Moving Averages for Financial Data Smoothing

Aistis Raudys, Vaidotas Lenčiauskas, and Edmundas Malčius

Vilnius University, Faculty of Mathematics and Informatics,
Naugarduko st. 24, LT-03225 Vilnius, Lithuania
aistis@raudys.com, edmundas.malcius@gmail.com

Abstract. For a long time moving averages has been used for a financial data smoothing. It is one of the first indicators in technical analysis trading. Many traders debated that one moving average is better than other. As a result a lot of moving averages have been created. In this empirical study we overview 19 most popular moving averages, create a taxonomy and compare them using two most important factors – smoothness and lag. Smoothness indicates how much an indicator change (angle) and lag indicates how much moving average is lagging behind the current price. The aim is to have values as smooth as possible to avoid erroneous trades and with minimal lag – to increase trend detection speed. This large-scale empirical study performed on 1850 real-world time series including stocks, ETF, Forex and futures daily data demonstrate that the best smoothness/lag ratio is achieved by the Exponential Hull Moving Average (with price correction) and Triple Exponential Moving Average (without correction).

Keywords: moving average, smoothing, filer, time series, smoothness, lag, hull, exponential, TRIX.

1 Introduction

Moving averages are one of the key tools used to analyse financial time series. In a nutshell, moving average is simple weighted sum (mean) calculated over selected historical price range. Financial data usually is noisy, if we choose to represent today's price as mean of today's price and 2 days before, all ups and downs will be averaged. Using more historical prices (increasing period), we can achieve more smoothed price that would show the trend, despite the price fluctuations. Let's define x_i as a price value at the time i. Let X= $x_1, x_2,..., x_p$ and p is the time series length. So most simple moving average at the time t would be

$$ma_t^n = \frac{1}{n}\sum_{i=1}^{n} x_{t-1} \text{ or } ma_t^n = \sum_{i=1}^{n} x_{t-i+1}\, \mathrm{w}_i\ ,$$

where $w_i = \frac{1}{n}, i = 1, \dots, n$, and integer n determines the averaging window width.

Moving averages are heavily used to show the trend in the noisy data. At the same time the smoothing plays role of regularisation, wide known in statistical data analysis. Smoothing is often improving stability of conclusion/predictions as well. As period (the window width), n, of moving average is increased, more noise can be filtered

T. Skersys, R. Butleris, and R. Butkiene (Eds.): ICIST 2013, CCIS 403, pp. 34–45, 2013.

from financial data, better smoothness is achieved. But sometimes price fluctuation is a trend reversal - not noise. Since moving average combines historical prices and a current price to get filtered price, for some time, depending on the period, it will show previous trend instead of a new one. This is called a lag. Raw price has zero lag, and n-bar moving average has $n/2$ bar lag. The most important question is how to reduce the lag, which causes missed trade opportunities or false trades, and keep reasonable smoothness remains unsolved problem, which immense armada of moving averages (ways to chose and techniques) try to solve.

In [1], the problems that moving averages in financial data suffer are well explained. The main one is that nondeterministic unknown process is generating financial time series. But to filter and smooth this data, we can use one of many defined moving average processes, which fit differently from time to time, and by definition can't be perfect. Still a lot of effort is thrown to invent and upgrade moving averages to get better results. Also complexity varies: from simple linear moving averages to higher order processes and neural networks.

One of approaches is to dynamically adjust period of moving average. Authors in [2] use reinforced learning in their described trading strategy to alter period of moving average on the fly. All trading system is able beat the market by about 50 percentage points, according to authors. In [4] it is also claimed, that using variable period moving averages is possible to achieve profit, even during financial crisis. In [9] authors profiled investor risk using multitude of factors. The idea of adaptive moving averages has been extensively discussed in [3] and some trading strategies involving adaptive element has been assessed in [16].

Artificial neural networks are widely used in time series analysis and forecasting. In [5] authors use recursive Elman neural network to calculate moving average. The average is later used in proposed stop loss – maximum return (SLMR) trading strategy. Authors claim big success due to optimizations by joining a SLMR trading strategy with a moving average calculation inside an Elman neural network.

