Nonparametric Kernel Estimation of Multiple Hedge Ratios

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Abstract:

It is possible for the traditional hedge ratio estimation to produce erroneous guidance to risk managers because of the restrictive assumptions. This study adopts nonparametric locally polynomial kernel estimation to exclude the assumptions. Results from the hog complex find that hedge ratios estimated by local polynomial kernel regression outperform naïve and GARCH models. Because of the potential assumption violations associated with the estimation and implementation of hedge ratios by GARCH models, LPK is a reasonable alternative for estimating hedge ratios to manage price risks.

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I. Introduction

The hedge ratio is traditionally estimated by simple regression such as OLS and SUR. However, this procedure is statistically inefficient because it ignores heteroskedasticity, which implies that estimates could not explain information inflow in prices. Since futures and spot prices exhibit time varying volatility, they should be represented by conditional heteroskedastic shocks (the conditional covariance of futures and cash prices), and this would lead to optimal hedge ratios that may change over time. With the development of autoregressive conditional heteroskedasticity (ARCH) and generalized ARCH (GARCH) models, various studies have been conducted to get efficient estimates of time varying hedge ratios (McNew and Fackler, 1994; Garcia, Roh, and Leuthold, 1995; Bera, Garcia, and Roh, 1997).

These analyses have been performed in a parametric context, involving a complete specification of the process of interaction among important variables, which may lead to error in the hedging decision. Specifically, these analyses are based on several assumptions. The functional form of the conditional mean of cash prices given futures prices is assumed to be known, usually linear. The conditional variance of cash prices given futures prices and the autocorrelation of error terms are assumed specified. The parametric joint density (data generating process) of cash prices given futures prices is assumed normal. Last, prices are often considered to be nonstochastic even though ARCH-type models incorporate stochastic features. Due to these assumptions, optimal hedge estimation in a parametric framework may not be robust to slight inconsistencies between data and any particular parametric specification. Thus, any misspecification in this regard may lead to erroneous estimation of the hedge ratio, usually exaggerating the variance.

This study develops and tests a new method for estimating time varying hedge ratios, a kernel-based nonparametric partial derivative estimator, specifically locally polynomial kernel regression, which reduces the number of arbitrary parametric restrictions. This estimator is consistent under most circumstances, such as rigid dependence of the error terms and an unknown functional form, and gives a better fit to the time series that often have fat tails. Thus, hedge ratios that are dependent on past values of conditioning variables can be consistently estimated within this nonparametric framework. The potential benefits of using this estimator are that it avoids the potential errors introduced by functional misspecification and expands the settings in which the estimator can be shown to be consistent, thus, permitting more efficient risk management (Ullah, 1988; Wand and Jones, 1995). This technique has never before been applied to optimal hedge ratio estimation and can be applied to both single commodity and simultaneous production decisions on outputs and inputs. As an example, it is applied here to the hog feeding complex.

If the functional form of the conditional mean of cash prices given futures prices is known, then the parametric approach will perform better or at least as well as the nonparametric approach. Therefore, a simple linear model and a GARCH-type model are developed to estimate time varying hedge ratios to compare with the nonparametric estimation.

The accuracy of hedge ratios, estimated by these three different methods, are evaluated in terms of maximizing expected return and variance reduction comparison. In addition, the accuracy of hedge ratios are evaluated and compared to various hedging strategies including naïve hedging, single-commodity hedging, and no hedging.

II. Hedging Rules

Hog producers face multiple price risks due to the volatile prices of live hogs and feed grains, and often achieve the objective of reducing these price risks by forward pricing through either the futures market or forward cash market.[1] Since buyers of hogs such as meat packers charge for their services, prices offered through forward cash contracts may be less than those offered using futures market, and hence the futures market is often preferred to the forward cash market. Another advantage of using the futures market comes from marketing flexibility.

In this study the feeding (final) stage of hog production (wean-to-finish) is considered because it is the main stage of hog production where large amounts of feed grains are consumed. It takes around 4 months to reach final market weight of hogs of about 225 pounds, a stage which begins when the hogs weigh about 60 pounds. Among various feed ingredients, corn is the major feed grain, and around 615 pounds per hog are fed during this final period.[2] Corn provides dietary energy in the form of carbohydrates and fat. The hedging decision framework is composed by two stages. The first stage, from t-6 to t-4, constitutes a planning period before feeding begins (t refers to when the output is marketed, and time is measured in months). At t-6, hedging occurs by simultaneously taking long positions in the input and a short position in the output in the futures market. Hedges on inputs are held for two months until the feeding begins. Corn is purchased for the feeding of hogs at t-4 in the cash market. At the same time, those input

[1] Forward pricing is not the only alternative to managing pricing risk. Floor pricing through the options market provides a minimum price while allowing the producer to take advantage of any higher prices. Forward pricing on the other hand will provide more price protection against lower prices than will floor pricing, but also precludes gains from higher prices.
[2] Another potentially important input is soybean meal. However, the amount of soybean meal consumed per hog is approximately only 10% of total feed grains while corn takes around 85%. Also, in a similar analysis for live cattle, Noussinov and Leuthold (1999) found that the coefficient for the soybean meal hedge ratio was insignificant and did not affect the overall hedging results. In addition, soybean meal adds a third dimension to the kernel estimation, which would make the procedure used in this study very complex.

hedges taken at t-6 are liquidated. After feeding, the live hogs are sold in the cash market at t and the associated output futures position held for six months is lifted.

