

8-11-2012

# Decisions under Risk, Uncertainty and Ambiguity: Theory and Experiments

Jimmy Martinez-Correa  
*Georgia State University*

Follow this and additional works at: [http://scholarworks.gsu.edu/rmi\\_diss](http://scholarworks.gsu.edu/rmi_diss)

---

## Recommended Citation

Martinez-Correa, Jimmy, "Decisions under Risk, Uncertainty and Ambiguity: Theory and Experiments." Dissertation, Georgia State University, 2012.  
[http://scholarworks.gsu.edu/rmi\\_diss/29](http://scholarworks.gsu.edu/rmi_diss/29)

This Dissertation is brought to you for free and open access by the Department of Risk Management and Insurance at ScholarWorks @ Georgia State University. It has been accepted for inclusion in Risk Management and Insurance Dissertations by an authorized administrator of ScholarWorks @ Georgia State University. For more information, please contact [scholarworks@gsu.edu](mailto:scholarworks@gsu.edu).

## PERMISSION TO BORROW

In presenting this dissertation as a partial fulfillment of the requirements for an advanced degree from Georgia State University, I agree that the Library of the University shall make it available for inspection and circulation in accordance with its regulations governing materials of this type. I agree that permission to quote from, to copy from, or publish this dissertation may be granted by the author or, in his/her absence, the professor under whose direction it was written or, in his absence, by the Dean of the Robinson College of Business. Such quoting, copying, or publishing must be solely for the scholarly purposes and does not involve potential financial gain. It is understood that any copying from or publication of this dissertation which involves potential gain will not be allowed without written permission of the author.

Jimmy Martínez-Correa

## **NOTICE TO BORROWERS**

All dissertations deposited in the Georgia State University Library must be used only in accordance with the stipulations prescribed by the author in the preceding statement.

The author of this dissertation is:

Jimmy Martínez-Correa

35 Broad Street NW, 11th Floor, Atlanta, GA 30303

The director of this dissertation is:

Glenn W. Harrison

Center for the Economic Analysis of Risk, Department of Risk Management and Insurance

35 Broad Street NW, 11th Floor, Atlanta, GA 30303

Decision under Risk, Uncertainty and Ambiguity: Theory and Experiments

BY

Jimmy Martínez-Correa

A Dissertation Submitted in Partial Fulfillment of the Requirements for the Degree

Of

Doctor of Philosophy

In the Robinson College of Business

Of

Georgia State University

GEORGIA STATE UNIVERSITY  
ROBINSON COLLEGE OF BUSINESS  
2012

Copyright by  
Jimmy Martínez-Correa  
2012

## ACCEPTANCE

This dissertation was prepared under the direction of Jimmy Martínez-Correa's Dissertation Committee. It has been approved and accepted by all members of that committee, and it has been accepted in partial fulfillment of the requirements for the degree of Doctoral of Philosophy in Business Administration in the J. Mack Robinson College of Business of Georgia State University.

H. Fenwick Huss, Dean

### DISSERTATION COMMITTEE

Glenn W. Harrison

James C. Cox

Ajay Subramanian

Eric R. Ulm

## ABSTRACT

Decision under Risk, Uncertainty and Ambiguity: Theory and Experiments

BY

Jimmy Martínez-Correa

April 27, 2012

Committee Chair: Glenn W. Harrison  
Major Academic Unit: Department of Risk Management and Insurance

I combine theory, experiments and econometrics to undertake the task of disentangling the subtleties and implications of the distinction between risk, uncertainty and ambiguity. One general conclusion is that the elements of this methodological trilogy are not equally advanced. For example, new experimental tools must be developed to adequately test the predictions of theory. My dissertation is an example of this dynamic between theoretical and applied economics.

The distinction between risk, uncertainty and ambiguity is a subtle and important one for individual decision-making Knight (1921, p. 19-20) was the first to explicitly make a distinction between two of the three concepts:

The essential fact is that “risk” means in some cases a quantity susceptible of measurement, while at other times it is something distinctly not of this character; and there are far-reaching and crucial differences in the bearings of the phenomenon depending on which of the two is really present and operating...It will appear that a *measurable* uncertainty, or “risk” proper, as we shall use the term, is so far different from an *unmeasurable* one that it is not in effect an uncertainty at all. We shall accordingly restrict the term “uncertainty” to cases of the non-quantitative type.

The measurable nature that Knight (1921, p. 198) confers to risk corresponds to situations in which “...all the alternative possibilities [states of nature] are known and the probability of occurrence of each can be accurately ascertained.” On the contrary, *uncertainty* corresponds to situations in which the possible states of nature and/or the probability of occurrence of each state are not foreseeable with complete confidence.

Knight’s distinction between risk and uncertainty has been deemed operationally useless by some economists,<sup>1</sup> the most likely reason being the implications for the theory of Subjective Expected Utility (SEU) due to Savage (1972). A decision-maker that follows the postulates of SEU theory, in particular *Postulate 2* or the *Sure-Thing Principle*, should treat equally a 50:50 chance of winning a prize in the roll of a fair die and a subjectively assessed 50:50 chance of winning the same prize if the price of a stock goes up. It is in this

---

<sup>1</sup>An example of this claim is the introduction by Borch (1968, p. xiii) in the proceedings of a conference held in 1966 and called *Risk and Uncertainty*: “It was Frank H. Knight who first used ‘risk’ and ‘uncertainty’ as two different, well-defined concepts. His book *Risk, Uncertainty and Profit*, which appeared in 1921, opened the way for systematic studies of the uncertainty elements in economics, and Knight’s terminology has been widely accepted by a whole generation of economists. It seems, however, that it no longer serves any useful purpose to distinguish between risk and uncertainty.”

probabilistic sense that risk is identified with uncertainty in the theory of SEU:

independent of the nature of the event, two events should be treated equally as long as they are assigned the same probability weight.

Ellsberg (1961) was the first to challenge the latter implication of the SEU theory, adding a critical nuance between risk, uncertainty and ambiguity. He used thought experiments to characterize situations in which the risk-uncertainty identity may not be satisfied. In doing so he resuscitated some of the arguments of Knight (1921) and others. Ellsberg (1961) used thought experiments to characterize situations in which the risk-uncertainty identity may not be satisfied. Relying on the psychology literature, Ellsberg (1961) chose the term *ambiguity*<sup>2</sup> to describe situations under which it might not be desirable to behave according to the postulates of SEU theory, or to describe cases in which these principles themselves might be inadequate to predict decision-makers' behavior.

Ellsberg (1961, p. 657) argued that the violations of the postulates in Savage (1972) might be related to the *ambiguity* of information which, he defines as “a quality on the amount, type, reliability and ‘unanimity’ of information giving rise to one’s degree of ‘confidence’ in an estimate of relative likelihoods.” Ellsberg (1961, p. 657) also considers ambiguity to be a condition that lies between two polar cases: *complete ignorance* and *risk*. In the latter, “a subject is willing to base his decisions on a definite and precise choice of a particular distribution,” while in the former case, a “decision-maker lacks any information whatever on the relative likelihoods. In ambiguous situations, like the famous Ellsberg’s two-urns

---

<sup>2</sup>According to Hamilton (1957, p. 200) the term “[i]ntolerance to ambiguity’ was first conceptualized by Frenkel-Brunswik (1949) who defined it as ‘preference for familiarity, symmetry, definiteness and regularity,... a tendency towards black-white solutions, over-simplified dichotomizing, premature, unqualified either/or solutions...’

thought experiment, decision-makers do not know enough about the problem to rule out a series of possible probability distributions.

Ellsberg (1961) identified the *Sure-Thing Principle* as a key postulate to explain the type of violations of SEU theory that he studied. Segal (1987, 1990, 1992) argues that the Independence Axiom in the Expected Utility (EU) theory in von Neumann and Morgenstern (1953), which is closely related to the Sure-Thing Principle, is best viewed as a combination of two axioms: the Compound Independence Axiom and the Reduction of Compound Lotteries (ROCL) Axiom.<sup>3</sup> Segal (1987, 1990) also proposes that ambiguous situations can be characterized as a compound lottery where the first stage is over the possible probability distributions, each of which assigns weights to the relevant states of nature. In his framework ambiguity sensitive preferences do not satisfy ROCL under subjective probabilities. In fact, many contemporary models of decisions under ambiguity, such as Gilboa and Schmeidler (1989), Klibanoff et al. (2005), Ghirardato et al. (2004), share the same feature: under ambiguity people may have multiple priors, and even if subjects are able to define a non-degenerate subjective probability distribution over those priors, they will not behave as if they “boil down” the distribution to the weighted average probability.

One possible characterization of distinction between risk, uncertainty and ambiguity can be proposed.<sup>4</sup> Risk corresponds to situations in which objective probabilities or subjective probabilities, “tamed” by ROCL, can be assigned to the possible states of nature.

---

<sup>3</sup>See Harrison, Martínez-Correa and Swarthout [2012] for a formal definition of the three axioms.

<sup>4</sup>The following definitions are my interpretations of how Harrison (2011a, p. §4) defines each of the three concepts.

Uncertainty corresponds to situations in which subjective beliefs do not satisfy ROCL. Ambiguity corresponds to situations in which there is not enough information to form a unique subjective belief distribution or, in an extreme case, to define a probability distribution at all. In the rest of this dissertation, using this terminology, by referring to objective probabilities we refer to situations in the risk domain, and when dealing with subjective probabilities we refer to situations under uncertainty and ambiguity that may or may not be in the risk domain depending on the validity of ROCL for the decision maker. One can restate this characterization of the distinction between risk, uncertainty and ambiguity in a way that puts the focus squarely on the validity of the ROCL axiom. Risk is defined as any situation in which the decision maker behaves as if ROCL applies to well-defined objective or subjective belief distributions. Uncertainty can then be defined as any situation in which the decision-maker behaves as if a well-defined objective or subjective belief distribution exists, but that ROCL does not apply to it. Hence the whole distribution matters for behavior, not just the weighted average of the distribution. Ambiguity can then be defined as any situation in which the decision maker behaves as if no well-defined objective or subjective belief distribution exists, and ROCL cannot logically be applied.

This dissertation is a collection of interrelated essays that are building blocks to the theoretical and empirical study of the subtle implications of the distinction between risk, uncertainty and ambiguity (RUA). Chapter 1 is a byproduct of the topic that motivated this thesis (insurance) and the literature surveyed in the rest of the dissertation, and reviews the literature on behavioral insurance. Chapter 2 theoretically studies individual

decision-making towards risk management and insurance under ambiguity. Chapter 3, 4 and 5 build the path to empirically study decisions under uncertainty and ambiguity. These chapters focus on testing ROCL with objective probabilities and identifying the necessary methodologies to test its validity in the domain of subjective probabilities. The latter objective is a vital step to rigorously study decisions under uncertainty and ambiguity. The following paragraphs explained in more detail the main findings of each chapter.

Chapter 1 reviews the theoretical, experimental and econometric applications of different behavioral assumptions in the field of insurance that can help characterize the difference between RUA. This chapter defines some methodological guidelines for the emerging field of Behavioral Insurance. Chapter 2 theoretically studies the impact of ambiguity on risk management and insurance decisions, and the optimality of insurance contracts. The tractable approach used in this chapter allows one to study the interaction between risk and ambiguity attitudes. In particular, when insurance decisions are made independently of other assets, for a given increase in wealth, both risk and ambiguity attitudes interact in nontrivial ways to determine the change in insurance demand. When a non-traded asset is introduced, this model predicts behavior that is inconsistent with some implications of classical portfolio theory that assumes SEU theory. It is possible to characterize a counterexample to a classical result in insurance economics where an insurance contract with a straight deductible is dominated by a coinsurance contract. Finally, it is found that a modified Borch rule characterizes the optimal insurance contract with bilateral risk and ambiguity attitudes, as well as heterogeneity in beliefs.

Chapter 3 presents experimental evidence on the validity of ROCL in the domain of

objective probabilities. Subjects do not violate ROCL when they are presented with only one choice that is played for money. However, when subjects are provided with many choices and only one choice is chosen for payoffs, they violate ROCL. An important methodological conclusion, that has been made by others in similar studies,<sup>5</sup> is that payment protocols can create distortions in experimental tests of basic axioms of decision theory. This chapter is a necessary prelude to rigorously study ROCL with subjective probabilities, and Appendix A of the chapter outlines the implied experimental procedures and hypothesis testing methods to evaluate the validity of ROCL under subjective probabilities.

Chapter 4 evaluates the binary lottery procedure for inducing risk neutral behavior. The main result is that the binary lottery procedure works robustly to induce risk neutrality when subjects are given one risk task defined over *objective* probabilities. Similarly, Chapter 5 evaluates the binary lottery procedure for inducing risk neutral behavior in a *subjective* belief elicitation task. Therefore, this is a natural extension of Chapter 4 to the domain of subjective probabilities, and both contribute methodological “stepping stones” to the test of ROCL for subjective beliefs. Chapter 5 provides evidence which supports the hypothesis that the binary lottery procedure induces linear utility in a subjective probability elicitation task that uses the Quadratic Scoring Rule. We also show that the binary lottery procedure can also induce direct revelation of subjective probabilities in subjects with certain Non-Expected Utility preference representations. These last two chapters are important building blocks to study decisions under RUA because inducing

---

<sup>5</sup>See for example Cox et al. (2011) and Harrison and Swarthout (2012).

linear utility in subjects provides degrees of freedom to experimenters to directly study subjective beliefs. The dissertation combines theory, experiments and econometrics to undertake the task of disentangling the subtleties and implications of the distinction between risk, uncertainty and ambiguity. One general conclusion is that the elements of this methodological trilogy are not equally advanced. For example, new experimental tools must be developed to adequately test the predictions of theory. My dissertation is an example of this dynamic between theoretical and applied economics.

## ACKNOWLEDGEMENTS

I'd like to thank my parents who taught me the value of hard work and discipline. Also, I want to thank my brothers and the rest of the family for also giving me their unconditional love and support.

I want to especially thank my advisor and mentor, Glenn W. Harrison. His guidance, support and insights have been instrumental to my dissertation and my career.

I owe special thanks to Ajay Subramanian for choosing me 5 years ago as a new Ph.D student in the program. His constant, and almost fatherly, support helped me, and the rest of the Ph.D students in my year, through highs and lows.

I also want to thank Eric Ulm and James C. Cox for being part of my dissertation committee. The long conversations I had with them and their extremely insightful comments raised many issues that I tried to answer in this dissertation. Special thanks to Todd J. Swarthout whose meticulous and systematic view of economic experiments has been enlightening to me.

I would like to thank Richard Phillips for taking really good care of his Ph.D students and the rest of the department. Additionally I would like to thank those that made this dissertation financially possible. In particular, I want to thank the Center for the Economic Analysis of Risk, CEAR, which contributed with funds for the experiments in this dissertation and to present my work in academic forums worldwide. I also want to thank those who donated to the RMI department. Their support is key to the development of the profession and their example will be followed. I am indebted to my good friends Mark Schneider, Carmen Brown, Ellie Diaz and Bettye Walker who were always there for me.

I would like to extend my appreciation to those good colleagues, friends and mentors that in one way or another helped me to become a better professional and person. Special appreciation goes, in random order, to Juan Pablo Zárate, Lisa Rutström, Daniel Bauer, Roberto Junguito, Omer F. Baris, Menna Bizunne, Camilo Zea, Ana Fernanda Maiuashca, Diego Vásquez, David Salamanca, Nancy Zamudio, Dairo Estrada, Esteban Gómez, Oscar Martínez, Erin Coffman, Hernán Vallejo, Michel Janna, Juanita Gonzales, Juan Pablo Arango, and the rest of my friends from Banco de la República and Fasecolda.

I would like to thank the Ph.D gang: Henny Jung, Fan Liu, Xue Qi, Thorsten Moenig and Nan Zhu. We all made it!

I heartfelt thank-you to my old and good friends: Diego Panqueva (R.I.P), Julián Palomino, Andrés Ramírez, Rafael Arturo Ramírez and Claudia Estrada-Hart.

My final and biggest thank-you goes to the love of my life, my wife Adriana, for her love and laughs that are my daily support.