Smoothing (blurring of the images or time series) was considered in statistical data analysis. Actually it is some sort of regularization. A degree of optimal smoothing depends on the velocity of time series changes: the more frequent are the changes the smaller window with (lag) should be chosen. In Parzen window multidimensional nonparametric features input density estimation used for classification purposes Marina Skurikhina [17] compared 13 functions of smoothing window shape (Gausian, trapezoidal, three angle, rectangular etc.). She found and that most of the smoothing functions were approximately equally effective. The Gaussian shape appeared the best. The rectangular shape appeared to be the worst one. The main effect of smoothing was obtained due to correct choose of lag - the window width. For financial data analysis we also have a number of diverse methods, however, there is a lack of comprehensive practical overview in the literature of moving averages for financial data smoothing, particularly paying attention to criterion “smoothens vs. lag ratio”. Authors try to fill this gap by analyzing numerous moving averages, on numerous instruments, their types and time frames.

The paper is organised as follows. In the next section we present evaluation methodology, next we describe moving averages and methods. In the following sections we present data description, taxonomy, experiments results and conclusions.

2 Evaluation Methodology

We evaluate two opposite properties of moving averages: smoothness and lag. Traders want optimal filter so trend following would be nice and one could avoid whipsaw trades. Usually moving averages take one parameter – period p of past prices to use. As period increases, lag grows and edges smoothen. In Fig. 1, we see two different period moving averages. When trend changes, red one responds very slowly, value is far from real price. But the line is smooth. Blue follows price more aggressively, stays close to the price, but is bumpier. Not all bumps represent reverse of trend.

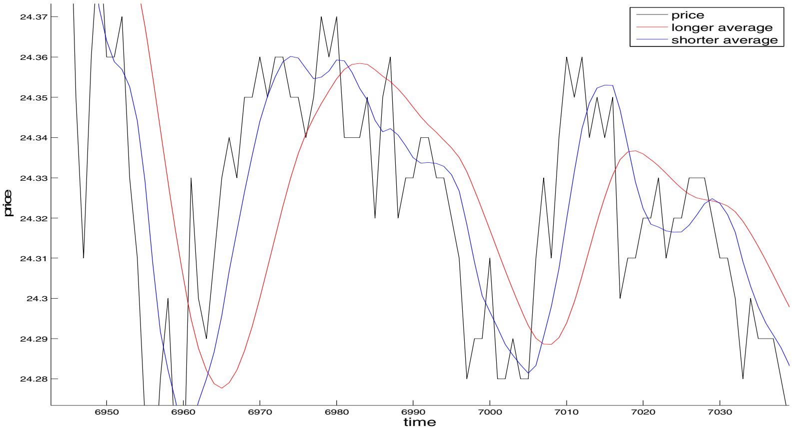

Fig. 1. Two MA (red and blue). Red one is long period MA hence more lagging and smoother, while blue is shorter period and less lagging and bumpier.

In this study we will investigate smoothness and lag in more detail. Assume we have prices X=(x_1, x_2, … x_p), here p is the length of time series. Moving average of length n at time t would be $ma(t,n) = \frac{1}{n}\sum_{i=0}^{n-1} x_{(t-i)}$. We define lag as a distance between current bar and moving average at that point, then at time t lag is: $lag_t = |x_t - ma(t,n)|$. For entire dataset, average lag is calculated like: $lag_T = \frac{1}{p}\sum_{t=1}^{p} lag_t$, here p is the number of data points in time series, x_i is the data point at pos i and ma_i is moving average of the n period at the position i. Lag estimate tells how moving average is lagging behind the price. Now smoothness needs to be measured. Assume we have moving average as vector of values (ma). First, we calculate how value changed from previous one:

$$dif_i = \begin{cases} i > 1, ma_i - ma_{i-1} \\ i = 1, 0 \end{cases}$$

Now $dif = \{dif_i\}, i = 1 \dots p$ vector holds values of how much moving average changed during each time period. If some value is negative, it indicates a negative change of direction. If growth is not constant, it creates bumps. We define smoothness as average of difference changes:

$$smo = \frac{1}{n-1}\sum_{i=2}^{p}|dif_i - dif_{i-1}|$$

Using mean of such vector, we average bumpiness/smoothness of our moving average. As we defined lag and smoothness we can calculate some moving averages, increasing period from 2 to 30, and evaluate their smoothness and lag. Results are visualised in Fig. 2. Having short period moving average is very bumpy – high smoothness value. But it follows price very well and the offset is low. As the period increases, average becomes very smooth, but lag increases. Alternatively, smoothness and lag can be plotted against each other, as in Fig. 3 (lower values are better).