Then, the returns from cash and futures transactions of hogs and corn at time t can be written as

$$R_t = P_{H,t} - P_{C,t-4} + \beta_{H,t-6}(F_{H,t-6} - F_{H,t}) + \beta_{C,t-6}(F_{C,t-4} - F_{C,t-6}), \tag{1}$$

where P and F stand for cash and futures prices, respectively, and H and C are abbreviated for hogs and corn respectively. $\beta_{H,t-6}$ and $\beta_{C,t-6}$ are hedge ratios for hogs and corn at time t-6.

This equation assumes no transaction costs. Assuming unbiasness in futures market, hedge ratios for multiproduct can be generated by the mean-variance framework. The variance of returns in (1) can be written as

$$\begin{aligned} Var(R_t) = {} & \mathrm{var}(P_{H,t}) + \mathrm{var}(P_{C,t-4}) + \beta_H^2 \,\mathrm{var}(F_{H,t}) + \beta_C^2 \,\mathrm{var}(F_{C,t-4}) - 2\,\mathrm{cov}(P_{H,t}, P_{C,t-4}) - 2\beta_H \,\mathrm{cov}(P_{H,t}, F_{H,t}) \\ & + 2\beta_C \,\mathrm{cov}(P_{H,t}, F_{C,t-4}) + 2\beta_H \,\mathrm{cov}(P_{C,t-4}, F_{H,t}) - 2\beta_C \,\mathrm{cov}(P_{C,t-4}, F_{C,t-4}) + 2\beta_H \beta_C \,\mathrm{cov}(F_{H,t}, F_{C,t-4}). \end{aligned} \tag{2}$$

Solving the first-order conditions of (2) for the optimal hedge ratios yields the following decision rules for hog producers:

$$\beta_C^* = \frac{[\mathrm{cov}(P_{H,t}, F_{H,t}) - \mathrm{cov}(P_{C,t-4}, F_{H,t})]\,\mathrm{cov}(F_{H,t}, F_{C,t-4}) + [\mathrm{cov}(P_{C,t-4}, F_{C,t-4}) - \mathrm{cov}(P_{H,t}, F_{C,t-4})]\,\mathrm{var}(F_{H,t})}{\mathrm{var}(F_{C,t-4})\,\mathrm{var}(F_{H,t}) - \mathrm{cov}(F_{H,t}, F_{C,t-4})^2}, \tag{3}$$

and

$$\beta_H^* = \frac{\mathrm{cov}(P_{H,t}, F_{H,t}) - \mathrm{cov}(P_{C,t-4}, F_{H,t}) - \beta_C \,\mathrm{cov}(F_{H,t}, F_{C,t-4})}{\mathrm{var}(F_{H,t})}. \tag{4}$$

Hedge ratios of single commodity is simply $\mathrm{cov}(P, F)/\mathrm{var}(F)$, where time subscripts for hedge ratios are omitted for simplicity.

III. Econometric Model and Data

Locally Polynomial Kernel Estimation

Since nonparametric estimation does not define the functional form, a general nonparametric regression model with n data points $\{(X_i, Y_i)\}_{i=1}^{n}$ follows as

$$Y_i = m(X_i) + u_i \tag{5}$$

where $m(x) = E(Y|X = x)$, $E(u|x) = 0$ and $X = \{F_t, Z_{t-l}\}$. The stochastic nature of the relationship is represented by the zero-mean random shock u_t. Y is easily replaced by P_t, and X is replaced by F_t and Z_{t-l}, where Z_{t-l} is other relevant information, and $l = 1, \cdots, m$. Our aim is to estimate the slopes of $\hat{m}(X_i)$. The conditional expectation of the partial derivative of P_t w.r.t. F_t can be derived to obtain hedge ratios,

$$HR_t = \frac{\partial E(P_t | F_t, Z_{t-1}, \ldots, Z_{t-m})}{\partial F_t}(F_t, Z_{t-l}) = \frac{\partial m}{\partial F_t}(F_t, Z_{t-l}). \tag{6}$$

The value of the function HR_t, which is dependent on F_t and Z_{t-l} at a particular point, gives the value of the partial derivative of the conditional expectation functional with respect to the concurrent futures price variable.

In the parametric approach, the critical assumptions of the response function $m(X_i)$ are known functional form and normality of error terms. Any misspecification in $m(X_i)$ causes serious consequences for econometric inference; for example, the estimators of the regression parameters can be seriously biased. Since the nonparametric approach does not rely on the assumptions underlying the parametric approach, it has more flexibility and is more efficient in estimating the complicated unknown response function $m(X_i)$.

Locally polynomial kernel (LPK) estimation is utilized in this study and is known to produce a good fit to a sample characterized by nonlinear relationships. The locally optimal hedge ratio can be achieved and represented by partial derivatives or derivatives in given directions. There are two important benefits of using LPK. First, it overcomes boundary problems, which often misleads a kernel estimation in an erroneous direction. Second, it is easy to get partial derivatives of $\hat{m}(X_i)$ because the purpose of LPK is to produce the partial derivatives of the estimated regression function in addition to a reasonable approximation to the unknown response function. This would thereby provide nonconstant and time-varying hedge ratios which account for all the relevant information of hog and corn prices. These hedge ratios obtained by LPK are extended into 3-dimension from 2-dimension, hedge ratios over futures price changes and time.

Local polynomial kernel estimates the regression function at a particular point by "locally" fitting a p^{th} degree polynomial to the data via weighted least squares.

To demonstrate the model, the model (5) is modified as follows,

$$Y_i = m(X_i) + s^{1/2}(X_i)u_i,\ i = 1, \dots, n \tag{7}$$

where, $m(x) = E(Y|X = x)$, $s(x) = Var(Y|X = x)$, and $\{u_i\}$ are i.i.d. The objective of this model is to estimate partial derivatives up to the p^{th} order $d^p m(x)/dX_i^p$ without imposition of $m(x)$ and $d^p m(x)/dX_i^p$ belonging to the parametric family of functions.