# TABLE OF CONTENTS

<b>ABSTRACT</b> . . . . .	<b>vi</b>
<b>ACKNOWLEDGEMENTS</b> . . . . .	<b>xiv</b>
<b>LIST OF FIGURES</b> . . . . .	<b>xxiii</b>
<b>LIST OF TABLES</b> . . . . .	<b>xxv</b>
<b>1 Behavioral Insurance: A Survey</b> . . . . .	<b>1</b>
1.1 Introduction . . . . .	2
1.2 Preliminaries on Behavioral Economics . . . . .	3
1.2.1 Stylized Facts . . . . .	3
1.2.2 Straw Men . . . . .	5
1.2.3 Behavioral Econometrics . . . . .	5
1.2.4 Hypothetical Bias . . . . .	7
1.2.5 wwSs: What Would Savage Say? . . . . .	9
1.3 Theoretical Models . . . . .	11
1.3.1 Risk . . . . .	11

1.3.2	Uncertainty and Ambiguity . . . . .	36
1.4	Experimental Evidence . . . . .	43
1.4.1	Estimating Preferences and Beliefs . . . . .	45
1.4.2	Letting the Data Speak for Itself . . . . .	49
1.4.3	General Methodological Issues for Experiments in Insurance Contexts	50
1.5	Further Topics . . . . .	55
1.5.1	Intertemporal Insurance: the Annuity Puzzle . . . . .	55
1.5.2	Information Problems . . . . .	63
1.5.3	Asset Integration . . . . .	64
1.5.4	Correlation Aversion . . . . .	66
1.5.5	Conclusions . . . . .	67
1.5.6	Appendix A: Basic Axioms . . . . .	67
<b>2</b>	<b>Insurance Decisions Under Ambiguity . . . . .</b>	<b>69</b>
2.1	Introduction . . . . .	70
2.2	Modeling Ambiguity and Attitudes towards Ambiguity . . . . .	76
2.3	Insurance Choices under Ambiguity . . . . .	80
2.3.1	Model for Coinsurance Demand under Ambiguity . . . . .	82
2.3.2	Model for Coinsurance Demand in the Presence of Ambiguity and a Non-traded Asset . . . . .	89
2.4	Optimal Insurance Contracts Under Ambiguity . . . . .	110

2.4.1	Optimality under Ambiguity of Insurance Contracts with a Straight Deductible: A Counterexample . . . . .	110
2.4.2	The Borch Rule Under Ambiguity . . . . .	111
2.5	Conclusions . . . . .	114
2.6	Appendix A. Proofs of Results . . . . .	115
2.7	Appendix B. Baseline Model: Insurance Demand under Risk . . . . .	122
2.8	Appendix C. Insurance Demand in the Presence of a Non-traded Asset under Risk . . . . .	125
<b>3</b>	<b>Reduction of Compound Lotteries with Objective Probabilities: Theory and Evidence . . . . .</b>	<b>127</b>
3.1	Introduction . . . . .	128
3.2	Theory . . . . .	131
3.2.1	Experimental Payment Protocols . . . . .	134
3.3	Experiment . . . . .	135
3.3.1	Lottery Parameters . . . . .	135
3.3.2	Experimental Procedures . . . . .	138
3.3.3	Evaluation of Hypotheses . . . . .	141
3.4	Non-Parametric Analysis of Choice Patterns . . . . .	143
3.4.1	Choice Patterns Where ROCL Predicts Indifference . . . . .	143
3.4.2	Choice Patterns Where ROCL Predicts Consistent Choices . . . . .	147
3.5	Estimated Preferences from Observed Choices . . . . .	153

3.5.1	Econometric Specification . . . . .	154
3.5.2	Estimates . . . . .	161
3.6	Conclusions . . . . .	165
3.7	Appendix A. Reduction of Compound Lotteries with Subjective Probabilities: Theoretical, Experimental and Methodological Issues . . . . .	171
3.7.1	Theoretical Issues . . . . .	172
3.7.2	Experiments . . . . .	179
3.7.3	Conclusions . . . . .	183
3.8	Appendix B. Instructions . . . . .	185
3.8.1	Instructions for Treatment 1-in-1 . . . . .	185
3.8.2	Instructions for Treatment 1-in-40 . . . . .	189
3.9	Appendix C. Parameters . . . . .	193
3.10	Appendix D. Related Literature . . . . .	203
3.11	Appendix E. Nonparametric Tests . . . . .	206
3.12	Appendix F. Additional Econometric Analysis . . . . .	215
3.12.1	Analysis of Data from the 1-in-1 Treatment . . . . .	215
3.12.2	Analysis of Data from the 1-in-40 Treatment . . . . .	215
3.13	Appendix G. Detailed Binomial Test Results . . . . .	220
3.13.1	Binomial tests for treatment 1-in-1 . . . . .	220
3.14	Appendix H. Detailed Fisher Exact Test Results . . . . .	222
3.15	Appendix I. Detailed McNemar Test Results . . . . .	225
3.16	Appendix J. Detailed Wald Test Results for Predictions . . . . .	234

<b>4</b>	<b>Inducing Risk Neutral Preferences with Binary Lotteries: A Reconsideration</b>	<b>238</b>
4.1	Introduction	239
4.2	Literature	242
4.2.1	Literature in Statistics	242
4.2.2	Literature in Economics	244
4.3	Theory	252
4.4	Experiment	254
4.5	Results	263
4.5.1	Do Subjects Pick the Lottery With the Higher Expected Value?	264
4.5.2	Effect on Expected Value Maximization	266
4.5.3	Effect on Expected Value Maximization	267
4.6	Conclusions	271
4.7	Appendix A. Instructions	273
4.8	Appendix B. Parameters of Experiments	281
4.9	Appendix C. Structural Estimation of Risk Preferences	283
<b>5</b>	<b>Eliciting Subjective Probabilities with Binary Lotteries</b>	<b>291</b>
5.1	Introduction	292
5.2	Theoretical Issues	295
5.2.1	Binary Scoring Rules for Subjective Probabilities	295

5.2.2	Subjective Belief Elicitation with Scoring Rules and the Binary Lottery	
	Procedure . . . . .	297
5.2.3	Non-Expected Utility Theory Preference Representations . . . . .	302
5.3	Experiments . . . . .	305
	5.3.1 Experimental Design . . . . .	305
	5.3.2 Evaluation of Hypothesis . . . . .	310
5.4	Results . . . . .	312
	5.4.1 Does the BLP Mitigate the Effects of Risk Aversion? . . . . .	312
	5.4.2 Does the BLP Improve Accuracy? . . . . .	315
5.5	Conclusions . . . . .	316
5.6	Appendix A. Instructions . . . . .	321
	<b>Bibliography . . . . .</b>	<b>332</b>

# LIST OF FIGURES

Figure 1.1	Symmetric Subjective Probability Distributions . . . . .	37
Figure 1.2	Asymmetric Subjective Probability Distributions . . . . .	38
Figure 1.3	ROCL at Work . . . . .	39
Figure 2.1	Portfolio Problem Interpretation of a Simple Insurance Decision . . . . .	97
Figure 2.2	Optimal Choice in the Risky Domain in the Presence of Ambiguity . . . . .	99
Figure 2.3	Optimal Choice in the Ambiguity Domain . . . . .	100
Figure 2.4	Risk/Ambiguity Trade-off and Optimal Choice . . . . .	102
Figure 3.1	Battery of 40 Lotteries Pairs Probability Coverage . . . . .	136
Figure 3.2	Tree Representation of a Compound Lottery and its Corresponding Actuarially-Equivalent Simple Lottery . . . . .	137
Figure 3.3	Choices Over Compound and Actuarially-Equivalent Lotteries . . . . .	140
Figure 3.4	Choices Over Simple and Compound Lotteries . . . . .	140
Figure 3.5	Choices Over Simple and Actuarially-Equivalent Lotteries . . . . .	141
Figure 3.6	Distribution of Parameter Estimates from the RDU Specification in the 1-in-1 Treatment Assuming Heterogeneity in Preferences . . . . .	168

Figure 3.7	Distribution of Parameter Estimates from the RDU Specification in the 1-in-40 Treatment Assuming Heterogeneity in Preferences . . .	170
Figure 3.8	Illustrative Subjective Beliefs . . . . .	172
Figure 3.9	Generalized Scoring Rule Using the Binary Lottery Procedure . . .	178
Figure 3.10	Distributional Scoring Rule Initialization . . . . .	183
Figure 3.11	Illustrative Distributional Scoring Rule Response . . . . .	183
Figure 3.12	Binary Scoring Rule Initialization . . . . .	184
Figure 3.13	Illustrative Binary Scoring Rule Response . . . . .	184
Figure 3.14	Default Simple Lotteries . . . . .	199
Figure 3.15	Estimated Functions from the RDU Specification in the 1-in-1 Treatment Assuming Homogeneity in Preferences . . . . .	218
Figure 3.16	Estimated Functions from the RDU Specification in the 1-in-1 Treatment Assuming Homogeneity in Preferences . . . . .	219
Figure 4.1	Default Binary Choice Interface . . . . .	256
Figure 4.2	Choice Interface for Points . . . . .	258
Figure 4.3	Choice Interface for Points with Expected Value Information . . .	262
Figure 4.4	Estimated Risk Attitudes in Treatments A and B . . . . .	270
Figure 4.5	Estimated Risk Attitudes in Treatments C and D . . . . .	271
Figure 5.1	Binary Scoring Rule Using the Binary Lottery Procedure . . . . .	299
Figure 5.2	Subject Display for Treatment M . . . . .	307
Figure 5.3	Subject Display for Treatment P . . . . .	308

Figure 5.4	Frequency of Reports by Session . . . . .	314
Figure 5.5	Estimated Densities of Reports by Session . . . . .	315
Figure 5.6	Empirical Cumulative Distribution of Distance of Reports from 50	319
Figure 5.7	Empirical Cumulative Distribution of Distance Pooling Data From All Sessions . . . . .	320

# LIST OF TABLES

Table 3.1	Default Simple Lotteries . . . . .	138
Table 3.2	Generalized Fisher Exact Test on the Actuarially-Equivalent Lottery vs. Compound Lottery Pairs . . . . .	145
Table 3.3	Binomial Probability Tests on Actuarially-Equivalent Lottery vs. Com- pound Lottery Pairs . . . . .	146
Table 3.4	Cochran Q Test on the Actuarially-Equivalent Lottery vs. Compound Lottery Pairs . . . . .	147
Table 3.5	Fisher Exact Test on Matched Simple-Compound and Simple-Actuarially- Equivalent Pairs . . . . .	150
Table 3.6	Bonferroni-Dunn Method on Matched Simple-Compound and Simple- Actuarially-Equivalent Pairs . . . . .	152
Table 3.7	McNemar Test on Matched Simple-Compound and Simple-Actuarially- Equivalent Pairs . . . . .	153
Table 3.8	Estimates of Source-Dependent RDU and EUT Models Allowing for Heterogeneity . . . . .	167

Table 3.9	Estimates of Source-Dependent RDU and EUT Models Allowing for Heterogeneity . . . . .	169
Table 3.10	Default Simple Lotteries . . . . .	196
Table 3.11	Simple Lotteries vs. Compound Lotteries (Pairs 1-15) . . . . .	200
Table 3.12	Simple Lotteries vs. Actuarially-Equivalent Lotteries (Pairs 16-30)	201
Table 3.13	Actuarially-Equivalent Lotteries vs. Compound Lotteries (Pairs 31-40) . . . . .	202
Table 3.14	Estimates of Source-Dependent RDU and EUT Model Allowing for Heterogeneity . . . . .	216
Table 3.15	Estimates of Source-Dependent RDU and EUT Model Allowing for Heterogeneity . . . . .	217
Table 4.1	Experimental Design . . . . .	255
Table 4.2	Observed Choice Patterns . . . . .	265
Table 4.3	Battery of Monetary Lotteries . . . . .	281
Table 4.4	Battery of Binary Lotteries . . . . .	282
Table 4.5	Estimation Results for Treatments A and B . . . . .	288
Table 4.6	Estimation Results for Treatments A, B, E and F . . . . .	289
Table 4.7	Estimation Results for Treatments C and D . . . . .	290
Table 5.1	Experimental Design . . . . .	306

# Chapter 1

## Behavioral Insurance: A Survey

*by Glenn W. Harrison and Jimmy Martínez-Correa*

### **Abstract**

Behavioral insurance applies alternative models of human behavior to insurance decisions. The use of alternative models allows an evaluation of the effect of different behavioral assumptions on inferences about insurance markets. We consider theoretical, experimental and econometric applications of different behavioral assumptions. The literature is reviewed and some gaps filled, providing a systematic overview of the field of behavioral insurance. We consider the effect of allowing for rank-dependent probability weighting, loss aversion, ambiguity and the presence of traded and non-traded assets on the classic theorems of insurance economics. We review experimental evidence on different facets of insurance from the laboratory and the field, including randomized control trials of alternative insurance products. Finally, we review the subtle econometric issues that arise when estimating models of behavior towards insurance, particularly when using naturally occurring data. A unifying theme of our review is the importance of allowing for alternative behavioral assumptions when undertaking welfare evaluations of insurance products and regulatory policies. Our review also contains numerous cautions about the subtleties of theoretical, experimental and econometric work in the field of behavioral insurance.

## 1.1 Introduction

The generic insurance product involves an agent giving up a certain amount of money ex ante some event in the expectation of being given some money in the future if something unfortunate occurs. Evaluation of the benefits of the product require that one specify risk and time preferences of the agent, since the benefits of the product are risky, and in the future, while the costs are normally certain and up front. We must also know the subjective beliefs that the agent used to evaluate the product, and then make some assumptions, explicit or not, with respect to uncertainty aversion or ambiguity aversion. Of course, there is a “revealed preference” argument that if the product is (not) taken up it was perceived to be a positive (negative) net benefit. But that is only the starting point of any serious evaluation of the product. From a normative perspective, what if the subjective beliefs were off, in the sense that the individual would revise them if given certain information? Behavioral insurance approaches this generic characterization of the insurance product by allowing a wide range of assumptions about behavior. Since the evolving literature on behavioral insurance grew out of experimental economics, behavioral economics and behavioral finance, we make some general comments about the nature of research in those areas as a necessary “preliminary” to detailed discussion of insurance. The norms of research, including standards of rigor in statements of theory and empirical tests, differs widely in these areas, and we take a clear stance on what methodologies should be employed. In particular, we stress the role of theory, rigorous experimental design, and structural econometrics. It is described the role of insurance discussions in the development

of decision theory under risk and uncertainty as well as the frontier of behavioral insurance. We emphasize that this survey focuses on the behavioral foundations of individual decision making under symmetric information. One of the purposes is then to survey the different individual behavioral assumptions which, with respect to classical approaches, might have very different implications for market and aggregate behavior in the presence of informational problems like moral hazard and adverse selection.

## 1.2 Preliminaries on Behavioral Economics

Behavioral economics is often presented as putting “people” back into economics, in the sense that the received paradigm had no essential role for agents that anyone would ever recognize as being human.

### 1.2.1 Stylized Facts

One characteristic of behavioral economics is a reliance on some “stylized facts” about behavior that are not as firmly established as one might like. Of course it is good to start with evidence, but sometimes the evidence is not available without the help of a theoretical structure or specific experimental procedures that bring with them potential confounds.

Data can sometimes be easy to measure, but rarely is it easy to interpret rigorously.

To illustrate the role of theory, if one pays subjects for one of  $K$  choice tasks chosen at random, inferences about the validity of expected utility theory require that one maintain the (mixture) independence axiom. There is an obvious circularity in having to invoke an

axiom that one then wants to reject. Many behavioral studies invoke something referred to as “the isolation effect,” which is often a *behavioral* assertion that a subject views each choice in an experiment as independent of other choices in the experiment. When used formally, this hypothesis is usually the same as the (mixture) independence axiom.

Nonetheless, appeal to the “isolation effect” is often invoked informally as “an empirical matter,” much as a magic talisman is used to ward off evil spirits.

To illustrate the role of experimental procedures, consider ground zero, the Allais Paradox. It is now well documented that experimental subjects simply do not fall prey to the Allais Paradox like decision-making lemmings when one presents the task for real payments and drops the word “millions” after the prize amount: see Conlisk (1989), Harrison (1994), Burke et al. (1996) and Fan (2002). Subjects appear to crank out the EV when given real tasks to perform, and the vast majority behave consistently with EUT as a result. This is not to claim that all anomalies or stylized facts are untrue, but there is a casual tendency in the behavioral economics literature to repeatedly assume stylized facts that are simply incorrect.

With respect to expected utility theory (EUT), the evidence suggests that some subjects violate the assumptions in some settings, but it is not universal. Mixture models allow one to identify the fraction of the population that violate EUT, and recent estimates place that around 50% in developed countries and undeveloped countries (Harrison and Rutström, 2008; Harrison et al., 2010). With respect to time preferences, there is little evidence for “hyperbolicky” discounting when one uses samples representative of the population rather than students, and stakes that avoid artefactual effects of rounding errors (Andersen et al.,

2011a,b). If one does use student populations, then there is again evidence for roughly 50:50 mixtures of exponential discounting and hyperbolic discounting (Coller et al., 2012).

### 1.2.2 Straw Men

The second issue is the misrepresentation of received theory. Studies that claim to use the expression *homo economicus* provide a clear sign in an academic context that someone is engaged in polemics rather than science. The details of some prominent examples are provided by Harrison (2011b, §3), and need not be aired yet again, but the manner in which real economists have abused the definition of “received theory” is staggering.

Actually, what is staggering is the manner in which this has been tolerated by consumers that know better. We prefer to address this by constructive discussion of what we can model, with care and qualification.

We also strictly avoid attaching the label “rational” to any set of axioms or theoretical structures. This is not because we have no interest in normative judgments, but simply to avoid a counterproductive bog of rhetoric.