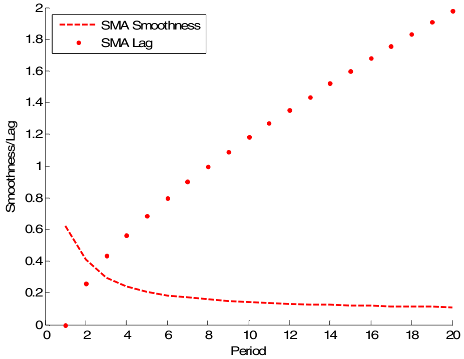

Fig. 2. Smoothness and lag of simple moving average can be plotted against each other (lower values are better)

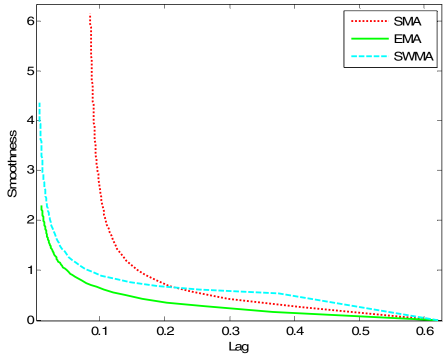

Fig. 3. In this figure we demonstrate how smoothness and lag are related. As MA period increases smoothness value is decreasing (smaller-better) and increases the lag.

Same situation is here, as lag grows, smoothness gets better and vice versa. Three different moving averages are plotted in Fig. 3, each calculated changing period from 2 to 30 over same dataset. One average looks slightly better. These two simple measurements can help to answer question, which moving average has best smoothness/lag ratio?

3 Moving Average Methods

In this paper we analyze 19 most popular moving averages (simple, exponential, weighted, sinus weighted, Spencers, median, Tilson, Hull, double exponential, TRIX/triple exponential, Ehlers, zero lag, Butterworth, Mesa, Savitzky-Golay, Kaufman, geometric, quadratic and harmonic moving average). In various sources one could find even more, but in most cases same average comes under different names. Proprietary moving averages (like Jurik Moving Average) are discarded from this paper.

Simple moving average (SMA) is well known and widespread. It gives equal weights to all past prices and by definition is just average of them. Although very simple, it can solve serious problems. It will be used as a benchmark to compare against other averages. For it's simplicity, formula is discarded.

Exponential moving average (EMA) gives exponentially diminishing weights to all past prices. This moving average is very well known and used, therefore formula is not included.

Weighted moving average (WMA) gives arithmetically diminishing weights for past prices, depending on length of the average.

Sinus weighted moving average (SWMA) is a weighted average, based on motivation, that price fluctuates following some unknown wave. As model, Sine wave is used to adjust price weights. SWMA is calculated using formula:

$$SWMA_n(X) = \frac{\sum_{n=1}^{m} \sin\left(n\frac{180}{6}\right) X_n}{\sum_{n=1}^{m} \sin\left(n\frac{180}{6}\right)}$$

Where m is period of moving average, X is list of prices with X_0 the most recent one.

Spencers 15 point moving average (SpMA) is another version of WMA used by actuaries. It is fixed 15 position mean with weights 3, -6, -5, 3, 21, 46, 67, 74, 67, 46, 21, 3, -5, -6, -3. The problem with this average is high lag.