The local polynomial kernel estimator $\hat{m}(x; p, h)$ at a point x is obtained by fitting the polynomial $\beta_0 + \beta_1^T(x) + \cdots + \beta_p^T(x)^p$ to the (X_i, Y_i) using weighted least squares with kernel weights $K_H(x - X_i)$. Attention will be devoted to the local linear least squares kernel estimator, which corresponds to fitting a degree-one polynomial (p=1). Then the multivariate polynomial is of the form, $\beta_0 + \beta_1^T(x)$ and $\beta_1 = (\beta_{11}, \cdots, \beta_{1d})^T$. The problem is to find arguments β that solve:

$$Min(Y - X_x\beta_x)^T W_x (Y - X_x\beta_x) \tag{8}$$

where $X_x = \begin{bmatrix} 1 & (x - X_1)^T \\ \vdots & \vdots \\ 1 & (x - X_n)^T \end{bmatrix}$, $Y = \begin{bmatrix} Y_1 \\ \vdots \\ Y_n \end{bmatrix}$, and $W_x = \text{diag}\{K_H(x - X_1), \cdots, K_H(x - X_n)\}$. H is a $d{\times}d$ symmetric positive definite matrix depending on n. K is a d-variate kernel, $K_H(v) = |H|^{-1/2} K(H^{-1/2}v)$. It assigns weight to a particular point Y_i for estimation at a particular point x depending on how far a data point X_i is from the prediction point x, and $x = (x_1, \cdots, x_2)$. In other words, the observations close to x have more influence on the regression estimate at x than those farther away. The kernel, K, is a continuous, bounded, and symmetric probability density function. The assumptions are as follows,

$$\int K(v)dv = 1, \int vK(v)dv = 0, \text{and} \int v^2 K(v)dv = k_2 .$$

The matrix H controls how weight is apportioned among closer and more distant data points. Each data point X_i gets its own weight. Under the common assumption $H = \text{diag}(h_1^2, \cdots, h_d^2)$, higher values of bandwidth tend to discount distance between X_i and x less than the lower values. As bandwidth gets smaller, the local linear fitting process depends heavily on those observations that are closest to x and tends to yield a wigglier estimate. Thus, very low h would correspond to an interpolation of the data and very high h would give a least squares fit of a p^{th} order polynomial.

Assuming the invertibility of $X_x^T W_x X_x$, (8) has a solution of

$$\hat{\beta} = \begin{bmatrix} \hat{\beta}_0 \\ \hat{\beta}_1 \end{bmatrix} = \begin{bmatrix} \hat{m}(x) \\ \hat{\beta}_1 \end{bmatrix} = \left(X_x^T W_x X_x \right)^{-1} X_x^T W_x Y . \qquad (9)$$

The prediction of the conditional expectation function $m(x)$ is given by the first element in (9). The remainder of the coefficient, $\hat{\beta}_1$, in the locally linear case, represents estimates of the first partial derivatives with respect to each of the variable, X_i. The local least squares estimator of $m(x)$ is then

$$\hat{m}(x;H) = e_1^T \left(X_x^T W_x X_x \right)^{-1} X_x^T W_x Y , \qquad (10)$$

where e_1 is the (d+1)×1 vector having 1 in the first entry and all other entries 0. Thus, the value of $\hat{m}(x;H)$ is the height of the fit $\hat{\beta}_0$. The hedge ratio at x is then,

$$HR_t = \hat{\beta}(x;H) = e_2^T \left(X_x^T W_x X_x \right)^{-1} X_x^T W_x Y , \qquad (11)$$

where e_2 is the (d+1)×1 vector with a 1 in its 2 coordinates and zero's elsewhere. A major advantage of (10) and (11) is that it is easy to visualize how the estimator is using the data when estimating m at a point x. The estimate $\hat{m}(x;H)$, which is $\hat{E}(Y|X = x)$, can be evaluated at any value of x to yield the nonparametric estimator of the regression function. Clearly, out-of-sample forecasts, conditional on a set of known X values, can be calculated using (10).

Explicit formulae can be driven from (10) and (11) to estimate the regression function and hedge ratios in (9) for local linear (p=1):

$$\hat{m}(x;H) = n^{-1} \sum_{i=1}^{n} \frac{\left[\hat{s}_2(x;H) - \hat{s}_1(x;H)(x - X_i) \right] K_H(x - X_i) Y_i}{\hat{s}_2(x;H)\hat{s}_0(x;H) - \hat{s}_1(x;H)^2} \text{, and} \qquad (12)$$

$$HR_t = \hat{\beta}(x;H) = n^{-1} \sum_{i=1}^{n} \frac{\left[\hat{s}_0(x;H)(x - X_i) - \hat{s}_1(x;H) \right] K_H(x - X_i) Y_i}{\hat{s}_2(x;H)\hat{s}_0(x;H) - \hat{s}_1(x;H)^2} , \qquad (13)$$

where $\hat{s}_j = n^{-1}\sum_{i=1}^{n}(x - X_i)^j K_H(x - X_i)$.[3]

Choice of Optimal Bandwidth

Kernel estimator is described as a sum of ‘bumps’ placed at the observations, and the kernel function K determines the shape of the bumps, the shape of the weights, while the window width h determines their width, the size of the weights. As h becomes large, the smoothness of estimation will increase.

The choice of optimal bandwidth, h, remains of prime importance for the analysis to produce a good fit to sample data, which controls the tradeoff between bias and variance. Among several, the plug-in approach is utilized to select h in this study.