### 1.2.3 Behavioral Econometrics

Beware of behavioral economists claiming to be able to estimate a latent structural parameter with one or two questions. One cannot say what the probability weighting parameter is without saying something about the utility function. Similarly, one cannot say what the loss aversion parameter of Prospect Theory is without saying something about probability weighting and utility curvature. And then one has to allow probability

weighting and utility curvature to differ over gains and losses. These all involve delicate structural inferences, which cannot be avoided by not caring for rigor.

For example, Fehr and Goette (2007) estimate a loss aversion parameter using what we will call a Blind Loss Aversion model of behavior, “extending” the Myopic Loss Aversion model of Benartzi and Thaler (1995). They ask subjects to consider two lotteries, expressed here in equivalent dollars instead of Swiss Francs:

*Lottery A:* Win \$4.50 with probability  $1/2$ , lose \$2.80 with probability  $1/2$ .

Otherwise get \$0.

*Lottery B:* Play six independent repetitions of lottery A. Otherwise get \$0.

Subjects could participate in both lotteries, neither, or either. Fehr and Goette (2007) assume that subjects have a linear utility function for stakes that are this small, relying on the theoretical arguments of Rabin (2000). They also assume that there is no probability weighting: even though Quiggin (1982, p. §4) viewed  $1/2$  as a plausible fixed point in probability weighting, most others have assumed or found otherwise. If one is blind to the effects of curvature of the utility function and probability weighting then the only thing left to explain choices over these lotteries is loss aversion. On the other hand, it becomes “heroic” to then extrapolate those estimates to explain behavior that one has elsewhere (p.304) assumed to be characterized by stakes large enough that strictly concave utility is plausible a priori. Of course, the preferred model (p.306) assumes away concavity and only uses the loss aversion parameter, but without explanation for why behavior over such stakes should be driven solely by loss aversion instead of risk attitudes more generally.<sup>1</sup>

---

<sup>1</sup>We use the term “risk attitudes” here in the broader sense of including possible effects from nonlinear

A second difficulty in behavioral econometrics is a tendency to use Ordinary Least Squares (OLS) for, well, everything. In some cases this is a safe and easy descriptive device, but in many other cases it can be dangerous. Imagine a setting in which subjects are choosing a portfolio allocation and over 95 data-generating process for the “now that I am contributing, how much” decision. And so on. These are all familiar issues, which we just conveniently forget.

A final difficulty in some quarters is the use of “parameter calibration” to fit a model, perhaps using some squared-error metric. This can be justifiable as approximating the point estimates from maximum likelihood tolerably well in many settings, but avoids any mention of sampling errors. Matters are compounded by these calibrated point estimates being used for hypothesis testing (e.g., a  $t$ -test on a vector of point estimates).

If academics do not object to statistical practices of this kind loudly and repeatedly in seminars, and they do not, then we get the quality of empirical behavioral research we pay for.

### 1.2.4 Hypothetical Bias

It is a pity to have to keep saying this, but hypothetical bias exists. In general one gets different answers if one asks hypothetical questions than if one asks comparable questions with real consequences. The abuse of experimental methods and results here is not due to subtle methodological error, but just plain bad scholarship that is uncritically tolerated.

One common *modus operandi* in behavioral economics is to cite some casual, hypothetical utility functions, probability weighting and loss aversion.

study of behavior that finds some odd and startling things, and then briefly mention that the same results are obtained when one uses real incentives, e.g., the literature on discounting anomalies, such as (Loewenstein and Prelec, 1992, §II). The problems here are not fixed by thinking more carefully from a methodological perspective, but by simply calling a spade a spade.

One also has to object that there have been decades of mischievous debate about hypothetical bias that has diverted resources inefficiently. We stress that this is of immediate policy significance in the area of environmental damage assessment (Harrison, 2006), and not just a procedural issue for backwater debate between experimenters. By all means do a hypothetical study to scope out an issue, if you must and are lazy, but don't leave it there if something interesting is discovered. Don't misquote assertions that one gets the same results: in general, and that is the key, one does not (Harrison and Rutström, 2008). Read (2005) is a wonderful, blunt satire of the arguments in favor of using hypothetical rewards, although some might read him as seriously advocating the positions he so brilliantly undermines.

The influential survey of Camerer and Hogarth (1999) is widely misquoted as concluding that there is no evidence of hypothetical bias in lottery choices. What Camerer and Hogarth (1999) actually conclude, quite clearly, is that the use of hypothetical rewards makes a difference to the choices observed, but that it does not generally change the inference that they are led to draw about the validity of EUT. Since the latter typically involves paired comparisons of response rates in two lottery pairs (for example, in common ratio tests), it is logically possible for there to be (1) differences in choice probabilities in a

*given lottery* depending on whether one uses real or hypothetical responses, and (2) no difference between the effect of the EUT treatment on lottery pair responses depending on whether one uses real or hypothetical responses.

Furthermore, Camerer and Hogarth (1999) explicitly exclude from their analysis the mountain of data from experiments on valuation that show hypothetical bias. Their rationale for this exclusion was that economic theory did not provide any guidance as to which set of responses was valid. This is an odd rationale, since the mere fact that hypothetical and real valuations differ so much tells us that at least one of them is wrong. Thus one does not actually need to identify one as reflecting “true preferences,” even if that is an easy task *a priori*, in order to recognize that there are differences in behavioral between hypothetical and real responses.

### **1.2.5 wwSs: What Would Savage Say?**

One of the great, and lasting, contributions of behavioral economics is that we now have many rich models of behavior, potentially allowing structural understanding of insurance behavior.

But we also realize that there are some basic confounds to reliable inference about behavior. These are not side technical issues. Risk attitudes can involve more than diminishing marginal utility, and we have no significant problems identifying alternatives paths to risk aversion through probability weighting. Loss aversion is much more fragile, until we can claim to know the appropriate reference points for agents. Time preferences can be characterized, and appear to hold fewer problems than early experimental studies

with lab subjects suggest.

But the 600 pound gorilla confound is the subjective belief that decision-makers hold in many settings. This is the one that is widely ignored.<sup>2</sup> What Savage (1971, 1972) showed was that, under some (admittedly strong) assumptions, one could infer a subjective probability and a utility function from observable behavior. The subjective probability and the utility function would each be well-behaved in the classical senses, but one could not, in general, make claims about the one without knowing or assuming something about the other.<sup>3</sup>

The suggestion is not that casual assumptions about possible subjective beliefs should be used to rationalize “rational behavior” in every setting, but that inferences about cognitive failures, and the need for nudges, hinge on our descriptive knowledge of what explains behavior. If we rule out some factor, then something else may look odd, just as a balloon bulges on one side if you press it on the other side. To take a simple example, assume that there is a risk premium, but one uses either a model that assumes that 100 percent of the observed behavior is due to diminishing marginal utility or a model that assumes that 100 percent of the observed behavior is due to probability pessimism. The first model will generate concave utility functions and impose zero probability weighting, and the second model will generate convex probability weighting functions and impose linear utility

functions: both will likely explain the risk premium tolerably well. But the two models can

---

<sup>2</sup>An important exception, from the behaviorist side of the Force, is Köszegi and Rabin (2008). They write (p.196ff.) about the “impossibility of Skinnerian welfare economics” in the absence of measurement of subjective beliefs.

<sup>3</sup>Machina and Schmeidler (1992) provide an important extension to show that “probabilistic sophistication,” in the sense of making decisions with underlying probabilities that obey the usual axioms of probability, did not require EUT. In particular, Rank-Dependent Utility models, with increasing probability weighting functions, would suffice.

have very different implications for the design of development policy that “works” in any interesting sense.

Of course, in some settings it is simply not possible to “go back to the well” and elicit information of this kind. However, there is no reason why one cannot use information from one sample, even from a different population if necessary, to condition inferences about another sample, to see the effect.

## **1.3 Theoretical Models**

We focus on the core setting in which there are no asymmetric information problems in the design of insurance. We later discuss extensions, but there are enough complexities in this core setting to warrant careful review.

### **1.3.1 Risk**

#### **Risk Attitudes, Utility Functions and Insurance Demand**

Bernoulli (1738, p. 29-30) is perhaps one of the first references in the literature analyzing insurance and its rationality both for the person offering the insurance coverage and the one buying it. Bernoulli used an expected utility argument, that was later axiomatized by von Neumann and Morgenstern (1953), vNM henceforth, to calculate the threshold in personal wealth that provides a rationale for a person to buy or sell insurance of commodities shipped by sea.

Bernoulli’s work is best known for explaining the “St. Petersburg paradox,” which

consisted in a person foregoing the possibility of playing a gamble with infinite expected value for a finite and not very large amount. He proposed that individuals make choices according to the “moral expectation” or expected utility of uncertain alternatives, where a concave logarithmic utility function is assumed to model the utility of wealth. This was a novel approach in his time when expected value was the accepted normative approach to make decisions between uncertain prospects. One assumption implicit in Bernoulli’s moral expectation argument is that the concavity of the utility function implies decreasing marginal utility, an still widely accepted behavioral assumption. However, by the 1950’s many academics were puzzle by observed behavior, such as people gambling, which contradicts the generality of decreasing marginal utility. For example, ubiquitous decreasing marginal utility would imply that a person would always reject a fair gamble in which it is equiprobable to win or lose one dollar. This brought into question the validity of EUT, even though decreasing marginal utility is not a requirement in the system of axioms in vNM.

Friedman and Savage (1948, p. 282) provided an explanation to seemingly puzzling observed choices such as people with lower income buying insurance but simultaneously buying lottery tickets. This is an issue that was extensively debated and it was considered that a “theory of uncertainty must account for the presence of both” (Arrow, 1951, p. 407). Friedman and Savage considered that the contradictory behavior can be explained within the classical EUT framework of vNM, if one departs from the assumption that individuals exhibit decreasing marginal utility at all levels of income, which is not a requirement at all in the vNM framework, but keeps the principle of expected utility maximization. Their

strategy to explain the contradictory behavior is to allow for a utility function that exhibits decreasing marginal utility at lower levels of income, and increasing marginal utility at higher levels of income. These imply a utility function that is concave for lower levels of income and convex for higher levels of income. A utility function with these features would explain why people choose to buy insurance to protect themselves from relatively small losses, but buy lottery tickets that could imply attaining relatively high levels of income. Markowitz (1952, p. 155) advanced a relatively similar hypothesis to explain the joint observance of insurance purchasing and gambling behavior. Markowitz also proposed a utility function of wealth that does not exhibit decreasing marginal utility everywhere, as in Friedman and Savage (1948), but that avoids counterintuitive implications of their approach. For example, the explanation in Friedman and Savage (1948) would imply that a sufficiently rich man would be willing to offer insurance even if this means losses in expected value terms or a sufficiently poor individual never gambling, both being implications that are quite debatable. Markowitz (1952, p. 156) also pointed out that his explanation implies a preference for positively skewed distributions of wealth. This means that individuals would exhibit a preference for distributions that offer low probability of very high realizations of wealth and relatively high probability of lower levels of wealth; in other words, the opportunity of a long shot is very valued by subjects.

The concave-convex nature of the utility was early tested in the seminal work of Mosteller and Noguee (1951, p. 404) who concluded that "... there is some support for the inflection-point analysis offered by Friedman and Savage, although this support is not wholly satisfactory- i.e., there is no contradiction, but the support is meager." Markowitz's

amendment to the Friedman-Savage utility function was tested by Weitzman (1965, p. 26) who used aggregate data from horse bets and found that the “results of this study show that the crowd at the race track behaves as if it were composed of a group of individuals each of whom possesses an identical utility function of the Markowitz variety.” However, Yaari (1965b) showed in an experimental setting that one doesn’t need to introduce the non-convexity<sup>4</sup> in individuals’ preferences to explain the co-existence of gambling and insurance in the subjects’ portfolio. Yaari (1965b, p. 290) found that attitudes towards probabilities played a role and that “one finds that some subjects tend to overstate low probabilities and to understate high probabilities.”

Quiggin (1993) provided a similar explanation to the insurance-gamble puzzle but within the Rank-Dependent Utility (RDU) model. He proposes (p. 88ff.) an explanation that involves both optimism towards probabilities (concavity in the probability weighting function) and outcome risk aversion (concave utility function):

Assuming that  $U$  is concave, the choices of an optimist facing uncertainty will reflect a balance between two conflicting forces. Optimism encourages risk taking but decreasing marginal utility discourages it. Since the effects of probability weighting are independent of the scale of the bet, optimism will tend to predominate when bets are small (that is, when all the outcomes are near the current wealth level). Outcome risk aversion will tend to predominate when bets are large. For example, an optimist faced with the opportunity to participate in an actuarially fair bet will always wish to put some money on (since EU analysis shows that behavior approaches risk neutrality as distributions collapse to the mean). However, as the stake increases outcome risk aversion will start to play a role. Thus, with small stakes, small winning probabilities and large prizes, there will be a trade-off between optimism and outcome risk aversion.

Machina (1982, p. 280-282) proposed the *fanning out* hypothesis which is capable of also

---

<sup>4</sup>We mean convexity in the set-theoretic sense of the word.

explaining the insurance-gambling puzzle by allowing subjects to have a tendency to prefer positively skewed distributions. For example, a subject exhibiting preferences that fan out would prefer to buy a lottery that induces a positively skewed distribution of wealth, if the initial wealth is assumed to be certain. Similarly, the same subject can exhibit a preference for insurance since in the simplest case it can transform a negatively skewed distribution into a symmetric certain one that assigns probability one to a given level of wealth.

Consequently, buying a lottery or insurance induce a distribution of wealth that is relatively positively skewed with respect to the initial distribution. Thus a subject satisfying the fanning out hypothesis of preferences can explain the seemingly contradicting the observation that people that buy insurance also gamble on lotteries. It is worth mentioning that this observation can be explained within the assumption of general risk aversion by allowing for indivisibility of expenditure and goods (Ng, 1965; Flemming, 1969), commitments in expenditure (Chetty and Szeidl, 2007; Chen and Mahani, 2011), making consumption and not wealth the argument of the utility function and introducing a borrowing constraint (Hakansson, 1970) and imperfection in the capital markets and intertemporal considerations (Kim, 1973).

The original formulation of prospect theory in Kahneman and Tversky (1979, p. 281), but also the cumulative version in Tversky and Kahneman (1992), can explain the popularity of insurance and gambling. They claim that the overweighting of small probability favors long shots like lottery tickets that pay large amounts with small probabilities over the expected value of the gambles. They also claim that people may prefer small losses represented, by an insurance premium, over large losses with small probabilities if they are

overweighted. Kahneman and Tversky (1979, p. 286) argue that the original version of Prospect Theory is able to predict both insurance and gambling for *small probabilities*, but they also claim that their “analysis fall far short of a fully adequate account for these complex phenomena.” They argue that there is evidence that purchase of insurance is often done in the medium range of probabilities but that small probabilities are often neglected below certain threshold. Slovic et al. (1977, p. 253), Kunreuther et al. (1978) and Schoemaker and Kunreuther (1979) found that in hypothetical situations people prefer to buy insurance for low-loss events that have high probability to purchase insurance for low-probability, high-loss events. Laury et al. (2009) replicated the results in Slovic et al. (1977) with hypothetical questions, but found the under-insurance of low probability/high-loss events, which is mainly based on field studies, does not hold in rigorous experimental conditions with real economic stakes. In fact Laury et al. (2009, p. 18) found that “individuals are more likely to purchase insurance for higher-consequence, lower probability events” when the framing of the hypothetical insurance situation is less abstract and the potential losses are clearly express in dollar amount. They also find that this insurance demand pattern is “strongest when incentives are real and subjects face the prospect of actually losing money” and conclude that “if there is a threshold for insuring against catastrophic events,” as hypothesize by Kunreuther and Pauly (2004, 2005), “then factors other than loss probability must be considered.”

## Optimal Risk-Sharing Arrangements and Insurance Decisions in the Absence of Other Risks

One of the landmark contributions in insurance economics is Borch (1962) who used the framework in Arrow (1964) to model a problem of a reinsurance market where risk is the only commodity and many reinsurers bargain to come to a Pareto-optimal risk-sharing agreement. The most important results are that any risk sharing agreement includes a mutuality principle (Gollier, 1992, p. 7) where all parties give their endowments up to a pool, thus only aggregate social risks matter since they cannot be diversified. A risk sharing rule that depends on risk tolerances and the Pareto weights, known as the Borch rule, is used to assign the risk allocations to each party.

Arrow (1963, p. 971-973) showed in a bilateral setting that, if both the insured and the insurer are risk averse expected utility maximizers, the Borch rule applies. In particular, the Pareto optimal risk-sharing agreement is a coinsurance contract in which the distribution of risk depends on the bargaining power and risk aversion of each party.

Machina (2000, p. 62-64) shows the robustness of the bilateral result in Arrow (1963) when subjects' preferences have a non-expected utility (non-EU) representation that generalized expected utility. Machina (2000, p. 65-70) also shows that the mutuality principle does hold in a general environment with many parties that exhibit non-EU preference representations; however, the original Borch's risk sharing rule is no longer sufficient in this environment, it is only a necessary condition. One needs to further assume that the non-expected utility indifference maps are outcome convex for the Borch rule to be sufficient for Pareto-optimality.