Double exponential moving average (DEMA) is whole different from described above. It is composite moving average and uses other moving averages to get the result [11]. In case of DEMA, the EMA is used. Also, DEMA is adaptive - it employs some mechanism to adapt to price swings dynamically. DEMA uses trick to get better smoothness by running moving average on itself. But this operation increases lag, so to counter this technique called twicing is used. It takes difference between price and moving average to adjust itself, making DEMA adaptive. Formula:

$$DEMA_n(X) = EMA_n(X) + EMA_n\big(X - EMA_n(X)\big)$$

where n is length of moving average and X is the prices.

Triple exponential moving average (TRIX) is similar to DEMA but uses exponential moving average three times:

$$TRIX_n(X) = EMA_n\left(EMA_n\big(EMA_n(X)\big)\right)$$

Zero lag moving average (ZMA) sounds like a perfect moving average [9]. But the only thing without lag is the price, which this adaptive and composite moving average uses to correct itself. In a nutshell, ZMA ads portion of price to EMA to counter lag, while giving up some smoothness. Formula (n – period, X – prices

$$if\ n > 1, \beta = 0{,}2$$

$$ZMA_n(X) = \alpha * (X_{n-1} + \beta * \big(X_n - ZMA_{n-1}(X_{n-1})\big) + (1-\alpha) * ZMA_{n-1}(X_{n-1})$$

Tilson moving average (TMA) is also known as T3. It is both composite and adaptive. It is build using EMA [11]. To make notion more readable, formula is decomposed. First one describes generalized DEMA average introducing parameters *n* and *v*. For Tilson moving average, *v* is 0,7. If *v* would be 1, then GD would be DEMA. To improve smoothness of TMA, moving average is applied over again.

$$TMA_n(X) = GD_n\left(GD_n\left(GD_n(X)\right)\right)$$

$$GD_n(X) = (1+v) * EMA_n(X) - EMA_n(EMA_n(X)) * v, where\ v = 0{,}7$$

Hull moving average (HMA) is composite moving average made from composing WMA of various period lengths [12]. Formula:

$$HMA_n\ (X) = WMA_{\sqrt{n}}\ (2 * WMA_{\frac{n}{2}}(X) - WMA_n(X))$$

Exponential Hull moving average (EHMA) is exactly the same as Hull MA but Exponential MA is used instead of Weighted MA:

$$EHMA_n\ (X) = EMA_{\sqrt{n}}\ (2 * EMA_{\frac{n}{2}}(X) - EMA_n(X))$$

Ehlers moving average (EhMA) is another adaptive moving average [8]. To use it, data must be first detrended subtracting SMA (of the same period as EhMA) from the price. Then EhMA coefficients are recalculated for each position, based on quadratic distance. This makes EhMA computational expensive with large periods over bigger datasets. Formula (*X* – detrended prices, *n* – period of EhMA) is gives for detrended prices, after applying EhMA result is obtained adding SMA back to it:

$$A = SMA_n(X); X = X - A; c_i = \sum_{m=1}^{n} (X_i - X_{i-m})\,;$$

$$EhMA'_n(X) = \frac{\sum_{i=0}^{n-1} c_i X_i}{\sum_{i=0}^{n-1} c_i}; EhMA_n(x) = EhMA'_n(x) + A$$

Butterworth moving average (BMA) came from analogue circuits' era [8]. Very well known there, works for trading as well. Formula (*n* – period, *X* - prices, *i* – current bar) to calculate current bar *BMA(i)*:

$$BMA(i) = (X_i + 2 * X_{i-1} + X_{i-2}) + a_1 * BMA(i-1) + a_2 * BMA(i-2);$$

$$\beta = 2.415 * \left(1 - \cos\left(\frac{360}{n}\right)\right); \alpha = -\beta + \sqrt{(\beta^2 + 2 * \beta)};$$

$$c_0 = \frac{\alpha^2}{4}; a_1 = 2 * (1 - \alpha); a_2 = -(1 - \alpha)^2;$$

Mesa moving average (MAMA) uses Hilbert transform to make EMA adaptive. Because of Hilbert transform this moving average has complex formula, only main parts will be given. By definition MAMA is EMA with variable alpha: *MAMA(i)* =

*alpha * Price + (1 – alpha) * MAMA(i-1),* where *alpha = FastLimit/DeltaPhase. FastLimit* is the upper bound of *a* and *DeltaPhase* is the rate of change of the Hilbert Transform homodyne discriminator. The alpha value is kept within the range of *FastLimit* and *SlowLimit.* This moving average focused on very short periods and tries to show cycles in them [3 p. 737].

