402	CSS	0.9394
CV	3456.2776	Std Mean	0.0016

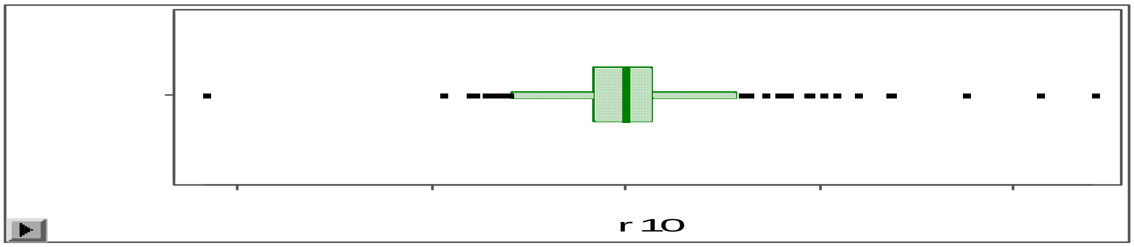
Quantiles

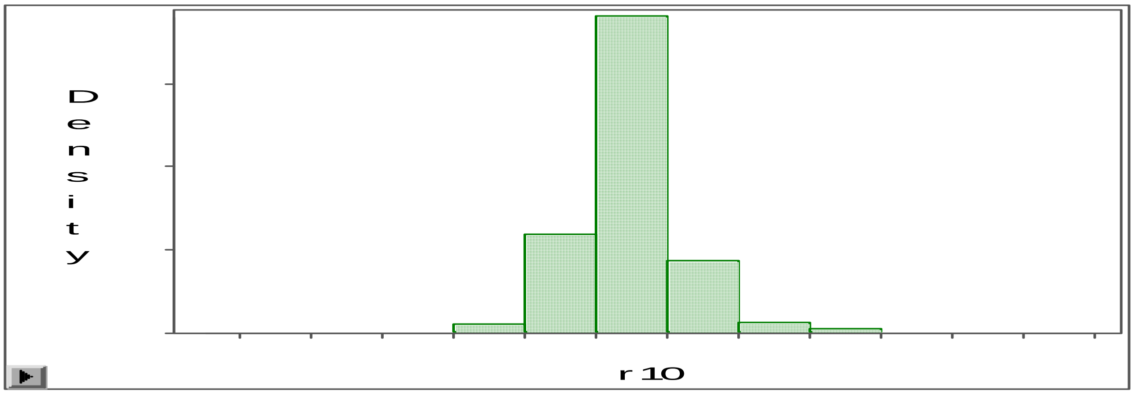
100% Max	0.1297	99.0%	0.1059
75% Q3	0.0201	97.5%	0.0909
50% Med	0	95.0%	0.0704
25% Q1	-0.0188	90.0%	0.0474
0% Min	-0.1579	10.0%	-0.0440
Range	0.2876	5.0%	-0.0620
Q3-Q1	0.0389	2.5%	-0.0833
Mode	0	1.0%	-0.1237

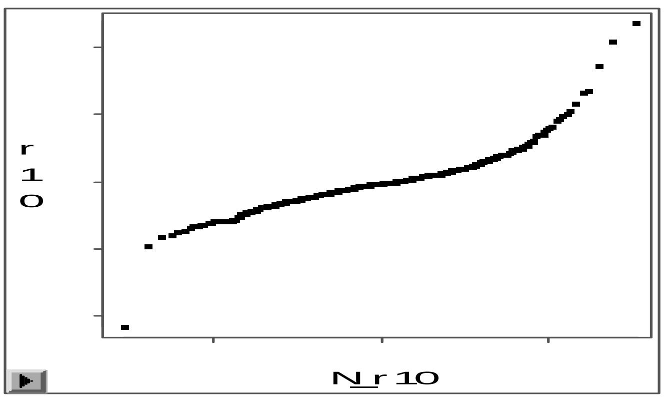
Tests for Normality

Test Statistic	Value	p-value
Shapiro-Wilk	0.961312	0.0000
Kolmogorov-Smirnov	0.081344	<.0100
Cramer-von Mises	1.285069	<.0050
Anderson-Darling	7.288738	<.0050

r10







Moments

N	599.0000	Sum Wgts	599.0000
Mean	0.0010	Sum	0.6197
Std Dev	0.0677	Variance	0.0046
Skewness	1.2202	Kurtosis	11.1655
USS	2.7399	CSS	2.7393
CV	6541.9368	Std Mean	0.0028

Quantiles

100% Max	0.4828	99.0%	0.2374
75% Q3	0.0262	97.5%	0.1554
50% Med	-0.0016	95.0%	0.1023
25% Q1	-0.0323	90.0%	0.0676
0% Min	-0.4330	10.0%	-0.0669
Range	0.9158	5.0%	-0.0910
Q3-Q1	0.0584	2.5%	-0.1154
Mode	0	1.0%	-0.1417

Tests for Normality

Test Statistic	Value	p-value
Shapiro-Wilk	0.869578	0.0000
Kolmogorov-Smirnov	0.113823	<.0100
Cramer-von Mises	2.414629	<.0050
Anderson-Darling	13.79627	<.0050