There is evidence of Borch's results both from the laboratory and from naturally occurring data. For instance, using data from three villages in India, Townsend (1994) tested what he calls the "full insurance" hypothesis in which all idiosyncratic risks are diversified and risk-sharing agreements depend on risk attitudes and bargaining power of the parties involved. Townsend (1994, p. 584) found that his hypothesis is rejected but individual consumption co-moves with the village's consumption average, thus consumption is not seriously affected by idiosyncratic risk. In a more recent paper, using data from Thai households, Chiapori et al. (2011, p. 1) control for heterogeneity and found that they cannot reject the hypothesis of full risk-sharing, where more risk averse households are insured by less risk averse households. In an experimental setting Bone et al. (1999) found the puzzling behavior that the risk-sharing agreements among trading partners was characterized by unconditional equal allocations across several prospects. In a follow-up study, Bone et al. (2004, p. 35) attributed this behavior partly to the complexity of the original task because they find that when giving simple choices to pairs of subjects with different elicited risk attitudes "partners largely favour ex ante efficiency over ex post equality."

Arrow (1963, p. 969-971) showed that if an insurer is willing to offer any insurance contract at a premium that depends only on the actuarial value of the loss, then the most preferred insurance contract of an expected utility-maximizer and risk averse individual is one with full insurance above a deductible. Vajda (1962) demonstrated a related result in the context of reinsurance that an insurance contract with a straight deductible provides the lowest variance to the insured. Raviv (1979) generalized Arrow's results and found that the

optimal contract involves a coinsurance arrangement above a deductible. Raviv also showed that the cost of insurance is the driving force of the deductible result and the coinsurance result is due to either risk sharing or cost sharing between the parties. Also, he finds that these results hold in the presence of several insurable losses and that only aggregate losses matter, thus assuming that losses are substitutable. This means that one dollar of a given loss is valued by a subject in the same manner that one dollar of another loss. Therefore, one can aggregate the monetary value of losses into one number. The importance of this assumption will be clear when we discuss multivariate risk aversion and its implications to insurance.

Gould (1969) and Schlesinger (1981) studied in an EUT framework the optimal deductible that should be taken in a given insurance contract, where the classic results state that individuals with a higher probability of a loss, more risk aversion or lower initial wealth will buy more insurance. Schlesinger (1985) studied the sufficient second order condition, extensively studied in Boyle and Mao (1983), for the choice of optimal deductible in insurance arrangements. Schlesinger (1985, p. 522) found that this sufficient condition depends on the “growth rate of the net price of insurance coverage and a utility weight that reflects a measure of risk aversion.”

It is a common observation that individuals have a tendency to choose a low deductible. For example, using aggregated data from auto insurance, Pashigian et al. (1966, p. 41) showed that there is “a sizeable group of individuals who select smaller deductibles than would be predicted from the expected-utility theory.” They suggest this behavior can be explained by subjective probabilities “if there is a systematic tendency to overestimate the

objective probabilities of an accident greater than the deductible” (p. 42).

However, there are other explanations to the demand for low deductibles that are related to the preference representations of individuals. Karni (1985) generalized the concept of risk aversion to state-dependent utility and found that higher risk aversion induces subjects to take lower deductibles. PT can also explain the demand for low deductibles. Schmidt (2012) studies in some detail the effect of different reference points for insurance demand within the PT framework and finds that wealth after buying full insurance seems to be a realistic reference point that can accommodate empirical evidence. Regret Theory (Loomes and Sugden, 1982; Bell, 1982; Sugden, 1993; Quiggin, 1994), which characterizes regret as not having chosen the ex post optimal choice, can also explain the demand for very low deductibles as shown by Braun Muermann (2004). Finally, the preference representation introduced by Gollier and Muermann (2012) can also rationalize the demand for low deductibles among other deviations from EUT like Allais paradoxes and the equity premium puzzle.

Mossin (1968) and Smith (1968) showed that if a decision maker is an expected utility maximizer and insurance is actuarially fair, in other words, that the proportional insurance loading is equal to zero, then full insurance is optimal. Since the loading is normally positive in practice, this means that partial coverage should be observed in practice.

Mossin (1968, p. 558) was aware that people choose full coverage quite often. In fact he offered several explanations for this behavior, namely: (1) people behave irrationally and do not bother to calculate the optimal coverage; (2) there can be uncertainty around the value of the insured asset; and (3) people might overestimate the probability of the loss.

Additionally, Mossin's and Smith's result may or may not hold depending on the subjects' preference representation. For instance, Razin (1976) analyzed insurance purchasing using the *minimax regret criterion* suggested by Savage (1951). Razin showed that if a subject follows this criterion, it is optimal to buy full coverage even if the insurance contract is not actuarially fair. Also,

Briys and Loubergé (1985) demonstrated that if an individual follows the Hurwicz (1951a) criterion, then the individual does not insure at all or buys full insurance even if the insurance loading is positive; however Lee and Pinches (1988) showed that by introducing risk aversion to the Hurwicz's criterion, partial insurance can also be optimal. Lee and Pinches (1988, p. 149) have a useful revision of the conditions for no insurance, partial or full insurance for the EUT model, Savage regret criterion, Hurwicz's criterion with risk neutrality and risk aversion, the minimax and the maximax rules. Machina (2000, p. 56) showed that Mossin's full coverage result does hold in a very general non-EU framework that assumes probabilistic sophistication. Machina (2000, p. 56 fn) recognizes that the robustness of the Mossin-Smith result under his generalized non-EU framework depends heavily on the insured's subjective probability of a loss coinciding with the probabilities of the market, that is, the probabilities used to price the insurance in actuarially-fair terms. Mossin (1968) also demonstrated that insurance is an inferior good if the decision maker exhibits decreasing absolute risk aversion (DARA). The latter result motivated interest in research to find conditions under which insurance is a Giffen good (Hoy and Robson, 1981; Briys et al., 1989). The interest in insurance contracts as inferior goods remained limited, probably, because following Arrow (1970), it was quickly recognized among economists that

insurance is a financial claim... [t]hus it does not seem really appropriate to apply to insurance concepts which were derived to categorize consumption goods (Loubergé, 2000, p. 9). Another reason might be that, as Hoy and Robson (1981, p. 51) claim, it seems empirically implausible for insurance to be a Giffen good since it would require unusually high loadings with respect to the probability of the loss.

However, it is a widely held view among practitioners that the demand for life and non-life insurance increases with wealth, which contradicts the behavioral implication of decreasing insurance demand of the widely accepted assumption of DARA. For example, Enz (2000) estimate the so called S-curve that documents a positive relationship across countries between income per capita and insurance penetration, which are used as proxies of wealth and insurance demand. A simple explanation would be that the size of the loss increases as wealth increases. Mossin (1968) assumed that the size of the loss is independent of changes in wealth. This assumption is arguably met in reality where increases in wealth normally increases the amount of loss the subject is exposed to (Chesney and Loubergé, 1986). For example, a subject that observes an increase in wealth may decide to buy a more expensive house which implies a larger potential loss in case of earthquake.

However, Foncel and Treich (2009) find that after controlling for the level of wealth, car insurance is a normal good and observed behavior in financial decisions is consistent with DARA. Cummins and Mahul (2004) offer a potential explanation by showing that an insurance contract with an upper limit on the coverage may behave as a normal good under the DARA assumption since the wealth effect is indeterminate. Aese (2007) provides conditions under which insurance can be a normal good if the decision problem endogenizes

the amount invested in the insurable asset. Under certain conditions insurance behaves as a normal good whenever the insurable asset is a normal good itself. Chen and Mahani (2011, p. 20) offer an alternative by assuming consumption commitments which can account for the normality of insurance at certain levels of wealth.

Another interesting explanation of insurance behaving as a normal good is the potential effect on insurance demand of liquidity constraints, at least for very poor households. It is relatively well-documented that poor households exhibit a low demand for insurance even though it might seem that they should be buying it since they are often also the ones more exposed to adverse random shocks.

Deaton (1990, 1991) argues that a model with liquidity constraints and prudent behavior, as defined by Kimball (1990), is a reasonable way to characterize consumption and saving behavior of poor farmers in least developed countries. If incomes are stationary and independent and identically distributed over time, as might be the case of a poor farmer, assets (e.g., harvested crops) play the role of a buffer stock and the decision maker adjusts saving to smooth consumption. The more prudent the individuals are and the more uncertainty in income, the greater the demand for precautionary saving in detriment of insurance purchasing. In fact, Moffet (1977) found that saving and insurance are substitutes in the absence of certainty. There is a close relationship between prudence and liquidity constraints. Carroll and Kimball (2005) explain why the introduction of a liquidity constraint increases the motive for precautionary saving. They argue that the latter arises from the concavity of the consumption function that uncertainty or liquidity constraints can induce. This relationship between prudence and liquidity constraints could

be used to explain the low take-up of insurance among poor households and an increase in insurance demand as the level of wealth increases. On one hand, Gollier (2001, p. 238) shows that prudence is a necessary condition for decreasing absolute risk aversion (DARA). On the other hand, Mossin (1968) shows that the coinsurance demand is decreasing in initial wealth if the utility function exhibits DARA. Both results suggest that an increase in prudence would decrease the demand for coinsurance. Because of the relationship explained in Carroll and Kimball (2005), an equivalent result can be derived. If liquidity constraints increase precautionary saving, then tighter liquidity constraints should decrease the demand for insurance or even make it disappear. The intuition is that a liquidity constraint induces individuals to save to smooth consumption and this competes with the incentives to buy insurance. As wealth increases the liquidity constraints could be relaxed and this should increase the insurance demand since less saving is needed to smooth consumption. This would make insurance behave as a normal good. However, it is likely that at some point risk attitudes exhibiting DARA could overcome the augmenting effect on insurance demand of relaxing the liquidity constraints. Nevertheless, it could be possible that some of the explanations mentioned above that explain the normality of insurance could be driving the increase in insurance demand at higher levels of wealth. To the best of our knowledge there is no paper that studies such a comprehensive model.

### **Probabilistic Insurance: A Novel Kind of Insurance?**

Kahneman and Tversky (1979), KT henceforth, constructed an example of an insurance contract, called probabilistic insurance, which EUT predicts would be preferred to a

traditional insurance contract that offers full insurance, but that intuition would suggest is riskier than regular insurance with complete coverage. Kahneman and Tversky (1979, p. 269) define probabilistic insurance in the following way:

In this program [the probabilistic insurance contract] you pay half of the regular premium. In case of damage, there is a 50 per cent chance that you pay the other half of the premium and the insurance company covers all the losses; and there is a 50 per cent chance that you get back your insurance payment and suffer all the losses. For example, if an accident occurs on an odd day of the month, you pay the other half of the regular premium and your losses are covered; but if the accident occurs on an even day of the month, your insurance payment is refunded and your losses are not covered.

An EUT maximizer with a concave utility function would preferred a probabilistic insurance over a contract that offers full insurance. A short proof of this claim is as follows. Suppose an individual with utility function  $u$  such that  $u' > 0$ ,  $u'' < 0$  and initial wealth position  $w$ . The subject is exposed to a potential loss  $x$  with probability  $p$ . The premium of a contract that offers full insurance,  $\pi$ , is defined such that it makes this contract barely worth its cost:

$$pu(w - x) + (1 - p)u(w) = u(w - \pi)$$

where the left (right) hand side of the equality is the expected utility of the individual without (with) insurance. Risk aversion, which is equivalent to concavity of the utility function under EUT, implies that the individual will prefer a probabilistic insurance contract over the full coverage contract:

$$p[0.5u(w - x) + 0.5u(w - 0.5\pi - 0.5\pi)] + (1 - p)u(w - 0.5\pi) > u(w - \pi)$$

where the left (right) hand side of the equality is the expected utility of the individual that takes the probabilistic (full) insurance contract.

Contrary to the EUT prediction, KT found in their hypothetical experiments that, out of 95 individuals, 80% preferred full insurance over probabilistic insurance, which is the opposite an EUT maximizer with concave utility function would do.

Kahneman and Tversky (1979, p. 270) claim that “[t]his is a rather puzzling consequence of the risk aversion hypothesis of the utility theory, because all insurance is, in a sense, probabilistic. The most avid buyer of insurance remains vulnerable to many financial and other risks which his policies do not cover.” They are able to explain this puzzling behavior within prospect theory by allowing the utility function to be convex in losses, or equivalently, by claiming that individuals exhibit risk loving behavior over losses with respect to a reference point. More recently, using real stakes, Herrero et al. (2006) found that subjects that are indifferent between full insurance and no insurance, prefer full insurance to probabilistic insurance and the latter to no insurance. The first finding is incompatible with EUT and the second is incompatible with the original Prospect Theory. Herrero et al. (2006) suggest that Regret Theory, as described in Loomes and Sugden (1982, 1987), can accommodate both results.

Segal (1988a) shows that the reluctance towards probabilistic insurance can be explained by his version of Quiggin’s RDU model without having to assume risk loving behavior or convexities in the utility function. Notice that a probabilistic insurance contract is basically a compound lottery in which an individual will pay only half the premium if no accident occurs but will be exposed to a 50/50 risky prospect in which the premium paid is

returned and no insurance repayment is done or the other half of the premium has to be paid and the total loss is covered. Segal demonstrated that the rejection of probabilistic insurance is consistent with risk aversion and a concave utility function if the individual has a RDU preference representation and either satisfies the reduction of compound lotteries or the compound independence axiom but not both.<sup>5</sup>

Wakker et al. (1997) generalized the particular version of probabilistic insurance shown by Kahneman and Tversky (1979) and find survey evidence of hypothetical questions in which people dislike their version of probabilistic insurance. Wakker et al. (1997, p. 7-8) claim that “most insurance policies are, in fact, probabilistic. Insurance policies typically specify some events (e.g., wars, ‘acts of god,’ contributory negligence) in which the consumer is not reimbursed for losses, whether or not the consumer was aware of these contingencies ex ante. Furthermore, there is always a possibility however remote—that the insurer will not pay for some other reason such a insolvency or fraud. Although such default risks are not explicitly acknowledge, they are present in any real insurance setting.” In fact, the unappealing nature of probabilistic insurance is also shared by Merton (1993, p. 43) who claims that “... many real world customers would consciously agree to accept non-trivial [default] risk on a \$200,000 life insurance policy in return for a large reduction in the annual premium, say from \$400 to \$300.” Doherty and Schlesinger (1990) showed that this risk of contractual nonperformance renders invalid most of the standard results in insurance economics. For example, if the risk of insolvency is non-negligible and the default

---

<sup>5</sup>Expected utility theory assumes the mixture independence axiom, which is in turn implied by the reduction of compound lotteries and the compound independence axiom. See Annex A for a detailed definition of these axioms.

is total, partial insurance is optimal even at actuarially fair prices, which is contrary to the Mossin (1968) and Smith (1968) full insurance result. Also, insurance as an inferior good when preferences exhibit DARA and more demand for insurance given higher risk aversion no longer hold. It is worth mentioning that this type of nonperformance in insurance contracts is conceptually related to what is called basis risk in hedging markets. For example, Clarke (2011) shows in a EUT framework and for a risk averse individual that zero demand for index insurance can be optimal depending on the transaction costs and the level of basis risk. Doherty and Richter (2002) show, in a mean-variance framework and in the presence of basis risk, that the availability of both an index-linked hedging product and gap insurance that covers the difference in payouts between the index product and actual losses can lead to efficiency gains.

### **Insurance as One Risk Management Strategy among Many and the Interaction of Many Risks**

Ehrlich and Becker (1972) is considered the first theoretical paper in risk management which considered insurance as one tool among many to deal with hazards and their economic costs. Ehrlich and Becker derived four important results.

First, when “market” insurance is not available, individuals engage in self-insurance (e.g., cash balances or savings) and self-protection (e.g., theft alarms) activities according to the costs and benefits associated with those activities. Ehrlich and Becker define self-insurance as an activity that is able to affect the cost associated to a potential loss, whilst

self-protection affects the probability of the loss itself.<sup>6</sup>

Second, market insurance and self-insurance are substitutes since both risk management strategies are able to redistribute resources from bad states of the world to good states of the world, thus affecting the size cost of the loss and smoothing consumption across states of nature. Third, market insurance and self-protection can be substitutes or complements. For example, the installation of a theft alarm at home might be enough to deter robbery so home insurance might not be necessary. On the contrary, a theft alarm might increase the demand for insurance if this self-protection strategy reduces the probability of a loss and the cost of insurance varies proportionally with this probability. An implication of this relationship is that if the price of insurance to cover certain risks is independent of expenditures on self-protection activities that reduce the probability of a loss, then the market will tend to consider those risks as uninsurable since moral hazard is more likely to arise. However, in contrast to the moral-hazard intuition, this relationship may also imply that the presence of market insurance may increase the self-protection activities.

Finally, Ehrlich and Becker's framework is able to explain the insurance-gambling puzzle. They consider that behavior of people buying insurance and gambling at the same time can be explained if the options available are sufficiently favorable. Therefore, Ehrlich and Becker (1972, p. 627) consider that "inferences about attitudes towards risk cannot be made independently of existing market opportunities: a person may appear to be 'risk avoider' under one combination of prices and potential losses and a 'risk taker' in another."