Savitzky-Golay moving average (SGMA) is polynomial smoother [15]. Given last *n* prices, i tries to fit *k* level polynomial over them using MSE. Then polynomial value is used as filtered value. SGMA has two parameters: *n* – period, and *k* –level of polynomial to fit.

Kaufman moving average (KAMA) is adaptive one, which alters alpha of EMA using smoothing constant *C* to achieve addictiveness [3 p.731]. Formula (*n* –period, *X* – prices, X_i – past price *i* bars back):

$$\mathrm{ER} = \frac{|\mathrm{X_i - X_{i-n}}|}{\mathrm{n} * \sum_{\mathrm{i=1}}^{\mathrm{n}} |\mathrm{X_n - X_{n-i}}|}; \mathrm{C} = (\mathrm{ER} * (0{,}6667 - 0{,}0645) + 0{,}645)^2;$$

$$\mathrm{KAMA(i)} = \mathrm{KAMA(i-1)} + \mathrm{C} * (\mathrm{X_i} - \mathrm{KAMA(i-1)}$$

KAMA adjust alpha using efficiency ratio of the market. It is ratio between direction and volatility. Constants 0,6667 and 0,0645 represent adaptivness range from 2 to 30 bars of EMA alpha value. These constants are suggested by author, so we will keep them.

Chande's variable index dynamic average (VIDYA) follows same concept as KAMA. VIDYA, however, uses relative volatility to adjust smoothing constant [3 p.736]. Formula (*s* – constant, representing 9 bar EMA smoothing constant, *C* – closing prices, *i* – current time, C_n – prices of recent n bars, C_m – prices of longer historic period m>n):

$$VIDYA(i) = k * s * C_i + (1 - k * s) * VIDYA(i-1)$$

$$s = \frac{2}{9+1} = 0.2 \quad, k = \frac{stdev(C_n)}{stdev(C_m)}$$

Other types of moving averages

Median moving average(MeMA) isn't weighted average, as by definition it is just median of a price range. So when calculating moving average, it just takes median element of a frame as average of frame. Formula is not included.

Geometric moving average (GMA) represents a growth function in which a price change from 50 to 100 is as important as a change from 100 to 200 [3 p.20]. Formula ($a_1..a_n$ – prices, *n* -period): $GMA = (a_1 * a_2 * \ldots * a_n)^{\frac{1}{n}}$

Quadratic moving average (QMA) is made from well known error estimator [3 p.21]. Formula(*a* – price, *n* - period) $QMA_n = \sqrt{\left(\frac{\sum a^2}{n}\right)}$

Harmonic moving average (HaMA) is time weighted mean, not biased towards higher or lower values as in the geometric mean [3 p.21]. Formula (*n* – period, a_i - prices):$HaMA = \frac{n}{\left(\sum_{i=1}^{n}\left(\frac{1}{a_i}\right)\right)}$

4 Taxonomy of Methods

Moving averages can be categorised into several groups depending on their behaviour. Most of the categorisation is based on properties of the weights. Some however are weightless (like Median MA) and cannot be assigned to any category. We characterised MA by three properties: positive/negative weights, look-back period and adaptiveness.

First categorisation is based on weights positivity. Weights can be positive only or positive and negative. If MA has negative weights it tries to reduce lag by correcting itself. This improvement also introduces overshooting on trend reversals. This can be seen from impulse response diagram. Each historical data point is weighted with positive weight and summed up afterwards. Other group of moving averages weights higher recent history and subtracts older history. This way it reduces lag but overshooting phenomenon appears on trend reversals.

Data dependant adaptive moving averages changes their behaviour based on the data, thus their smoothness and lag varies. Impulse response diagram may not correctly reflect their weighting scheme.