Plug-in bandwidth selectors are based on the simple idea of “plugging in” estimates of the unknown quantities that appear in formulae for the asymptotically optimal bandwidth (Wand and Jones, 1995).

The value of h should be guessed first to obtain a preliminary estimate of $m(x)$. For simplicity, the errors are homoskedastic with common variance, σ^2, then the natural selection of asymptotically optimal bandwidths as *pilot* bandwidth becomes

$$h_{\text{AMISE}} = \left[\frac{d\int K(z)^2\,dz}{\mu_2(K)^2}\right]^{\frac{1}{d+4}}\left[\frac{\sigma^2}{n\theta_{22}}\right]^{\frac{1}{d+4}},$$

with $\theta_{rs} = \int m^{(r)}(x)m^{(s)}(x)f(x)dx$ and $m^{(q)}$ is a q^{th} derivative. A estimator for θ_{22} is

$$\hat{\theta}_{22} = n^{-1}\sum_{i=1}^{n}\hat{m}^{(2)}(X_i;g)^2,$$

where g is a bandwidth. Meanwhile a estimator for σ^2 is

[3] Details on the proof are available from the author.

$$\hat{\sigma}^2(\lambda) = v^{-1}\sum_{i=1}^{n}\left[Y_i - \hat{m}(X_i;\lambda)\right]^2 ,$$

where λ is a bandwidth, $v = n - 2\sum_{i=1}^{n} w_{ii} + \sum_{i=1}^{n}\sum_{j=1}^{n} w_{ij}^2$ and $w_{ij} = e_1^T (X_{X_i}^T W_{X_i} X_{X_i})^{-1} X_{X_i}^T W_{X_i} e_j$.

Wand and Jones (1995) suggested estimating $m''(x)$ to avoid the problem that g and λ are dependent on another bandwidths, and so on. Use this estimate of $m''(x)$ to choose the bandwidth. Then direct plug-in rules for selection of smoothing parameters are in the form of

$$\hat{h}_{\mathrm{DPI}} = \left[\frac{d\int K(z)^2\,dz}{\mu_2(K)^2}\right]^{\frac{1}{d+4}}\left[\frac{\hat{\sigma}_1^2(\lambda)}{n\hat{\theta}_{22}(g)}\right]^{\frac{1}{d+4}} .$$

This procedure is iterative until it reaches convergence.

GARCH Model

In this study, the *BEKK* technique is employed in BGARCH and MGARCH specification.[4] Time series diagnostics led to the following econometric specification of the model, which has ARMA and exogenous variables in the conditional mean:

$$\begin{aligned} \Delta P_t &= C + \sum_{i=1}^{r}\phi_i P_{t-i} + \sum_{i=0}^{l}\beta_i F_{t-i} + \sum_{i=1}^{s}\theta_i e_{t-i} + e_{tP} \\ \Delta F_t &= C + \sum_{i=1}^{r}\phi_i F_{t-i} + \sum_{i=0}^{l}\beta_i P_{t-i} + \sum_{i=1}^{s}\theta_i e_{t-i} + e_{tF} \end{aligned} , \qquad (9)$$

where C is constant mean, P and F are cash and futures prices, and < denotes changes in prices. $e_t = [e_{tP}\ e_{tF}]^T \sim MN(0,V_t)$ and $V_t = AA^T + A_1(e_{t-1}e_{t-1}^T)A_1^T + B_1 V_{t-1} B_1^T$. A, A_1, B_1 are $d \times d$ matrices, and V_t is symmetry and non-negative-definiteness of the conditional covariance matrix.

The log likelihood function for the *BEKK* model (Engle and Kroner, 1995) is:

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[4] This model is referred to as BEKK specification due to the results obtained by Baba, et al (1989).

$$L(\theta) = -\frac{TN}{2}\log 2\pi - \frac{1}{2}\sum_{i=1}^{T}\left(\log\left|V_t\right| + e_t^T V_t^{-1} e_t\right), \tag{10}$$

where θ denotes all unknown parameters in e_t and V_t, T is the sample size, and N is the number of mean equations. Conditional normality has been assumed.

Data Description

The hog producer is assumed to begin planning for, and subsequently feeding, a new lot of hogs every week. Percentage changes in weekly (Wednesday) cash and futures closing prices are used for January 1990 to June 1999 (last two years for lean hogs), providing 493 number of observations: 332 for live hogs and 161 for lean hogs.[5] Wednesday is selected because on that day both cash and futures trading is active with relatively high and stable trading volume. Omaha cash and central Illinois bid prices serve as the cash prices for hogs and corn, respectively, because of their relatively high volume and wide acceptance as market barometers.

Lean hog prices are converted to live hog prices by multiplying by 0.74 to get overall hog hedge ratios. Lean hog values represent the carcass, averaging a 74% yield from live hogs. Futures contracts selected are those that will be the nearby ones when hedges are lifted, and these futures positions are maintained throughout the hedging period without adjustment. Data during the delivery months are not used.

IV. Estimation Results and Nonconstant Hedge Ratios

Primary Time Series Analysis

The presence of a unit root is tested in each price level and change by performing Augemented Dickey-Fuller (ADF) and Phillips-Perron (PP) tests. These tests confirm that all data are stationary except hog futures. Hence, these data are converted into percentage changes to be stationary. Jarque-Bera normality tests on percentage changes of each series indicate that the null hypothesis of normality is rejected for corn cash, hog cash, and hog futures. The normality hypothesis is again rejected for the parametric joint density of cash given futures in the GARCH model, implying an assumption violation. Conditional heteroskedasticity is identified under the assumption of normality, but since the assumption is broken, this result is not valid.