---

<sup>6</sup>Ehrlich and Becker recognize that many of the risk management strategies that people have at their disposal can potentially affect both the cost and the probability of the loss. However, for explanation purposes, they assume that there is a separation between self-insurance and self-protection activities.

Using the EUT framework, several authors expanded the analysis in Ehrlich and Becker (1972) in different directions. Dionne and Eeckhoudt (1985) found that more risk aversion increases self-insurance activities but does not necessarily increase self-protection. Briys and Schlesinger (1990, p. 466) provided an explanation for these results: Self-insurance unambiguously reduces risk, while self-protection does not. Consequently, it is no surprise that an increase in risk aversion unambiguously increases the level of self-insurance, but sometimes it may decrease the level of self-protection. However, Hiebert (1989, p. 300-301) showed that self-insurance activities does not necessarily increase with more risk aversion if the effectiveness on loss mitigation of these activities is uncertain. Briys et al. (1991) take the analysis one step further and analyze the main tools of risk management when their reliability cannot be guaranteed, in other words, that there is as chance that these tools might not work as expected in the case of a loss, just like an insurer might default on its clients because of solvency issues. In contrast to Ehrlich and Becker (1972), Briys et al. (1991, p. 47) find that: (i) the “riskiness” of final wealth is not necessarily reduced by an increase in market insurance or self-insurance when they are not fully reliable; and (ii) market insurance and self-insurance might be complements without full reliability. Finally, Sweeny and Beard (1992) study the comparative statics of self-protection when initial wealth and the size of the loss changes.

From a Non-EUT perspective many of the results in Ehrlich and Becker (1972) hold. In fact, Quiggin (1991, p. 340) showed that a wide range of comparative static results under EUT can be extended to RDU, because this model “may be regarded as expected utility with respect to a transformed probability distribution.” Konrad and Skarpedas (1993),

Machina (1995, 2000) and Courbage (2001) showed that the main results in Ehrlich and Becker (1972) hold under RDU theory, generalized smooth Non-EU and Yaari's Dual Theory.

The framework in Ehrlich and Becker can be used to study how policies or activities that change risks can affect individuals welfare. Shogren and Crocker (1991, p. 6, 9 and 11), using the EUT model, analyze the *ex ante* value of reducing risk when there are self-protection possibilities and find three important results. First, when self-protection influences the probability and/or the severity of a loss, the *ex ante* valuation of a reduction in risk is a function of risk attitudes and the marginal rate of technical substitution between self-protection and hazard concentrations. This implies that willingness to pay for reductions in risk cannot be studied just by looking at observable expenditures in self-protection and exposure to risk. This also implies that one cannot simply sum the unweighted compensating or equivalent variation to study the societal impact of a policy that affects a risk individuals are exposed to. Second, even under very intuitive conditions such as individuals' pecuniary costs of hazards being convex to risk, increased exposure to a hazard does not necessarily mean that the individual needs to be compensated. The intuition behind this result is that a change in exposure to risk induced by self-protection may have effects both on the probability and the severity of the loss. There are no *ex ante* reasons to believe that self-protection affects both probability and severity in the same direction. Finally, intuitive and simple conditions on the costs of risks are not sufficient to guarantee an unambiguous response of self-protection expenditures to changes in risk. This implies that observed expenditures in self-protection are no necessarily a lower bound on

the subject's ex ante value of a reduction in risk. Intuitively this may happen because these expenditures may not be necessarily increasing in risk.

Quiggin (1992, p. 41) claims that, under certain intuitive conditions, the negative results in Shogren and Crocker will not hold and the "standard willingness-to-pay approach to valuing environmental hazards is valid under fairly general conditions." These are decreasing absolute risk aversion and a separability condition. The former is an assumption that deserves discussion but is widely accepted by economists, however, the latter implies that self-protection activities mitigates the individual's exposure to risk but does not affect the risk itself. In a subsequent response to Quiggin's results, Shogren and Crocker (1999) claim that the separability assumption is problematic and consider that risk is endogenous in many situations. This implies that self-protection activities can mitigate the consequences of risk to the individual but can also affect the general level of risk itself. If this is in fact the case, then the analysis welfare analysis of risk reduction cannot avoid the identification of risk attitudes. This is an important example in which experiments can help identify risk attitudes in order to carry out welfare analysis.

A modern perspective in the analysis of financial decisions was formally introduced by Mayers and Smith (1983), but recognized earlier by Gould (1969, p. 151), who proposed that insurance decisions should be analyzed in the presence of traded and non-traded assets like human capital. The introduction of a non-traded asset can significantly change some of the standard results in insurance economics, such as a wealthier individual buying less insurance.

A non-traded asset is closely related to the concept of uninsurable background risk, which

is of great relevance to the analysis of decisions under risk. As pointed out by Schlesinger and Doherty (1985), incomplete markets induce the presence of uninsurable background risks that can affect the standard results in the analysis in insurance decisions. Doherty and Schlesinger (1983) studied the robustness of the standard results to the presence of uninsurable background risk that is independent to the insurable loss. They found that under certain restrictive conditions the Mossin-Smith theorem holds and more risk averse people will choose a lower deductible. Turnbull (1983, p. 217) also showed that in the presence of many risks, the Arrow-Pratt measure of risk aversion is not a sufficient statistic to describe individuals' behavior in insurance purchasing. Doherty (1984, p. 209) showed in an EUT framework that the Mossin-Smith theorem only holds if the covariance between the non-traded asset and the insurable loss is negative. The intuition is that even if the insurance premium is fair, the DM might still not be willing to fully insure if she can compensate high losses with high realizations of her human capital. However, if the covariance is negative, the individual might want to fully insure if a health shock affects negatively her productivity, which would undermine her human capital. Moreover, in the presence of a non-traded asset circumstances may arise where a risk averse individual prefers a coinsurance arrangement to an actuarially equivalent insurance contract with straight deductible.

Many of the later studies in background risks focused on finding conditions under which unambiguous comparative statics can be obtained. For example, Eeckhoudt and Kimball (1992) derived a sufficient condition on utility functions called *standard risk aversion* which guarantees that under less restrictive condition, the presence of background risk

(independent or not) will induce a higher demand for insurance. Standard risk aversion is satisfied if both absolute risk aversion and absolute prudence are decreasing in wealth. Gollier and Pratt (1996, p. 1109) derived a restriction on utility functions called *risk vulnerability* which is equivalent to “the condition that an undesirable risk can never be made desirable by the presence of an independent, unfair risk.”

### **General Characterizations of Risk aversion, Changes in Risks and Insurance Decisions**

There is an important line of research that analyzes decisions under risk and uncertainty by studying very general definitions of risk aversion and the stochastic orders that the individual's preference representation satisfies.

This approach demonstrate that several of the results derived under EUT apply to preference representations that satisfy risk aversion in the sense of the mean-preserving contraction property (MPC). Gollier (2000) has a rigorous analysis of the following stated results. First, if transaction costs are linear in the actuarial value of the potential loss, the insurance loading is zero and the preference representation exhibits risk aversion in the MPC sense, then it is always optimal to fully insure. This is the main result in Mossin (1968). Second, the optimality of an insurance contract with a deductible derived by Arrow (1963) is maintained if there is linearity in costs and the subject exhibits risk aversion in the sense of MPC. Third, also under linear costs and risk aversion in the MPC sense, the optimality of an umbrella policy that covers aggregate losses in the presence of many risks is maintained, a result derived by Gollier and Schlesinger (1995) under EUT. There is an

important strand of literature that studies the comparative statics of insurance demand when there is a change in the distribution of the loss, a problem that was originally studied by Rothschild and Stiglitz (1970, 1971). The most general result was obtained by Gollier (1995, 1997) who demonstrated under the EUT model that an increase in risk in the sense of “greater central riskiness” (CR) is a necessary and sufficient condition for all risk averse individuals to decrease their exposure to the source of risk, thus raising the demand for insurance. Interestingly, other characterizations of increased riskiness that has been studied in the literature, like one proposed by Rothschild and Stiglitz (1971), are especial cases of CR. Eeckhoudt and Gollier (2000) have a very comprehensive exposition of different definitions of an increase in risk and its consequences.

The advantage of these approaches are evident, they allow to derive very general results under very few restrictions. Although this approach might be very useful for objective probabilities, they have both theoretical and practical limitations when dealing with subjective probabilities. First, from a theoretical perspective, this approach requires that individuals are probabilistically sophisticated as defined by Machina and Schmeidler (1992). Second, from an applied perspective, in order to make any comparative static a researcher would have to know the subjective probabilities that an individual has in mind when making decisions. There are ways to elicit subjective probabilities in the laboratory, which we review in the next section, but most of these methods require the specification of the preference representation. Thus, from an operational point of view, the advantages of this approach are limited.

### 1.3.2 Uncertainty and Ambiguity

One new program of research is studying insurance decisions under uncertainty, which Ellsberg (1961) originally referred to as ambiguity. The latter corresponds to a situation in which an individual is not sure about the true distribution of the states of nature and a decision maker is usually assumed to be averse to this situation. We first offer a statement of the differences between risk, uncertainty and ambiguity, which stresses the role of the ROCL axiom. There are many studies that use the expression “uncertainty” for what we would call subjective risk, or use the expression “ambiguity” for what we would call it uncertainty. This actually matters less than the formal characterization that is being referred to, and often the context makes that characterization clear. But many researchers in insurance do not seem to know the formal differences, and the literature is peppered with instances of polysemy, where one word is used to mean many things.

#### Defining Risk, Uncertainty and Ambiguity

The evaluation of insurance involves more than just the evaluation of objective risk. Even when actuaries are called in to offer probabilities of alternative outcomes, there is a significant element of subjectivity. Does anything change when we allow for subjective beliefs?

Unfortunately, yes and no. Nothing changes if we assume, following Savage (1971), that decisions are made as if one obeys the ROCL axiom. But things change radically if one does not make that assumption. This seemingly technical issue is actually of great significance for the evaluation of insurance, and is worth explaining carefully.

Figure 1.1 illustrates the situation. Assume that the subjective beliefs are symmetric, with mean one-half as shown by the solid, vertical line. But they vary in terms of the underlying distribution, as shown in the four panels of Figure 1.1. Some are just more or less precise than others, and one is bimodal. Under ROCL, all would generate decisions with the same outcome, since all have the same (weighted) average. Something nags at us to say that behavior ought to be different under these different sets of beliefs, but ROCL begs to differ.

Figure 1.1: Symmetric Subjective Probability Distributions

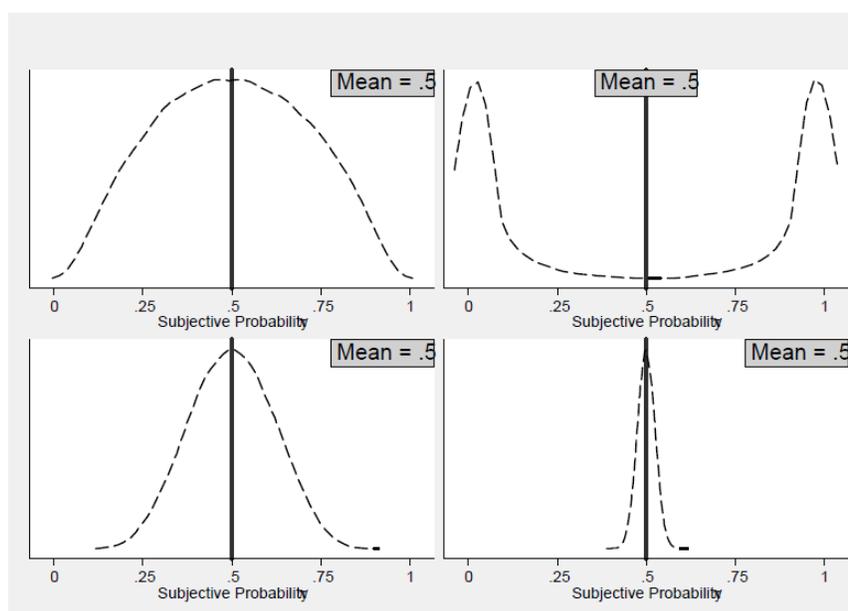


Figure 1.2 raises the stakes by considering asymmetric distributions. Again, ROCL is a strong, identifying assumption. Together, Figures 1.1 and 1.2 remind us that Savage (1971) did not assume that people *had* degenerate subjective probabilities that they held with certainty, he only assumed that under ROCL they behaved *as if* they did. We often forget that linguistic methodological sidestep, and confuse the “as if” behavior for what was actually assumed. In some cases the difference does not matter, but here it does. The

reason is that when we have to worry about the underlying nondegenerate distribution, when ROCL is not assumed, then we have moved from the realm of (subjective) risk to uncertainty. And when the individual does not even have enough information to form any subjective belief distribution, degenerate or non-degenerate, we are in the realm of ambiguity.

Figure 1.2: Asymmetric Subjective Probability Distributions

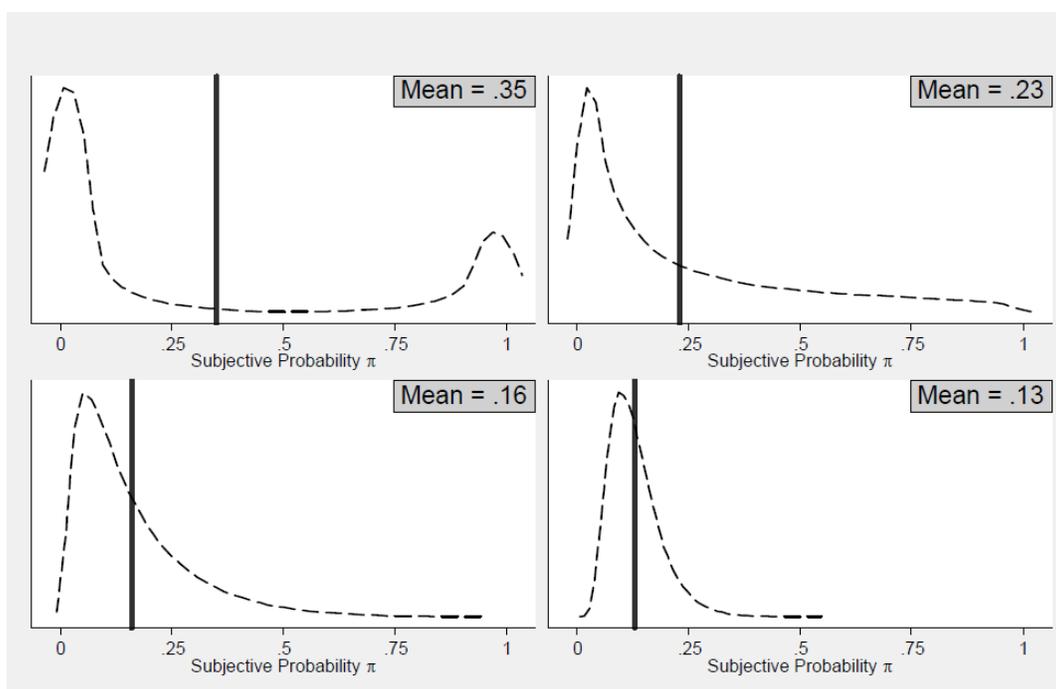
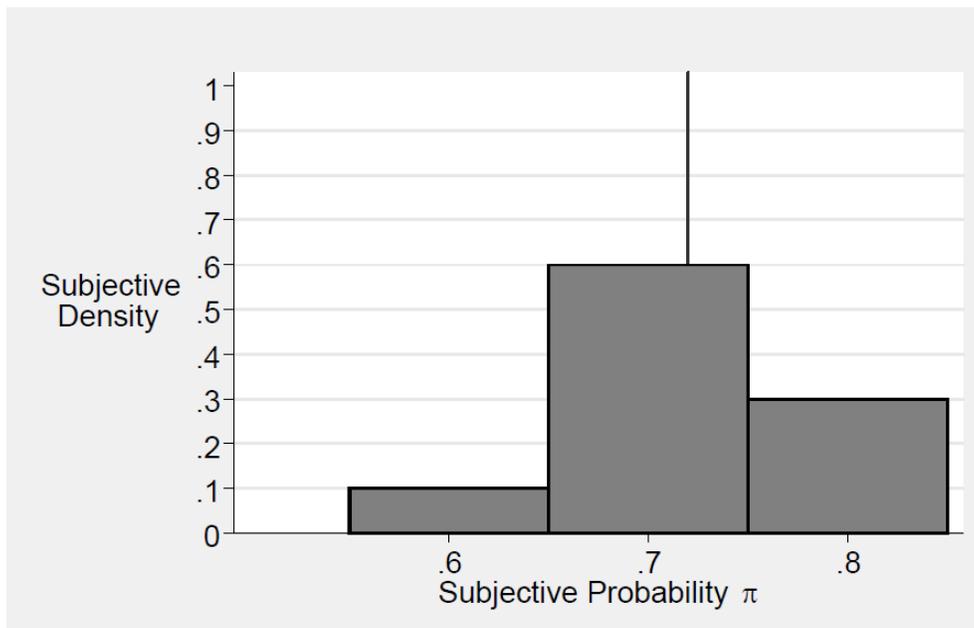


Figure 1.3 allows a simple illustration of how ROCL allows one to collapse these disparate, non-degenerate distributions into one degenerate weighted average. Figure 1.3 displays a three-point discrete, non-degenerate, subjective distribution over a binary event in which the individual holds subjective probability  $\pi = 0.6$  with “prior” probability 0.1,  $\pi = 0.7$  with “prior” probability 0.6, and  $\pi = 0.8$  with “prior” probability 0.3, for a weighted average  $\pi = 0.72$ . Now consider a lottery in which one gets  $\$X$  if the event occurs, and  $\$x$

otherwise. Then the subjective expected utility (SEU) is

$$0.1 \times 0.6 \times U(X) + 0.1 \times 0.4 \times U(x) + 0.6 \times 0.7 \times U(X) + \\ 0.6 \times 0.3 \times U(x) + 0.3 \times 0.8 \times U(X) + 0.3 \times 0.2 \times U(x)$$

Figure 1.3: ROCL at Work



which collapses to

$$(0.1 \times 0.6 + 0.6 \times 0.7 + 0.3 \times 0.8) \times U(X) + (0.1 \times 0.4 + 0.6 \times 0.3 + 0.3 \times 0.2) \times U(x)$$

and hence to

$$0.72 \times U(X) + 0.28 \times U(x)$$

under ROCL. So the non-degenerate distribution in Figure 1.3 can be boiled down to a degenerate subjective probability of 0.72 under ROCL: an impressive identifying restriction.