Fixed length/infinite length – look-back period. Some moving averages use exactly the same number of historical data points to calculate smoothed value (Simple MA, weighted MA). The other group references all the values to the beginning (Exponential MA). Latter has a problem of MA calculated on different length of data may not be the same.

5 Data Description

In this study we used solely real-life data. No synthetically generated time series or processes [6] have been used. The aim of this paper is to empirically evaluate large number of MAs on large-scale real-life data. We used daily Stock data, ETF data, Futures data and foreign exchange (Forex) data. Initially we planned to use intraday data (1 min, 5 min, 60 min frequency) but we faced overnight and weekend non trading hours problem, then a big gap in the data can occur. MA in such cases may behave inadequately so we decided to restrict to daily data for the current research. We obtained time series data from Tradestation Securities. Summary of our data is presented in Table 1. For each time series we preformed experiments on all time series, where we selected best moving average for each of them.

Table 1. Summary of the data used in the analysis process

	Stocks	**ETF**	**FOREX**	**Futures**	**Total**
Instruments	630	1092	34	94	1850
Days	4524	5105	4379	5111	5117
Start	2001-01-01	1999-05-19	2001-05-21	1999-05-20	1999-05-20
End	2013-05-22	2013-05-10	2013-05-17	2013-05-17	2013-05-22
Total days	1990795	1518000	82157	282580	3873532

Stock data is a series of prices stock was traded on the exchange. Historical stock prices are adjusted at the point they pay a dividend. Next day after dividend payment historical data is moved down by the amount of dividend stocks paid. Hence some stocks that paid unusually big dividend at some point in the history may have negative price. In our study we selected the most popular and liquid stocks. We filtered stocks with highest trading volume and with the recent price above 10 USD.

Exchange Traded Funds (ETF) are instruments traded on the main stocks exchanges and representing some index or other investable assets. They cover most of the investment universe: Equity, Bond/Fixed Income, Commodity, Currency, Alternative, Inverse instruments, Leveraged instruments and Real Estate across the globe. The excellent source for more information on ETF is http://etfdb.com/.

Foreign exchange market (Forex) is probably the most liquid market in the world. It trades trillions of dollars every day. It is decentralised market where every broker trades separately and synchronises prices between each other in real-time. We used all major currency pairs traded by typical currency broker. The data we used came from Tradestation Securities.

Futures are vanilla derivative instruments traded in regulated futures exchanges such as CME, EUREX, ICE, etc. We included only US and European futures in this study. Future it is a contract to buy or sell specific underlying instrument at specific price at some point in the future. Futures usually have an expiration date on a monthly or quarterly basis. Hence long and continues data is composed of multiple contracts by sticking them together and adjust the difference at the sticking point - moving the history up or down depending on the price difference at the point of joining.

6 Experiments

In this paper we empirically compared various MA on real world datasets. To compare two moving averages we used Simple MA as a benchmark. We selected 5, 10, 21, and 63 periods as a benchmark periods for smoothness. These are most common periods representing a week, two weeks a month and a quarter.

$$S_n^{SMA} = Smoothness(SMA_n)$$

where n=(5,10,21,63). At mentioned periods, we measured smoothness of SMA and selected other MA with the same or better smoothness. For example for Exponential MA (EMA):

$$S_g^{EMA} \leq S_n^{SMA}$$

$$g = \arg\min_{\mathrm{g}} S_g^{EMA} \leq S_n^{SMA}$$

$$S_{SMA5}^{EMA} \leq S_5^{SMA},$$

then we measured their lag

$$L_{SMA5}^{EMA} = lag(EMA_g)$$

In Table 2, we present a relationship between SMA periods and periods of other MA. It can be used as a reference to select desired smoothness. This is very useful reference as authors where not able to find any literature that contains such reference. Winner selection was performed using voting. For each stock, ETF, forex or future and for each smoothness level (SMA equivalent n=5,10,21,63) we selected the best (smallest lag) MA as a winner. Later we counted the wins and selected the most often winning MA as a final winter for the category. More information can be seen in the Table 2. So for example, we have 1000 stocks in our database, for each stock we have 4 smoothens levels (SMA equivalent n=5,10,21,63) so in total we can have 4000 winners. For each out of 19 MA we selected a winner in each smoothness level.