Model Selection

Model choice for kernel regression methods is the subject of ongoing research. Three methods are used: R-square, approximate F-test, sum of squared residuals (SSR), and cross-validation. The following models are selected among several,

$$P_t = f(F_t, P_{t-i}, F_{t-j}),$$

where $i = j = 1$ for corn and $i = j = 1, \cdots, 3$ for hogs in single commodity hedging. Also $i = 1$ and $j = 1, \cdots, 2$ are selected for both corn and hogs in multiproduct hedging.

In this study two types of models are considered with various lags. One is the traditional GARCH model with constant mean, C, and the other is ARMA and exogenous variables in the conditional mean. Using AIC and BIC model selection, the second model is selected with ARMA (1,1) and lag 1 of futures prices for both single-commodity and multiproduct hedging,

$$\begin{aligned}\Delta P_t &= C + \phi_1 P_{t-1} + \beta_1 F_{t-1} + \theta_1 e_{t-1} + e_{t,P} \\ \Delta F_t &= C + \phi_1 F_{t-1} + \beta_1 P_{t-1} + \theta_1 e_{t-1} + e_{t,F}.\end{aligned}$$

AIC and BIC model selection is again used to select simple linear models as follows,

$$P_t = g(F_t, P_{t-i}, F_{t-j}),$$

where $i = j = 1$ for corn, and $i = 1$ and $j = 1, \cdots, 3$ for hogs in single commodity hedging. Also $i = j = 1$ is selected for both corn and hogs in multiproduct hedging.

Estimated Hedge Ratios

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[5] The final live hog contract is December 1996 and the first lean hog contract is February 1997.

Figures 1 through 6 present the estimated hedge ratios of two alternatives, Local Polynomial Kernel (LPK) (figures 1 through 4) and GARCH (figures 5 and 6), which are nonconstant and time varying hedge ratios. Unlike GARCH, hedge ratios produced by LPK are extended into three-dimension. They, however, are traced into two 2-dimension figures for easy understanding of how hedge ratios are related to the percentage changes in futures prices and time. Both corn and hogs are clearly nonconstant for both alternatives. Figures 1 and 2 show the relationship between hedge ratios and percentage futures price changes as given by LPK. These ratios give an idea to hog producers the many different positions (hedge ratios) they might take in the futures market as futures prices change. For example, if a hog producer wants to hedge the input, corn, and expects corn futures price to change by 5.2% (0.052) in two months from today, the producer needs to take a position in the corn futures market by 0.90 as indicated in figure 1. Figures 3 and 4 given by LPK and figures 5 and 6 given by GARCH present the relationship between hedge ratios and estimated time period.

The estimation results for single-commodity and multiproduct hedges from OLS, LPK and GARCH during the estimation years from January 3, 1990 to June 30, 1999 are reported in table 1. The hedge ratios using LPK reported here are estimated at the mean level of each price series. That is, $P_t = f(\overline{F}_t, \overline{P}_{t-i}, \overline{F}_{t-i})$, where $i = 1, \cdots, 3$. As we go from single commodity to multiproduct hedging, LPK produces bigger hedge ratios for corn (0.94 to 1.16) and smaller hedge ratios for hogs (0.79 to 0.55). In the mean time, GARCH generates smaller hedge ratios for both commodities (1.11 to 0.93 for corn and 1.05 to 0.62 for hogs) and OLS remains at the same ratios (0.95 for corn and 0.91 for hogs).

For single commodity hedging, LPK generates the smallest estimated hedge ratio (0.94 for corn and 0.79 for hogs), both being significantly less than one. Meanwhile, BGARCH produces the largest hedge ratio (1.11 for corn and 1.05 for hogs), which is significantly larger than one for corn and but not statistically different from one for hogs. Interestingly, the corn hedge ratios for all three alternatives are significantly different from one while only the hog hedge ratio generated by LPK is significantly different from one. Thus, if only hogs are hedged based on a simple regression and BGARCH, the returns defined in (1) are not statistically different from a naïve hedge.

Unlike the single commodity hedging, MGARCH in the multiproduct case yields the smallest hedge ratios for both corn. Corn is statistically indistinguishable from the naïve hedge, while hogs are significantly less than one. Hog producers should over hedge corn and under hedge hogs at the same time when LPK is used. The estimated hedge ratios from LPK and MGARCH for hogs are far less than the one, different from OLS in the multiproduct hedging.

Hedging Effectiveness

The hedging effectiveness of the various models is examined based on two measures: the proportional reduction in the unhedged variance of returns and the proportional increase in the unhedged return. The larger the reduction in variance and the larger the increase in return, the higher the degree of hedging effectiveness. Equation (1), with hedge ratios specified by the various procedures, is used to calculate the weekly return and its variance. The variance of returns also is calculated for a naïve hedge which offsets the spot price risk by taking futures positions in corn and hogs based on the fixed proportions of the production technology.

The results of the in-sample and out-of-sample hedging are presented in tables 2 and 3. Two sample periods are used to test hedging efficiency; period of live hog trading only from January 3, 1990 to May 30, 1996 and the full sample from January 3, 1990 to June 30, 1999. During the in-sample period, based on 280 observations for live hogs shown in table 2 and 436 observations for all hogs in table 3, no dramatic difference is found in returns across the alternative models within either sample period. Unhedged returns at the mean and variances show the highest values of ($369.29, 0.0874) and ($364.41, 0.1209) for live hog and full sample periods, respectively.[6] These variances lead to the dramatically wide ranges of return, $180 to $561.60 and $72 to $573.60 per hog, for the two respective periods. Larger variance reduction is associated with Naïve, SLPK, and BGARCH models relative to OLS and multiproduct hedging models. Among alternatives, SLPK performs well for both measures and for both sample periods. It outperforms BGARCH in both measures, and produces larger reduction in variance than multiproduct hedging models do and similar decrease in mean return to multiproduct hedging models for both sample periods. Thus, it seems that SLPK is a good alternative, which balances fairly high return and variance reduction on the mean-variance frontier.