How we relax ROCL is a matter for important, foundational research. Although it has taken half a century for the implications of Ellsberg (1961) to be formalized in tractable ways, we are much closer to doing so. One popular approach is the smooth ambiguity model of Klibanoff et al. (2005), KMM henceforth, with important parallels in Davis and Paté-Cornell (1994), Ergin and Gul (2009), Nau (2006) and Nielsen (2010). Another popular approach is the  $\alpha$ -MEU model due to Ghirardato et al. (2004), generalizing the Maxim Expected Utility model of Gilboa and Schmeidler (1989) and the Bayes-Minimax criterion of Hurwicz (1951a,b).

To provide a concrete example, we can illustrate the smooth ambiguity model with some simple numbers. Let  $CE(\pi = 0.6)$  be the certainty equivalent of the lottery  $0.6 \times U(X) + 0.4 \times U(x)$ ,  $CE(\pi = 0.7)$  be the certainty equivalent of the lottery  $0.7 \times U(X) + 0.3 \times U(x)$ , and  $CE(\pi = 0.8)$  be the certainty equivalent of the lottery  $0.8 \times U(X) + 0.2 \times U(x)$ . Then the evaluation of the lottery can be written

$$0.1 \times \phi(U(CE(\pi = 0.6))) + 0.6 \times \phi(U(CE(\pi = 0.7))) + 0.3 \times \phi(U(CE(\pi = 0.8))),$$

where  $\phi$  is a function defined over the domain of  $U(\cdot)$ . Akin to the properties of  $U(\cdot)$  defining risk attitudes under EUT or SEU, the properties of  $\phi(\cdot)$  define attitudes towards the uncertainty over the particular subjective probability value.<sup>7</sup> If  $\phi$  is concave, then the decision-maker is uncertainty averse; if  $\phi$  is convex, then the decision-maker is uncertainty

---

<sup>7</sup>In the original specifications  $\phi$  is said to characterize attitudes towards ambiguity, but the earlier definition of risk, uncertainty and ambiguity makes it apparent why one would not want to casually confound the two. One would only be dealing with ambiguity in the absence of well-defined prior probabilities over the three subjective probability values 0.6, 0.7 and 0.8.

loving; and if  $\phi$  is linear, then the decision-maker is uncertainty neutral. The familiar SEU specification emerges if  $\phi$  is linear, since then ROCL applies after some irrelevant normalization. The overall evaluation of the lottery depends on risk attitudes and uncertainty attitudes, and there is no reason for the decision-maker to be averse to both at the same time. An important econometric corollary is that one cannot infer attitudes toward uncertainty from observed choice until attitudes toward risk are characterized. One final point is that in the analysis of attitudes towards uncertainty, aversion towards it is normally assumed, much like in the analysis of risk aversion is assumed. The latter approach is a normative view of how to make decisions under risk that started with Bernoulli (1738) and became the norm after Pratt (1964). However, there is nothing in the axiomatic derivation of EUT of von Neumann and Morgenstern (1953) that restricts individuals to be risk averse since the utility function need not be concave as was argued by Friedman and Savage (1948). A similar claim can be made about attitudes towards uncertainty: people can behave as if they were uncertainty averse, neutral or loving. Ellsberg (1961) is normally interpreted as suggesting that uncertainty aversion is a reasonable response to uncertainty; however he also suggested (Ellsberg, 2001, p. 199-209) that there are situations where subjects might consider it favorable to behave as if they were uncertainty lovers. More recently, Machina (2011, p. 2) has emphasized the same issue and provide thought experiments that can be implemented in a laboratory in which uncertainty loving is a “reasonable” heuristic to deal with the situation. There is some evidence from a field experiment by Wakker et al. (2007, p. 1779) in which uncertainty seeking was found in the losses domain. The main point is that researchers should

approach the analysis of decisions under uncertainty from an agnostic point of view and let attitudes towards uncertainty vary across all domains.

### **Implications for Insurance Demand**

One important research challenge that lies ahead is that different models may produce different predictions of insurance behavior, because of the different behavioral assumptions of the models. This is inevitable given the myriad ways that ROCL can be relaxed.

For example, Cherbonnier and Gollier (2011) found that decreasing concavity of the utility function is necessary and sufficient in the  $\alpha$ -MEU model to guarantee decreasing aversion.

This would imply that higher wealth would decrease the demand for insurance. However, they also show that in the KMM model one has to impose restrictions on both risk and uncertainty attitudes, as well as on the uncertainty structure, to guarantee that an increase in wealth will increase the demand of an ambiguous asset. The Borch rule is another classic result that is robust depending on the framework that one uses to model decisions under uncertainty. Martínez-Correa (2012) showed that a modified Borch rule characterizes the optimal contract under uncertainty when one allows for bilateral risk and uncertainty aversion and differences in beliefs. However, Wener (2011) shows that the multiple-priors MEU model by Gilboa and Schmeidler (1989) implies that uncertainty averse individuals might not participate in risk sharing agreements.

Another important line of research that is underdeveloped is the robustness of standard results to the introduction of uncertainty and non-traded assets/background risks. For example, using the KMM framework, Alary et al. (2010) found that uncertainty aversion

always raises the demand for self-insurance but may decrease the demand for self-protection, that it also increases the optimal insurance coverage, and show a case in which the optimal insurance contract is one with a straight deductible, the one originally derived by Arrow (1963). However, Martínez-Correa (2012), also using the KMM model, showed that an insurance contract with a straight deductible may be dominated by an equivalent coinsurance contract if the decision maker owns a non-traded asset. An important related line of research that is basically undeveloped is analyzing the original framework in Mayers and Smith (1983) in the presence of ambiguous traded and non-traded assets, insurable and uninsurable uncertainties.

Since any insurance problem can be written as a portfolio choice problem, an important result in Martínez-Correa (2012) is that, in the presence of uncertainty, the optimal risk return allocation will not always be located in the efficient part of the classical mean-variance frontier. The reason is that high incentives to self-hedge create a trade-off between the variance and uncertainty of wealth that a decision maker must solve. Thus, an individual could appear to be “inefficient” from the perspective of classical portfolio analysis that assumes (subjective) expected utility maximization, although such action is optimal in the presence of uncertainty.

## 1.4 Experimental Evidence

Experiments can help inform the evaluation of insurance in two ways. The first is by providing some guidance as to latent structural parameters needed to understand observed

behavior and to complete a welfare evaluation. The second is by bypassing the need for all of this structure, in an agnostic manner, and “letting the data speak for itself” with minimal theoretical assumptions.

It is worth identifying the various types of experiments in wide use. Harrison and List (2004) propose a taxonomy to help structure thinking about the many ways in which experiments differ. At one end of the spectrum are *thought experiments*, which can be viewed as the same as any other experiment but without the benefit of execution (Sorenson, 1992). Then there are conventional *laboratory experiments*, typically conducted with a convenience sample of college students and using abstract referents. Then there are three types of field experiments. *Artefactual field experiments* are much like lab experiments, but conducted with subjects that are more representative of a field environment. *Framed field experiments* extend the design to include some field referent, in terms of the commodity, task, or context. *Natural field experiments* occur without the subject knowing that they have been in an experiment. Then we have *social experiments*, where a government agency deliberately sets out to randomize some treatment. Finally, there are *natural experiments*, where some randomization occurs without it being planned as such: serendipity observed. Randomization can be used in every one of these types, and is more a method of conducting experiments rather than a defining characteristic of any one type of experiment in the field, as some have suggested. Nor are these categories intended to be hard and fast: one can easily imagine intermediate categories, such as the *virtual experiments* of Fiore et al. (2009), with the potential of generating both the internal validity of lab experiments

and the external validity of field experiments.<sup>8</sup>

### 1.4.1 Estimating Preferences and Beliefs

There are three fundamental, behavioral “moving parts” in almost any decision of importance concerning insurance: risk attitudes, time preferences, and subjective beliefs.

Experimental economists now have a robust set of tools to elicit each of these, although controversies remain, as expected in foundational concepts such as these.

Risk attitudes refer to the risk premium that individuals place on lotteries. The familiar diminishing marginal utility explanation of EUT provides one characterization of the risk premium, and allows a wide range of flexible utility functions to be estimated. But it is a simple matter to also allow for probability weighting to explain the risk premium:

“pessimistic” attitudes towards probabilities can just as easily account for risk aversion.<sup>9</sup>

Similarly, it is possible to extend the estimation to allow for sign-dependent preferences, whereby losses are evaluated differently than “gains.” We add quotation marks for losses and gains because the Achilles Heal of sign-dependent models is the specification of the

---

<sup>8</sup>A virtual experiment (VX) is an experiment set in a controlled lab-like environment, using either typical lab or field participants, that generates synthetic field cues using Virtual Reality (VR) technology. The experiment can be taken to typical field samples, such as experts in some decision domain, or to typical lab samples, such as student participants. The VX environment can generate internal validity since it is able to closely mimic explicit and implicit assumptions of theoretical models, and thus provide tight tests of theory; it is also able to replicate conditions in past experiments for robustness tests of auxiliary assumptions or empirically generated hypotheses. The VX environment can generate external validity because observations can be made in an environment with cues mimicking those occurring in the field. In addition, any dynamic scenarios can be presented in a realistic and physically consistent manner, making the interaction seem natural for the participant. Thus the VX builds a bridge between the lab and the field, allowing the researcher to smoothly go from one to the other and see what features of each change behavior.

<sup>9</sup>The logic is easy to see. Assume lotteries defined solely over gains, and a linear utility function just to remove the effect of diminishing marginal utility. Then if the weighted probability is always equal to or less than the actual (objective or subjective) probability, the EU based on these weighted probabilities will be less than the EV based on the actual probabilities, hence there is a risk premium.

reference point, and this is the subject of considerable debate. All of these approaches simply decompose and explain the risk premium in different ways, and build on the approach before it. Experimental and econometric methods for the estimation of risk attitudes using all of these approaches are relatively well-developed: see Harrison and Rutström (2008) for an extensive survey.

There is also considerable evidence that behavior towards risky lotteries is not characterized by just one model of decision making under risk. Mixture specifications in rich and poor countries, in the lab and the field, show a remarkable combination, close to 50:50, of *both* EUT *and* non-EUT characterizations (Harrison and Rutström, 2009; Harrison et al., 2010). This finding is likely to vary from domain to domain, and population to population, but offers a much richer characterization of behavior than the usual approach favored by economists.<sup>10</sup>

Most of the effort to go into estimating “risk attitudes” is actually directed at estimating utility functions. So a trivial byproduct of that effort is to be able to generate estimates of higher-order concepts such as “prudence” and “temperance”. Although it is possible to generate lotteries that identify preferences driven solely by prudence or temperance (e.g., Ebert and Wiesen, 2011), these design typically require that subjects satisfy ROCL, which is a strong assumption and appears to limit the generalization to non-EUT models such as RDU.

Recent extensions include attention to the problem of the presence “background risk”

---

<sup>10</sup>In effect, the usual methodological approach is akin to running a horse race, declaring a winner, maybe by a nose, and shooting all of the losing horses. The fact that one of these losers might have done better on a different, wetter track is ignored.

affecting decisions over foreground risk (e.g., Harrison et al., 2007). For example, it makes little sense to evaluate the value of a statistical life without worrying about the confound of compensating differentials for non-fatal injuries: what does not kill often injures. A further extension to multi-variate, or multi-attribute, risks promises greater insight into risk management over traded and non-traded assets in the individual's portfolio (e.g., Andersen et al., 2011b).

Time preferences are also now relatively well understood. The first generation of experiments used loose procedures by modern standards, often relying on the elicitation of present values using Fill-In-The-Blank (FIB) methods that have notoriously poor behavioral properties. This literature is characterized by the need to use scientific notation to summarize estimated astronomic discount rates, a sure sign that something was wrong with behavior, experimental design, or inferential methods. Frederick et al. (2002) summarize the literature up to this point. The second generation of experiments moved towards binary choice tasks to ensure incentive compatibility, albeit at the loss of information precision (if the FIB methods behaved the way theorists advertized them, which was not the case), and stakes that were more substantial. Inferred discount rates were now at the level of consumer credit cards: high, but believable (e.g., Coller and Williams, 1999; Harrison et al., 2002). The third generation of experiments recognized that discount factors equalize time-dated utility, and not time-dated money, so one needed to account for diminishing marginal utility when inferring discount factors. This is a simple matter of theory, from the conceptual definition of a discount factor. Jensen's Inequality does the rest theoretically: inferred discount rates must be lower if one has a concave

utility function than if one assumes a linear utility function. Appropriate experimental designs and econometric inferences then simply quantify this insight from theory, with a dramatic reduction in estimated discount rates down to 10% or even lower (e.g., Andersen et al., 2008).

Quite apart from the level of discount rates, there appears to be no support for “hyperbolicky” specifications of the discounting function in field data (e.g., Andersen et al., 2011a). This does not mean that exponential specifications are appropriate for all populations, just that the monolithic presumption in favor of non-exponential specifications is not supported by the data.

Subjective beliefs can be elicited using scoring rule procedures that have a venerable tradition, such as Savage [1971]. These procedures do require that one correct for risk attitudes, and only directly elicit true subjective beliefs under the assumption of risk neutrality. But it is a relatively simple matter to condition inferences about beliefs on the estimated risk attitudes of individuals, by combining experimental tasks that allow one to identify the risk attitudes independently of the task that elicits subjective beliefs (e.g., Andersen et al., 2010, 2012).<sup>11</sup> One can also use generalizations of these scoring rules to elicit whole subjective probability distributions, rather than just one subjective probability (e.g., Mathieson and Winkler (1976) for the theory). This area is the least developed of the three, but the experimental tools are in place for rigorous elicitation, and are being widely applied.

---

<sup>11</sup>This joint estimation approach does require the maintained, identifying assumption that risk attitudes over objective probabilities are the same as risk attitudes over subjective probabilities. Although consistent with subjective expected utility, this assumption is considered controversial by some, such as Abdellaoui et al. (2011).

It should be stressed that there are also many loose claims about how one can elicit risk attitudes, time preferences, and subjective beliefs “on the cheap” with simpler methods. In some cases these are hypothetical survey methods, with no theoretical claim to be eliciting anything of interest. In other cases these are experimental methods that rely, as noted, on tasks that are simply not incentive compatible: subjects could exploit the experimenter, for gain, by deliberately misrepresenting their true preferences. Or experimenters use FIB elicitation methods that have known behavioral biases, as noted above. The fact that experimenters assert that these problems did not arise says nothing about whether they do. The existence of relatively transparent, incentive compatible methods leads one to wonder why one would risk using other methods.

It is appropriate that all of these methods were first developed in laboratory environments, and that the econometric procedures for estimation of preferences and beliefs first refined in that setting. Lab experiments give us control, if designed and executed correctly. If we cannot identify the conceptually correct measure in that setting, we cannot hope to do so in more complicated field settings. But there is a relatively easy bridge between the lab and the field, as stressed by Harrison and List (2004), so that both are complementary ways to make inferences (Harrison et al., 2012).