Table 2. Corresponding periods of a MA that has similar smoothness and lag to that of SMA

		By Smoothness				By Lag			
No.	**Title**	**P5**	**P10**	**P21**	**P63**	**P5**	**P10**	**P21**	**P63**
1	Simple	5	10	21	63	5	10	21	63
2	Butterworth	50	62	28	59	52	55	19	57
3	Double exponentional	10	19	31	49	15	32	82	277
4	Exponentional	5	10	16	24	8	17	40	151
5	Hull	12	17	28	39	12	28	67	301
6	Sine weighted	4	7	11	17	3	9	23	83
7	Spencers 15 point	2	5	10	15	1	2	2	61
8	T3	4	6	9	12	5	9	19	80
9	Triangular	5	9	14	20	2	6	15	54
10	Chande's variable index	5	10	17	30	8	17	41	180
11	Weighted	6	10	16	26	8	18	40	134
12	ZERO lag	5	12	25	70	8	25	244	162
13	Geometric	5	10	22	94	5	9	23	60
14	Exponential Hull	8	14	19	29	14	30	78	342
15	Median	19	61	50	146	2	8	21	78
16	Harmonic	5	11	22	89	5	11	23	60
17	TRIX	2	3	4	5	3	5	10	39
18	Ehlers`					13	30	66	303
19	Savitzky-Golay	67	124	208	299	30	80	168	396

7 Results

We present results in the Table 3 below. Table is composed of 4 parts, each for different type of time series. Rows represent different moving averages and columns represent 4 smoothness levels equivalent of SMA p=5,10,21,63. The number in the table indicates how many times that moving average had smallest lag in comparison to other ones (note that smoothness is the same). Last column "Tot." Summarises win count. We sorted the list with highest win count at the top of the table.

As can be seen winning algorithms are Exponential Hull and TRIX. TRIX is the leader between stocks and EHMA is everywhere else. For Futures, Forex and ETF TRIX is the second best algorithm.

Table 3. Results of winning moving averages

Futures

Name	P5	P10	P21	P63	Tot.
Exp. Hull	13	37	39	42	131
TRIX	31	30	16	20	97
Dbl. exp.	11	3	2	1	17
Butterwo.	0	0	2	4	6
ZERO lag	2	1	0	1	4
Exp.	3	0	0	0	3
T3	0	0	1	0	1
Weighted	0	1	0	0	1

Stocks

Name	P5	P10	P21	P63	Tot.
TRIX	270	235	287	280	1073
Exp Hull	2	277	291	231	801
Butterworth	0	2	15	101	118
ZERO lag	22	0	1	0	23
Weighted	6	0	0	0	6
Double exp.	5	0	0	0	5
Hull	0	1	2	2	5
Exp	4	0	0	0	4

Forex

Name	P5	P10	P21	P63	Tot.
Exp. Hull	12	15	16	15	58
TRIX	4	11	10	12	37
ZERO lag	7	1	2	1	11
Exp.	6	0	1	1	8
Double exp.	0	3	2	2	7
Butterwo.	0	0	0	1	1
Chande v. i.	0	0	0	1	1
T3	0	0	1	0	1

ETF

Name	P5	P10	P21	P63	Tot.
Exponential Hull	48	131	223	227	659
TRIX	52	85	81	113	331
Double exp.	68	24	22	22	136
ZERO lag	27	15	8	8	58
Exp.	19	8	4	8	39
T3	1	3	2	0	6
Butterworth	0	0	0	1	1

8 Conclusions

In this paper we compare 19 the most popular moving averages used in practical trading and determine the most suitable according the criteria "smoothens vs. lag ratio". We performed large-scale study by testing all the MAs on 1850 real-world daily time series from following domains: Stock, ETF, Futures and Forex. We compared all MA at 4 different smoothness levels equivalent of a simple MA 5, 10, 21 and 63 days and selected the best one for each category and each time series. Finally we counted which one won most of the time. Two best moving averages identified: Exponential Hull Moving Average (EHMA), next followed by a Triple Exponential Moving Average (TRIX). EHMA uses a correction term to reduce lag and is different in that from TRIX. Correction term subtracts older history to reduce lag of the moving average but introduces "overshooting" behaviour in trend reversals. For stocks TRIX showed the best results as stocks tend to be more volatile and have frequent trend reversals where correctionless MA is more accurate. For all other time series EHMA was the winner. All other methods are far behind the two winners.