MGARCH performs slightly better than MLPK for both measures, return and variance, and for both in-sample periods but is hard to conclude that MGARCH is superior to MLPK. Caution is needed to prefer MGARCH to MLPK because the major assumption of GARCH, normality of error terms, has been violated. Thus, using MGARCH based on the violated assumption may cause erroneous results in managing price risk. In the mean time, since LPK does not depend on any parametric assumptions, it is more useful for firms to use in price risk management. Hence, LPK is likely to be a reliable method to estimate hedge ratios because no critical assumptions are required.

The out-of-sample results, which are based on 52 observations for the live hog period and 57 observations for the full period, are different. Hedge ratios of each model are one-week ahead forecasted based on the estimated hedge ratios by each method with keeping the same number of

[6] Additional costs beyond corn are not considered here.

observations. For example, 437th and 438th hedge ratios of LPK are forecasted by $\hat{E}(Y|X = x)$ using 1 to 436th and 2 to 437th observations, respectively. Out-of-sample performance for the two periods is depicted by models in figures 7 and 8 to visualize how the alternatives relate to each other.

Often, naïve hedging has been found to surpass current available models in variance reduction such as GARCH, and is found to perform better than both single and multiple GARCH and OLS for the live hog period in table 2; 43% versus 38.43%, 33.06%, 36.57%, and 33.47% for BGARCH, SOLS, MGARCH, and MOLS, respectively. Both SLPK and MLPK, however, outperform naïve model in variance reduction during the live hog period, showing around 53.51% and 45.45% of reduction. Larger variance reduction leads to larger decrease in return relative to unhedged, and smaller variance reduction in variance results in smaller decrease in return. For example, around 17.78% and 10.17% of decreases in return are produced by SLPK and OLS, respectively, which yield 53.51% and 33.06% of variance reduction. Since SLPK results in large reduction in return even though it leads to the largest reduction in variance, MLPK is a good alternative, which balances variance reduction and less decrease in return on the mean-variance frontier.

For the out-of-sample of the full sample period, MLPK and MGARCH outperform single hedging models in terms of variance reduction. MGARCH performs slightly better than MLPK, 48.34% and 46.62%, respectively, but again MLPK might be better to use because of the violated assumption in the GARCH model. In addition, MLPK produces the largest increase in return, 27.30% while MGARCH results in 13.10% increased return. Therefore, MLPK is the best alternative, which produces the biggest mean return and a large reduction in variance.

Naïve hedging generates larger variance reduction than other single hedging models but only slightly better than SLPK. However, SLPK might be better than naïve hedging for balancing return and variance. SLPK realizes 25.69% increase in return relative to unhedged while naïve hedging shows 20.57% increase. In other words, SLPK increase return and decrease variance, 4.25% and 1.6%, respectively, relative to naïve.

The out-of-sample full period is when hedging instruments are most needed because being unhedged is much riskier than in earlier periods. The unhedged produces the smallest mean return and highest variance, showing the range of return, $16.80 to $321.60 per hog, and all hedging alternatives tested here lead to increased return and reduction in variance. This might be due to the fact that hog prices are highly unstable in this latter period: hog prices dramatically fell and plummeting to 57-year lows of less than $10/cwt in December 1998. Thus, using the futures market helps limit possible losses in this volatile market.

Mixed results are found between in- and out-of-sample. SLPK might be a better alternative on the mean-variance frontier relative to naïve, GARCH and OLS for both in-sample periods because it shows good combinations of return and risk that a hog producer could assume. MLPK is a better alternative for the same reasons with SLPK for both out-of-sample periods, which is consistent with the results that Garcia, Roh, and Leuthold (1995) and Tzang and Leuthold (1990) have found. Multiproduct hedging leads to the balance of larger reduction in variance and increase in return (or less decrease in return), indicating the importance of incorporating multiple price risk in the estimation of hedge ratios.

V. Conclusion

This study has examined the use of nonconstant and time-varying optimal hedge ratios for the hog industry. A nonparametric, locally polynomial kernel approach is used and compared to parametric approaches, BGARCH and MGARCH, and OLS models. Nonparametric models have not previously been applied to hedge ratio estimation, and used in price risk management. For the in-sample periods, SLPK is found to be a better alternative on the mean-variance frontier relative to naïve, GARCH and OLS.

MLPK is a better alternative for both out-of-sample periods because it shows good combinations of return and risk that a hog producer could assume. MGARCH performs nearly as well as MLPK. This study, however, suggests taking a special care when using MGARCH as a price risk management tool because one of the parametric assumptions, normality, is violated. Because of the potential assumption violations associated with the estimation and implementation of hedge ratios by GARCH models, LPK is a reasonable alternative for estimating hedge ratios to manage price risks.

This study suggests a new method, locally polynomial kernel, to estimate nonconstant hedge ratios, which is independent of parametric assumptions. This technique can have broad application to many types of agribusiness firms, and needs to be tested in other situations. Further study is currently underway to see whether or not LPK performs reasonably well compared to GARCH when normality is maintained.