### **1.4.2 Letting the Data Speak for Itself**

Randomized evaluations, inspired by the Randomized Control Trials (RCT) literature in health, have become popular in economics. They involve the deliberate use of a randomizing device to assign subjects to treatment, or the exploitation of naturally

occurring randomizing devices. Good reviews of the methodology are contained in Duflo (2006), Duflo and Kremer (2005), Duflo et al. (2007), and Banerjee and Duflo (2009). Complementary econometric strategies are well described in Angrist and Pischke (2009). One of the claimed advantages of randomization is that the evaluation of policies can be “hands off,” in the sense that there is less need for maintained structural assumptions from economic theory or econometrics. In many respects this is true, and randomization does indeed deliver, on a good, asymptotic randomizing day, orthogonal instruments to measure the effect of treatment. This has been well known for a long time in statistics, and of course in the economics experiments conducted in laboratories for decades. But it is apparent that the case for randomization has been dramatically oversold: even if the original statements of the case have the right nuances, the second generation of practitioners seem to gloss those. Words such as “evidence based” or “assumption free” are just marketing slogans, and should be discarded as such. Excellent critiques by Rosenzweig and Wolpin (2000), Keane (2010), Heckman (2010), Deaton (2010), and spirited defenses by Imbens (2010), cover most of the ground in terms of the statistical issues.

### **1.4.3 General Methodological Issues for Experiments in**

#### **Insurance Contexts**

There is an issue of *framing* that must be carefully dealt with when conducting experiments in the context of insurance. Tversky and Kahneman (1986, p. S253) emphasized the importance of violations of the *principle of invariance* which implies that “different representations of the same choice problem should yield the same preference.”

There is experimental evidence that framing an insurance task as an abstract gamble or as a loss makes a difference to subjects. Hershey and Schoemaker (1980), using hypothetical questions, found that their subjects exhibited more risk aversion in choices that were presented in an insurance context than in mathematical equivalent choices presented as standard gambles. Moore and Eckle (2003, p. 11) elicited, using real stakes, certainty equivalents of prospects using two frames, “describing gambles as abstract ‘lotteries’ or as ‘investment/insurance’ decisions.” Moore and Eckle (2003, p. 21) found that there is no significant “difference between gambles framed as lotteries for possible losses versus insurance decisions” but found a preference for prospects framed as investment decisions as opposed to abstract lotteries (p. 19). The logical conclusion is that, at least in the pool of subjects of Moore and Eckel, the framing of a prospect matters for individuals. Similarly, Laury et al. (2009, p. 18) find that subjects are more likely to buy insurance when their task is framed in a “less abstract context and express losses in dollar terms.”

An important body of experimental literature studies insurance choices by eliciting valuations with different market institutions such as auctions. For example, Shogren (1990) studies the difference in subjects’ valuations of different risk-reduction mechanisms by using Vickery sealed-bid second-price auctions and found that subjects tend to value more self-protection to self-insurance and private mechanisms were valued more than collective mechanisms. Shogren (1990) also found that subjects tend to overweight-low probability insurable events as implied by the higher valuation with respect to events with higher probability and values of risk-reduction mechanisms change rapidly with repeated exposure to the market. McClelland et al. (1993) and Di Mauro and Maffioletti (1996) study,

respectively, low-probability events and risk-reduction strategies by eliciting valuations in auction environments. Sarin and Weber (1993) studies the effect of uncertainty in insurance markets. Using the sealed bid auction and the double oral auction, they found that aversion to uncertainty around probabilities does not vanish in market settings and claim that this result might be relevant to specialized markets like insurance where uncertainty might be acute.

Although market institutions are valid and important ways to elicit responses from individuals about many insurance situations, they might not be environments in which many other insurance decisions are taken. Reinsurance contracts or corporate insurance might be situations in which clients might compete and bargain to get coverage from insurers. In contrast, most of real individual insurance choices are of the take-it-or-leave-it type without little bargaining process with the insurer, although menus might be provided and pressure on the insurance provider might come from the possibility of getting similar coverage from another competing insurer. Therefore, methods of elicitation that resemble this take-it-or-leave-it nature might be important for the study of insurance choices and comparisons with other methods of elicitation might uncover behavioral differences between methods and potential confounds. For instance, using data from binary choices, Ganderton et al. (2000) found no evidence of the bimodal distributions of willingness to pay for low-probability insurance. McClelland et al. (1993) elicited using auctions. Ganderton et al. (2000) used dichotomous choices to estimate the willingness to pay for insurance in a logit regression that models the probability of buying insurance using the methods developed by Cameron (1988).

The Random Incentive Lottery Mechanism (RLIM) is probably the payoff mechanism most widely used in experiments of individual choice under risk and uncertainty. Many experiments that study insurance decisions also use the RLIM, except for a few cases as Deck (2001) who provided subjects with only one choice to make. In an experiment where the subjects have to complete several tasks, this mechanism dictates that at the end of the experiment one task is chosen at random to be play out for real, while the other tasks are not. By using the RLIM, an experimenter hopes to avoid portfolio effects, which arise when all tasks are played at the end of the experiment. Also, wealth effects can be avoided by not having to pay all tasks sequentially. A by-product of this procedure is that higher stakes can be considered.

The universal validity of the RLIM as an incentive compatible payoff mechanism has been put into question recently. Cox et al. (2011) and Harrison and Swarthout (2012) found evidence that payment protocols affect observed behavior and can create confounds. For instance, the RLIM could make an individual behave as if she was more or less risk averse than she would normally behave. Harrison and Swarthout (2012) propose to use experimental designs that provide subjects with one and only one choice and control for heterogeneity at the econometric analysis stage. This issue imposes important challenges for the study of insurance choices in the laboratory given that many interesting questions include intertemporal dimensions; and unless one is willing to make intertemporal comparisons across subjects, an experimenter would have to elicit several choices from the same individual.

We have discussed the relevance of the distinction between risk, uncertainty and ambiguity

for the study of insurance decisions. This is an interesting and promising line of research but the empirical validation of these models is not trivial because one has to identify, depending on the model, many factors: subjective beliefs, attitudes towards risk and uncertainty and, potentially, intertemporal preferences. There has been a few attempts to directly estimate attitudes towards uncertainty in experimental environments. Becker and Brownson (1964) is probably the first attempt to estimate a preference functional that takes into account uncertainty attitudes. They estimate the preference functional under uncertainty proposed by Ellsberg (1961, p. 664). More recently, Ahn et al. (2011) and Hey et al. (2010) attempt to make a systematic comparison of many of the models available that deal with uncertainty. One problem with this approach is that the researcher has to make identifying assumptions over the behavioral moving parts, such as the set of probabilities that an uncertainty non-neutral individual perceived as reasonably possible, which might affect the results. For instance, even if the experimenter tells the subject that the probability of a certain event happening is guaranteed to be within certain range, it is still very possible that the subject distrusts the experimenter and make an adjustment to the stated probability interval to account, from her perspective, for the possibility that the experimenter is trying to rig the experiment to minimize its actual cost. In fact, this is an issue that was originally acknowledged by Ellsberg (1961, p. 658) and Ellsberg (2001, p. 131), and also by others: Brewer (1963, p. 161), Brewer and Fellner (1965, p. 661), Schneeweiss (1973) and Kadane (1992), that can create serious confounds in experiments since a subject distrustful of the experimenter cannot be distinguished from one that is uncertainty averse. This is particularly relevant if one wants to study insurance decisions in

the presence of a background risk or uncertainty, because distrust in the experiment can be confounded with the latter. Also, distrust in the experimenter might induce subjects to predominantly choose a coinsurance contract over a contract with a deductible if they think that the experimenter is going to rig the experiment such that the amount of the loss is never going to be above the deductible.

## 1.5 Further Topics

### 1.5.1 Intertemporal Insurance: the Annuity Puzzle

One of the most studied insurance issues in economics is what is usually referred to as the annuity puzzle. Modigliani (1986, p. 307) drew attention to this issue in his Nobel Prize acceptance speech: "... it is a well-known fact that annuity contracts, other than in the form of group insurance through pension systems, are extremely rare. Why this should be so is a subject of considerable current interest. It is still ill-understood."

The "puzzling" nature of the topic arises from the discrepancies between reality, where low take-up of annuities is observed in many contexts, and classic theory (Yaari, 1965a), which predicts that under certain restrictive assumptions full annuitization of wealth is the optimal asset allocation for retirement savings. To be explicit, the set of assumptions in Yaari (1965a) are: (1) individuals are EU maximizers with inter-temporally separable and additive utility, (2) the only source of uncertainty is the time of death, (3) individuals have no bequest motive, (4) life annuities are available at actuarially fair prices.

However, it must be recognized that the low take-up of annuities is not a generalized

phenomenon. For instance, in Chile about two-thirds of retirees buy annuities (Mitchell and Ruiz, 2009, p. 19). For the United States, Benartzi et al. (2011, p. 151) found that as much as 88% of retirees of certain retirement plans choose to buy annuities when given the choice.

Explanations of the Annuity Puzzle within Classical Frameworks There is a large literature that attempts to explain the puzzle within the classical framework (for reviews see Brown, 2007; Benartzi et al., 2011). Babel and Merrill (2006, p. 3) summarizes some of the possible explanations: (1) a bequest motive; (2) health shocks; (3) self-insurance through family or other networks; (4) Social Security crowding out private annuitization; (5) premiums above actuarially fair prices; (6) adverse selection; (7) regulatory barriers for retirement plan sponsors; (8) high-profile failures of insurance companies; (9) irrevocability and illiquidity; (10) imperfect information; and (11) erosion of purchasing power due to inflation.

Several studies have relaxed some of the original assumptions of Yaari (1965a) and found that full annuitization of retirement wealth is still optimal, which has strengthened the perceived “puzzling” nature of the phenomenon. For instance, Davidoff et al. (2005, p. 1578) found that

*...with complete markets, the result of full annuitization extends to many periods, the presence of aggregate uncertainty, actuarially unfair but positive annuity premiums, and intertemporally dependent utility that need not satisfy the expected utility axioms. This generalization of Yaari holds, so long as markets are complete. Thus, if the puzzle of why so few individuals voluntarily annuitize is to be solved within a rational, life-cycle framework, we have shown that the answer does not lie in the specification of the utility function, per se, (beyond the issue of a bequest motive)... The results do demonstrate, however, the importance of market completeness...*

We return to the role of additivity of the intertemporal utility function below.

We have emphasized throughout the importance of incomplete markets for insurance decisions. Schlesinger and Doherty (1985, p. 402) claim that with market incompleteness, uninsurable background risks might invalidate many theoretical results when complete markets are assumed. Babbel and Merrill (2006, p. 1) examine the possibility of insurer default, and find that even a small default risk, which can be thought of as background risk, can have important implications for annuity purchase decisions.<sup>12</sup> Peijnenburg et al. (2010) modeled explicitly market incompleteness and the presence of background risk in a comprehensive model of optimal decumulation of retirement wealth. They find that neither “incomplete annuity markets nor background risk lead to a sizeable reduction of optimal annuitization levels,” if individuals are able to save part of their annuity income to insure against shocks.

This line of research could benefit from the accumulated knowledge about insurance decisions in the presence of background risk, but will have to face several challenges such as choosing an appropriate measure of statistical dependence between different sources of risk.

It has long been recognized in the insurance literature (Schlesinger and Doherty, 1985, p. 420) that correlation as a measure of association is only of limited use.

---

<sup>12</sup>Parallels can be made to the explanations of the Ellsberg Paradox offered by Kadane (1992) and Schneeweiss (1973): just add a small chance that the experimenter is playing a strategic game with the subject, and picking realizations from the ambiguous urn to reduce his expected payout. Then no subject would rationally select to bet on realizations from the ambiguous urn.

## Behavioral Explanations of the Annuity Puzzle

There is a growing body of literature that uses deviations from the behavioral assumptions of the standard theory to explain the low take-up of annuities. A non-exhaustive list of the most salient behavioral explanations to the phenomenon are ( for a review see Brown, 2007; Mitchell and Utkus, 2004): (1) complexity of products and financial literacy; (2) misleading heuristics: Insurance is for bad situations; (3) regret aversion; (4) the illusion of control; (5) uncertainty aversion; (6) self-control, hyperbolic discounting and their implication to retirement saving; (7) framing, inertia and default choices; (8) mental accounting and loss aversion: annuities are a gamble; and (9) Cumulative Prospect theory, probability weighting and loss aversion.

One important shortcoming of Non-EUT models (e.g., uncertainty aversion models, Cumulative Prospect Theory, the Rank-Dependent Utility) that is usually not recognized is that these frameworks are not naturally endowed with tools to deal with inter-temporal problems such as the decision to annuitize or not one's wealth at the moment of retirement. Hu and Scott (2007) is an exception who model the inter-temporal nature of annuities within a Prospect Theory type. Hu and Scott (2007, p. 9-11) use this approach to make a CPT evaluation of an annuity as a one stage prospect where the probabilities are the odds of survival at each age and the outcomes are the discounted value of the payout flows.

There are at least three related theoretical approaches to solve this deficiency.

The first approach is to think of annuities as complex compound lotteries and model the reluctance to take up these products as violations of the Reduction of Compound Lotteries axiom. The latter is one of the key postulates both of EUT as axiomatized by von

Neumann and Morgenstern (1953) and the Subjective Expected Utility theory as formalized by Savage (1972). Before we explain this approach, following Segal (1988b, 1990, 1992), we distinguish between three axioms: Reduction of Compound Lotteries (ROCL) axiom, Compound Independence Axiom and Mixture Independence Axiom. The usefulness of these distinctions will become evident soon. ROCL states that a decision-maker is indifferent between a compound lottery and the actuarially-equivalent simple lottery in which the probabilities of the two stages (or more) of the compound lottery have been multiplied out. To use the language of Samuelson (1952, p. 671), the former generates a compound income-probability-situation, and the latter defines an associated income-probability-situation, and that “...only algebra, not human behavior, is involved in this definition.” The reason these three axioms are important is that the failure of MIA, a key axiom in EUT, does not imply the failure of CIA and ROCL. It does imply the failure of one or the other, but it is far from obvious which one. Indeed, one could imagine some individuals or task domains where only CIA might fail, only ROCL might fail, or both might fail. Moreover, specific types of failures of ROCL lie at the heart of many important models of decision-making under uncertainty and ambiguity. Appendix A provides a more formal statement of these axioms.

Therefore, a natural candidate to model the reluctance for annuities as violations of ROCL is the family of models that deal with uncertainty and ambiguity, for example, Hurwicz (1951a,b) and Ghirardato et al. (2004), Gilboa and Schmeidler (1989), Schmeidler (1989) and Klibanoff et al. (2005). From a behavioral perspective, Harrison et al. (2012b) study ROCL with objective but this axiom remains to be studied in the subjective probability

domain in which uncertainty and ambiguity play a key role, as emphasized in Ellsberg (1961). He showed in his thought experiments that the observed hypothetical choices contradict the Sure-thing principle in Savage (1972), which has its equivalent in MIA of EUT. Since the latter is implied by ROCL and CIA, aversion towards uncertainty may well be due to violations of ROCL. Camerer and Weber (1992) and Halevy (2007) offer further empirical evidence in line with Ellsberg's findings.

The relevance of uncertainty perception, and attitudes towards it, can be seen in the particular case of an individual that prefers a lump-sum over an annuity product (Hu and Scott, 2007; Brown, 2007, p. 21). This behavior can be rationalized by an individual that perceives a life annuity as an ambiguous prospect for various reasons. For instance, the individual might have the subjective belief that the annuity provider may default but he is unsure about the true distribution of such event. Similarly, the individual might be uncertain about his probability of death. Therefore, an individual might prefer a lump-sum amount to a life annuity if the perceived uncertainty around this product and his attitudes towards uncertainty are such that the certainty equivalent of the ambiguous annuity is smaller than the lump-sum amount. To the best of our knowledge only d'Albis and Thibault (2009) attempt to explain the low take-up of annuities through aversion towards mortality uncertainty. However, as they recognize, the problem must be studied in an inter-temporal context that allows for uncertainty. This is a relatively new avenue of research, but there have been advances on this front such as Epstein and Schneider (2003) and Klibanoff et al. (2009), who axiomatized the inter-temporal versions of the multiple-priors Maximin Expected Utility and the smooth ambiguity aversion model,

respectively. Further refinements can include other traded and non-traded assets that can be ambiguous or not, as well as non-insurable background risks. It is worth noting that uncertainty and attitudes towards uncertainty can be used to formally model the intuition in the behavioral literature that people perceive annuities as a bad deal (Benartzi et al., 2011, p.156).

A second approach is to use axiomatic foundations that allow non-EUT models to evaluate prospects that are inter-temporal in nature. Sarin and Wakker (1998, p. 88) provided an axiomatic approach that identifies conditions under which non-EU models can be used in problems of dynamic choice under uncertainty and which is summarized in the following excerpt:

For the evaluation of decision trees, we propose a new condition, sequential consistency. It is a meta-principle for the folding back procedure. Suppose the decision maker commits himself to using a family of models  $M$ , say the rank-dependent family. Thus he uses the family  $M$  to evaluate certainty equivalents in the folding back procedure. The meta-principle requires that he uses this family  $M$  to evaluate decision strategies in the single-stage tree as well. In other words, it requires that the evaluation of a strategy obtained by using a family  $M$  through the folding back procedure should coincide (in the sense of giving the same certainty equivalents) with a direct evaluation of the same strategy in the single-stage tree using the same family  $M$ .