We also created a reference table where we link different moving averages to the smoothness of Simple MA. The other table references lag to a SMA lag. This can be used by practitioners trying to replace one MA with other one with the same lag or the same smoothness.

For the future work we already did preliminary research. Our aim is to create a tailor-made moving averages for specific time-series that would have lowest lag for a given level of smoothness. We plan to create two versions, one with positive only weights and other with positive-negative weights (i.e. with correction term). We estimate that tailor-made moving average will have better smoothness and lag characteristics than current winners EHMA and TRIX. Our inference supports conclusions in [17] where classification accuracy was used as a performance criterion. We also plan include other criteria in the analysis: forecasting accuracy, maximum profit (in case of trading system), minimum risk or similar criteria.

Acknowledgments. This work was supported by the Research Council of Lithuania under grant MIP-018/2012.

References

1. Hamilton, J.D.: Time series analysis, vol. 2. Princeton University Press, Princeton (1994)
2. Tan, Z., Quek, C., Cheng, P.Y.K.: Stock trading with cycles: A financial application of ANFIS and reinforcement learning. Expert Systems with Applications 38(5) (2011)
3. Perry, J.: Kaufman, New Trading Systems and Methods, 4th edn. John Wiley & Sons (2005)
4. Ni, Y.-S., Lee, J.-T., Liao, Y.-C.: Do variable length moving average trading rules matter during a financial crisis period? Applied Economics Letters (2012)
5. Marques, N.C., Gomes, C.: Maximus-AI: Using Elman Neural Networks for Implementing a SLMR Trading Strategy. In: Bi, Y., Williams, M.-A. (eds.) KSEM 2010. LNCS, vol. 6291, pp. 579–584. Springer, Heidelberg (2010)
6. Ruseckas, J., Gontis, V., Kaulakys, B.: Nonextensive Statistical Mechanics Distributions And Dynamics of Financial Observables From The Nonlinear Stochastic Differential Equations. Advances in Complex Systems 15(suppl. 1) (2012)
7. Jurgutis, A., Simutis, R.: An investor risk profiling using fuzzy logic-based approach in multi-agents decision support system. In: Proceedings of the 17th International Conference on Information and Software Technologies, Kaunas (2011)
8. John, E.: Cybernetic Analysis for Stocks and Futures, pp. 213–227. John Wiley & Sons (2004)
9. John, E.: Rocket Science for Traders, 245 pages. John Wiley & Sons (2001)
10. Kirkpatrick, C.D., Dahlquist, J.R.: The Complete Resource for Financial Market Technicians, pp. 39–50. Financial Times Press (2006)
11. Tillson, T.: Smoothing Techniques For More Accurate Signals. Stocks & Commodities 16, 33–37 (1998)
12. Hull, A.: Hull moving average, `http://www.justdata.com.au/Journals/AlanHull/hull_ma.htm`
13. John, E.: Cybernetic Analysis for Stocks and Futures, pp. 213–227. John Wiley & Sons (2004)
14. John, E.: Rocket Science for Traders. John Wiley & Sons (2001)
15. Person, P.-O., Strang, G.: Smoothing by Sawitzky-Golay and Legendre filters, `http://persson.berkeley.edu/pub/persson03smoothing.pdf`
16. Ellis, C.A., Parbery, S.A.: Is smarter better? A comparison of adaptive, and simple moving average trading strategies. Research in International Business and Finance 19(3), 399–411 (2005)
17. Skurichina, M.: Effect of the kernel functional form on the quality of nonparametic Parzen window classifier. In: Raudys, S. (ed.) Statistical Problems of Control, vol. 93, pp. 167–181. Institute Mathematics and Informatics, Vilnius (1991) (in Russian)