Table 1. Estimated Hedge Ratios for Corn and Hogs (1990 – 1999)

		Single Commodity Hedge BGARCH(1,1) for GARCH		Multiproduct Hedge MGARCH(1,1) for GARCH	
		Corn	Hog	Corn	Hog
OLS	Hedge Ratio	0.95	0.91	0.95	0.91
	t-Ratio (β = 0)	36.99	10.07	36.19	9.85
	t-Ratio (β = 1)	1.86	0.89	1.89	0.90
LPK	Hedge Ratio	0.94	0.79	1.16	0.55
	t-Ratio (β = 0)	24.83	6.31	27.79	1.96
	t-Ratio (β = 1)	1.66	1.73	3.82	1.63
GARCH	Hedge Ratio	1.11	1.05	0.93	0.62
	t-Ratio (β = 0)	30.21	14.30	7.27	6.43
	t-Ratio (β = 1)	3.07	0.69	0.57	3.94

Table 2. Hedging Effectiveness, Live Hog Period

(The unit of return is dollar)

	In-Sample (1/3/1990 ~ 5/30/1995)				Out-of-Sample (6/1/1995 ~ 5/30/1996)			
	Return/hog	PI	Var/lb	PR	Return/hog	PI	Var/lb	PR
Unhedged	369.29		0.0874		385.39		0.0484	
Naïve	360.93	-0.0226	0.0477	0.4542	337.51	-0.1242	0.0276	0.4300
SLPK	364.23	-0.0137	0.0467	0.4657	316.88	-0.1778	0.0225	0.5351
BGARCH	361.01	-0.0224	0.0477	0.4542	337.35	-0.1247	0.0298	0.3843
SOLS	364.09	-0.0141	0.0567	0.3513	346.18	-0.1017	0.0324	0.3306
MLPK	364.00	-0.0143	0.0539	0.3833	357.88	-0.0714	0.0264	0.4545
MGARCH	364.26	-0.0136	0.0538	0.3844	367.82	-0.0456	0.0307	0.3657
MOLS	364.02	-0.0143	0.0564	0.3547	365.60	-0.0514	0.0322	0.3347

PI, PR and Var are percentage increase, percentage reduction, and variance, respectively. S and M denote single commodity hedging and multiproduct hedging, respectively.

Table 3. Hedging Effectiveness, Full Sample Period

(The unit of return is dollar)

	In-Sample (1/3/1990 ~ 5/30/1998)				Out-of-Sample (6/1/1998 ~ 6/30/1999)			
	Return/hog	PI	Var/lb	PR	Return/hog	PI	Var/lb	PR
Unhedged	364.41		0.1209		228.80		0.0813	
Naïve	360.32	-0.0112	0.0517	0.5724	275.87	0.2057	0.0500	0.3850
SLPK	361.47	-0.0081	0.0509	0.5790	287.59	0.2569	0.0508	0.3752
BGARCH	359.97	-0.0122	0.0510	0.5782	283.49	0.2390	0.0566	0.3038
SOLS	361.28	-0.0086	0.0590	0.5120	268.13	0.1719	0.0576	0.2915
MLPK	361.91	-0.0069	0.0699	0.4218	291.26	0.2730	0.0434	0.4662
MGARCH	361.92	-0.0068	0.0694	0.4260	258.78	0.1310	0.0420	0.4834
MOLS	361.27	-0.0086	0.0712	0.4111	268.56	0.1738	0.0538	0.3383

PI, PR and Var are percentage increase, percentage reduction, and variance, respectively. S and M denote single commodity hedging and multiproduct hedging, respectively.

Figure 1: Hedge Ratios for Corn Using LPK

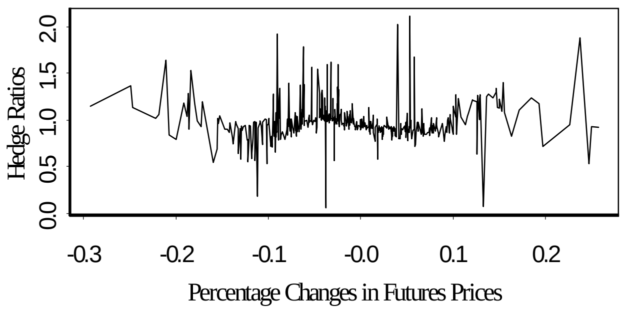


Figure 2: Hedge Ratios for Hogs Using LPK

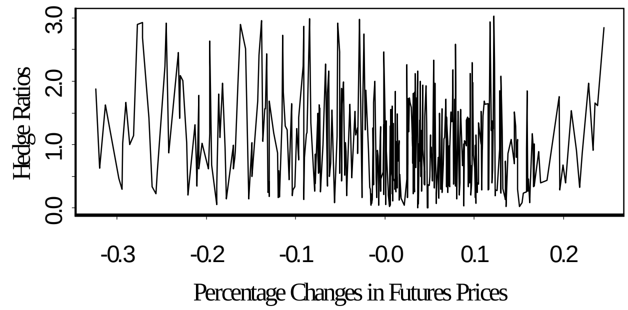


Figure 3: Hedge Ratios for Corn Using LPK

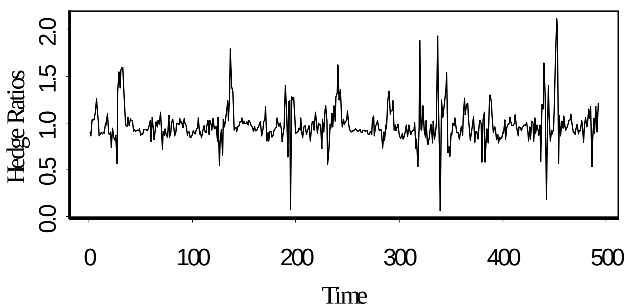


Figure 4: Hedge Ratios for Hogs Using LPK

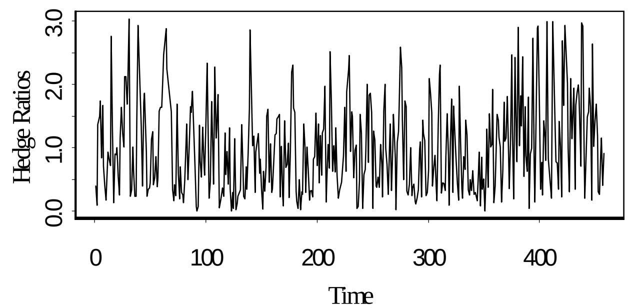


Figure 5: Hedge Ratios for Corn Using GARCH

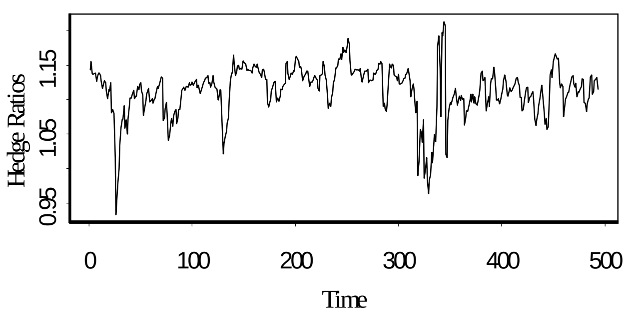


Figure 6: Hedge Ratios for Hogs Using GARCH

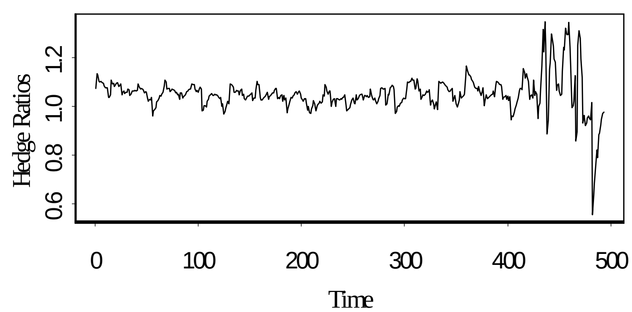


Figure 7: Out-of-Sample for Live Hog Period

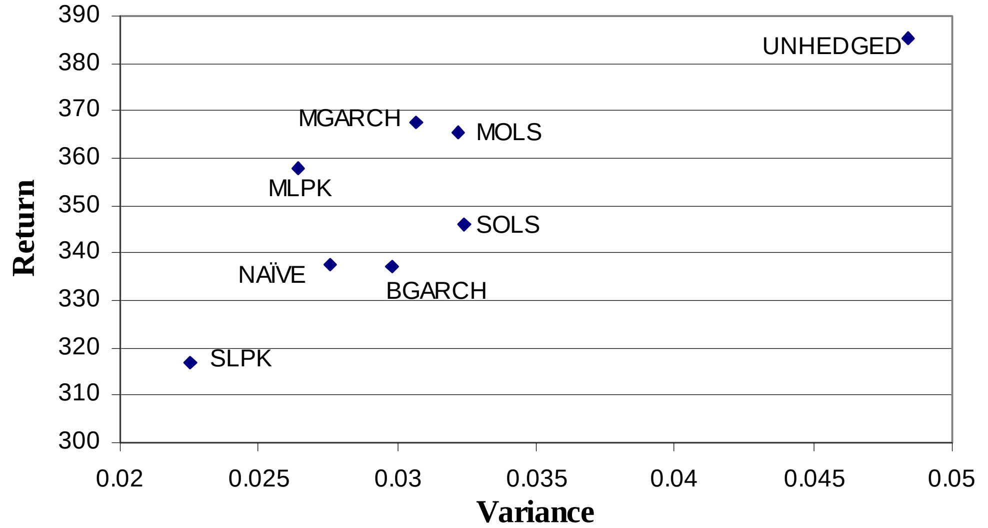
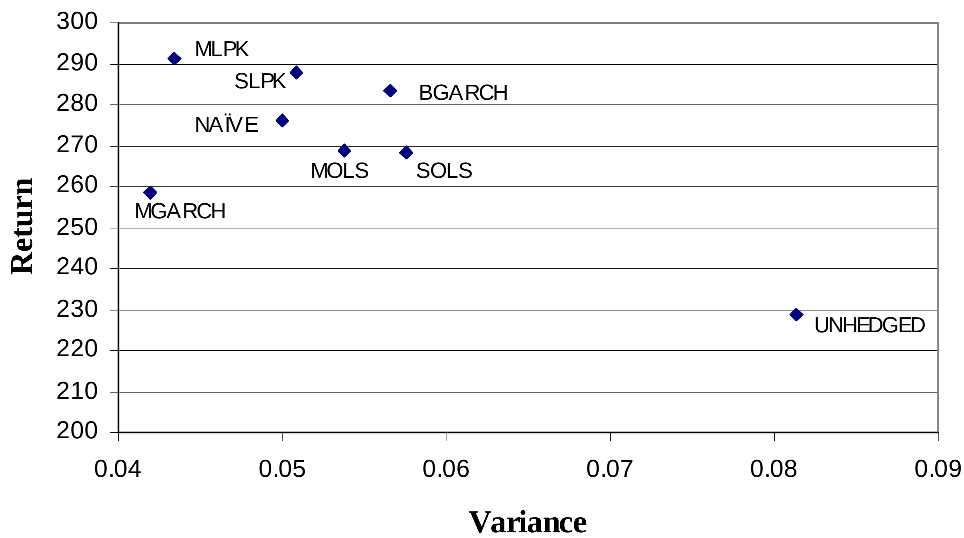


Figure 8: Out-of-Sample for Full Sample Period



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