An important caveat in Sarin and Wakker (1998, p. 96) is that not all families of models can apply the same preference representation in each stage of the tree. For instance, they derived a corollary which shows that the rank-dependent model can only be used in the second stage of a decision tree but must maximize EU in the first stage. An alternative approach that does not suffer from this issue is the one developed by Segal (1988b). He derived a version of the Rank-Dependent utility model by Quiggin (1982) that can deal

with compound lotteries. Segal drops ROCL but keeps CIA which allows the RDU model to be able to evaluate compound lotteries in a folding back procedure. The latter consists of estimating certainty equivalents within stages and applying RDU preferences on them to do the folding back. In contrast with the collorary in Sarin and Wakker (1998), this approach allows to consistently apply one type of preferences in each stage of the compound lottery. Finally, another approach claims that people dislike uncertainty, just like in Ellsberg (1961), and behave in a similar manner when presented with a compound lottery. Smith (1969, p. 329) conjectured this relationship:

I would conjecture that the kind of choice behavior revealed by the Ellsberg counterexample is not just characteristic of ‘known’ versus ‘unknown’ probability situations. I suspect that the preference for Urn II [the one with known composition] gambles over Urn I [the one with unknown composition] gambles would be little changed for many, if not most, subjects, if they were guaranteed that the number of red balls in Urn I had been determined by a random draw from the integers 0-100. Urn I is then a 50-50 gamble except that it is a compound gamble, rather than a simple gamble as in Urn II.

Halevy (2007, p. 531) found evidence of this claim. He found a tight association between violations of ROCL and uncertainty aversion. To model his conjecture, Smith (1969, p. 328-329) proposed a preference representation that keeps the SEU structure but allows the utility function to vary across prospects. This could explain why people may prefer risky over ambiguous gambles as in Ellsberg’s thought experiments. The argument relies on the assumption that subjects potentially view or experience differently each random processes that they might face, which is why this line of research is known as source dependence models. A different brand of source dependent models was recently developed by Abdellaoui et al. (2011). Building upon Tversky and Kahneman (1992) and Tversky and

Fox (1995), they emphasized the importance of sources of uncertainty as an explanation of behavior under uncertainty in a non-SEU framework. The key component in their model is the probability weighting function, which they allow to vary across sources of uncertainty. They claim that this preference representation is completely characterized by a subjective probability distribution, the von-Neumann-Morgenstern utility function and the source dependent weighting function. Abdellaoui et al. (2011) argue that the importance of the concept of decision weights is fundamental to explain several behavioral patterns in decisions under uncertainty. Source dependent models should be explored since they predict that different financial products as annuities or stocks can be perceived as different sources and treated differently which has the potential to explain puzzles in decisions under risk and uncertainty.

## 1.5.2 Information Problems

The presence of informational problems in insurance contracts certainly has an important impact on the analysis of insurance demand and provision. A full assessment of this literature is beyond the scope of the present survey. However, we make some comments of immediate relevance for the topics surveyed in this document.

First, there is a huge body of literature analyzing moral hazard and adverse selection problems in insurance markets that mostly assumes EUT. However, the analysis of these problems assuming different preference representations like PT or RDU and preferences sensitive to uncertainty and ambiguity remains underdeveloped. The analysis of these problems, which started with Arrow (1963), Akerlof (1970) and Rothschild and Stiglitz

(1976), has been masterfully surveyed by Dionne and Doherty (1992), Winter (2000) and Dionne et al. (2000). A perfect example of this research is Huang et al. (2007) who study the market implications of individuals with preference representations sensitive to regret. Second, some informational problems might be difficult to study in the laboratory. In particular, fraud in insurance contexts is an issue difficult to study because the experimenter has to be careful not to nudge fraudulent behavior and lose control of the experiment. Experimenters interested in this topics can benefit from the experimental literature on public goods and the elicitation mechanisms develop in this literature.

### **1.5.3 Asset Integration**

A common assumption in many models of insurance, even those that consider portfolios and related risk management strategies, is perfect asset integration within a given time period. This amounts to the assumption that there exist perfect markets that allow all assets to be traded, and aggregated into one scalar value of wealth. This scalar wealth is then used as the sole argument of some utility function. The same issue arises in an intertemporal context. When imperfect capital markets exist, it is no longer possible for the individual to aggregate time-dated wealth or consumption into one aggregate.

When imperfect markets are assumed, things change fundamentally. Pye (1966) considered the implications for the optimal investment rule of a company, and demonstrated that imperfect capital markets implied that there no longer existed a “utility free” investment rule, such as implied by the Fisher Separation Theorem. That utility free rule held that production and consumption decisions could be separated, and that one does not need to

know the utility function of the agent in order to identify optimal investment and production. Pye (1966) and Hirshleifer (1970, ch. 7) showed that when capital markets were imperfect, in general one could not define the intertemporal budget constraint without knowing the utility function of the individual.

More generally, imperfect markets force one to consider multivariate risk aversion when evaluating insurance demand. Since it is then no longer possible to aggregate to a scalar wealth measure, one must pay attention to the utility evaluation of two or more components of wealth with tools of multivariate risk aversion. Generalizations of the one-dimensional Arrow-Pratt measure of risk aversion have been proposed by Kihlstrom and Mirman (1974), Duncan (1977) and Karni (1979). Kihlstrom and Mirman (1974) posed the issue of multivariate risk aversion under the restrictive assumption that the ordinal preferences underlying two expected utility functions exhibit the same preferences over non-stochastic outcomes. In this case they propose a scalar measure of total risk aversion that allows one to make statements about whether one person is more risk averse than another in several dimensions, or if the same person is more risk averse after some event than before.

If one relaxes this assumption, which is not an attractive one in most applications, Duncan (1977) shows that the Kihlstrom and Mirman (1974) multivariate measure of risk aversion naturally becomes matrix-valued. Hence one has vector-valued risk premia, and this vector is not “direction dependent” in terms of evaluation. Karni (1979) shows that one can define the risk premia in terms of the expenditure function, rather than the direct utility function, and then evaluate it “uniquely” by further specifying an interesting statistic of

the stochastic process. For example, if one is considering risk attitudes towards a vector of stochastic price shocks, then one could use the mean of those shocks.

#### 1.5.4 Correlation Aversion

A closely related literature defines multi-attribute risk aversion where the utility function is defined over more than one attribute. In this context, Keeney (1973) first defined the concept of conditional risk aversion, Richard (1975) defined the same concept as bivariate risk aversion, and Epstein and Tanny (1980) defined it as correlation aversion.<sup>13</sup> There are several ways to extend these pairwise concepts of risk aversion over two attributes to more than two attributes, as reviewed by Dorfleitner and Krapp (2007).

One attraction of the concept of multiattribute risk aversion is that it allows a relatively simple characterization of the functional forms for utility that rule out multiattribute risk attitudes: additivity. To see the significance of this for insurance demand, consider time-dating as the attribute in question. If one assumes the popular additive intertemporal utility function, one rules out correlation aversion. In this case, as is well know, a-temporal risk preferences and the intertemporal elasticity of substitution cannot be estimated or calibrated independently: one is the inverse of the other. But with non-additive intertemporal utility functions, one can immediately separate “risk preferences” and “time preferences.” And one can then talk about individuals having preferences for how risk is resolved over time, the essence of any insurance contract. That is, preferences for how risk is resolved over time can be distinct from preferences for how risk is resolved at any given

---

<sup>13</sup>Several studies note that the core concept appeared as early as de Finetti (1952), but this was written in Italian and we cannot verify that claim.

point of time, and hence be a separate behavioral determinant of the demand for insurance. Controlled experiments provide a way to identify and estimate the degree of correlation aversion, and Andersen et al. (2011c) present evidence that it exists and is significant for the Danish population.

### 1.5.5 Conclusions

Behavioral economics has generated a rich set of alternative models of decision-making. Not all of them are valid, and not all of the anomalies that one hears about have rigorous support. It is critical that the emerging field of behavioral insurance trade carefully among the theories, experimental designs, and econometrics that behavioral economics provides. The “behavioral moving parts” of the demand for insurance demand a careful, structural evaluation of the way in which decisions are made. This conclusion holds obviously for the descriptive evaluation of why people demand insurance in some settings and not others. But it assumes a critical role when we move beyond descriptive analysis to evaluate the welfare effects of alternative insurance products and insurance policies. Our survey points to the importance of theorists being listened to by experimenters and econometricians interested in insurance behavior. It also generates a derived demand for theorists to be more directly involved in the study of behavior.

### 1.5.6 Appendix A: Basic Axioms

Following Segal (1988b, 1990, 1992), we distinguish between three axioms. In words, the **Reduction of Compound Lotteries** (ROCL) axiom states, using the notation to be used to state all axioms, let  $X$ ,  $Y$  and  $Z$  denote simple lotteries,  $A$  and  $B$  denote compound lotteries,  $\succ$  express strict preference, and  $\sim$  express indifference. Then the ROCL axiom

says that  $A \sim X$  if the probabilities and prizes in  $X$  are the actuarially-equivalent probabilities and prizes from  $A$ . Thus if  $A$  is the compound lottery that pays “double or nothing” from the outcome of the lottery that pays \$10 if a coin flip is a head and \$2 if the coin flip is a tail, then  $X$  would be the lottery that pays \$20 with probability  $0.5 \times 0.5 = 0.25$ , \$4 with probability  $0.5 \times 0.5 = 0.25$ , and nothing with probability 0.5. From an observational perspective, one would have to see choices between compound lotteries and the actuarially-equivalent simple lottery to test ROCL.

The **Compound Independence Axiom** (CIA) states that a compound lottery formed from two simple lotteries by adding a positive common lottery with the same probability to each of the simple lotteries will exhibit the same preference ordering as the simple lotteries. So this is a statement that the ordering of the two constructed compound lotteries will be the same as the ordering of the different simple lotteries that distinguish the compound lotteries, provided that the common prize in the compound lotteries is the same and has the same (compound lottery) probability. It says nothing about how the compound lotteries are to be evaluated, and in particular it does not assume ROCL. It only restricts the preference ordering of the two constructed compound lotteries to match the preference ordering of the original simple lotteries.

The CIA says that if  $A$  is the compound lottery giving the simple lottery  $X$  with probability  $\alpha$  and the simple lottery  $Z$  with probability  $(1 - \alpha)$ , and  $B$  is the compound lottery giving the simple lottery  $Y$  with probability  $\alpha$  and the simple lottery  $Z$  with probability  $(1 - \alpha)$ , then  $A \succ B$  iff  $X \succ Y \forall \alpha \in (0, 1)$ . So the construction of the two compound lotteries  $A$  and  $B$  has the “independence axiom” cadence of the common prize  $Z$  with a common probability  $(1 - \alpha)$ , but the implication is only that the ordering of the compound and constituent simple lotteries are the same.<sup>14</sup>

Finally, the **Mixture Independence Axiom** (MIA) says that the preference ordering of two simple lotteries must be the same as the actuarially-equivalent simple lottery formed by adding a common outcome in a compound lottery of each of the simple lotteries, where the common outcome has the same value and the same (compound lottery) probability. So stated, it is clear that the MIA strengthens the CIA by making a definite statement that the constructed compound lotteries are to be evaluated in a way that is ROCL-consistent. Construction of the compound lottery in the MIA is actually implicit: the axiom only makes observable statements about two pairs of simple lotteries. To restate Samuelson's point about the definition of ROCL, the experimenter testing the MIA could have constructed the associated income-probability-situation without knowing the risk preferences of the individual (although the experimenter would need to know how to multiply).

---

<sup>14</sup>For example, Segal (1992, p. 170) defines the CIA by assuming that the second-stage lotteries are replaced by their certainty-equivalent, “throwing away” information about the second-stage probabilities before one examines the first-stage probabilities at all. Hence one cannot then define the actuarially-equivalent simple lottery, by construction, since the informational bridge to that calculation has been burnt.

## Chapter 2

# Insurance Decisions Under Ambiguity

*by Jimmy Martínez-Correa*

### Abstract

I study the impact of ambiguity on insurance decisions and the optimality of insurance contracts. My tractable approach allows me to study the interaction between risk and ambiguity attitudes. When insurance decisions are made independently of other assets, for a given increase in wealth, both risk and ambiguity attitudes interact in nontrivial ways to determine the change of coinsurance demand. I derive sufficient conditions to guarantee that the optimal coinsurance demand is decreasing in wealth. When a non-traded asset is introduced, my model predicts behavior that is inconsistent with the classical portfolio theory that assumes Subjective Expected Utility theory; however, it provides hints to a possible solution of the under-diversification puzzle of households. I also identify conditions under which more risk or ambiguity aversion decreases the demand for coinsurance. Additionally, I show a counterexample to a classical result in insurance economics where an insurance contract with straight deductible is dominated by a coinsurance contract. Finally, I find that a modified Borch rule characterizes the optimal insurance contract with bilateral risk and ambiguity attitudes and heterogeneity in beliefs.

## 2.1 Introduction

Ellsberg (1961) studied the distinction between risk and ambiguity and its relevance for decision-making theory. He used a thought experiment to show that under certain situations many reasonable people tend, even after reflection, not to comply with the Savage (1972) postulates for subjective expected utility (SEU) theory.

Through his thought experiment, Ellsberg (1961) highlighted the importance of ambiguity and attitudes towards it. Camerer and Weber (1992) and Halevy (2007) are examples of studies that provide empirical evidence consistent with Ellsberg's findings.

I answer the following questions: What is the impact of ambiguity on the optimal coinsurance demand? Does this analysis change if the individual owns a non-tradable asset (e.g., Human capital)? What is the optimal insurance contract in the presence of ambiguity when both the insurer and insured might be ambiguity averse? Does this analysis have implications for portfolio theory?

A particular advantage of my approach is its tractability, which allows us to analyze the interaction between risk and ambiguity attitudes. However, this comes at the cost of generality because the approximation used to solve the problem is constructed to perform well in the *small*. Since Pratt (1964) the notion of small risks is relatively well understood. However, the concept of *small ambiguity* (or uncertainty)<sup>1</sup> is less straightforward<sup>2</sup> and is characterized by the convergence of the remainder in the approximation used here. My approach might be of special interest for experimental economics since the stakes and

---

<sup>1</sup>I use ambiguity and uncertainty as equivalent terms. However, there are important subtle differences between the two concepts but it is not the purpose of this study to describe the distinction between them. I follow the customary approach in the recent literature and use the term ambiguity (e.g., Gollier, 2009.)

<sup>2</sup>For a discussion on uncertainty in the small please see Maccheroni et al. (2011b).

conditions in the laboratory are appropriate to generate small risks and/or uncertainties.

A clear prediction of my model is that, for a given increase in risk aversion, ambiguity averse individuals will increase their coinsurance demand less rapidly than ambiguity neutral agents. The intuition is that higher ambiguity aversion makes a marginal increase in insurance more valuable. Therefore, when risk aversion increases and more insurance is required to reduce variance, a smaller increase in insurance is needed because the additional coverage provides the double benefit of reducing risk and ambiguity.

Moreover, the response of the optimal coinsurance demand to changes in initial wealth will depend on the attitudes towards risk *and* ambiguity. For instance, given an increase in wealth, a subject with constant absolute risk aversion could still decrease her insurance demand if she exhibits decreasing absolute ambiguity aversion. I derive sufficient conditions to guarantee that the optimal coinsurance demand is decreasing in wealth.

A topic that is often overlooked in the analysis of insurance and portfolio choice is the presence of non-traded assets, such as Human capital. Cochrane (2007, p. 78) claims that:

I have emphasized outside income [e.g., the return of human capital]..., even though it is rarely discussed in the modern portfolio theory literature. I think it's the most important and most overlooked component of portfolio theory, and that paying attention to it could change academic theory and the practice of the money management industry in important ways.

Mayers and Smith (1983) were the first to emphasize the importance of studying insurance decisions in the presence of traded and non-traded assets. Doherty and Schlesinger (1983) and Doherty (1984) studied a similar problem where the insurance demand can be affected by a background risk that is not insurable.

A non-traded asset introduces a new dimension to the insurance analysis under ambiguity.

An individual is able to “*self-hedge*” if she can compensate high losses with high realizations of her non-traded human capital.

When ambiguity matters to a decision-maker and there exists a non-traded asset, an increase in risk aversion may or may not increase the demand for insurance depending on the incentives to “self hedge.” This result is a counterexample to the Pratt-Arrow result which claims that higher risk aversion decreases the demand for the risky asset. This might happen when a marginal increase in insurance also increases the variance of wealth. Hence, an increment in risk aversion induces a reduction in the insurance demand. However, since an insurance decision problem can be interpreted as a portfolio problem, there is a much deeper result that has implications for portfolio theory and that drives this counterintuitive comparative static for risk aversion.

An important result is that, in the presence of ambiguity, the optimal risk-return allocation will not always be located in the efficient part of the classical mean-variance frontier. The reason is that high incentives to self-hedge create a trade-off between the risk and ambiguity dimensions of wealth that a decision maker must solve. Thus, an individual could appear to be “inefficient” from the perspective of classical portfolio analysis that assumes SEU utility maximization, although such action is optimal in the presence of ambiguity. I show that the counterintuitive comparative static for risk aversion that I described above can only arise when the optimal choice of an ambiguity averse individual lies in the “inefficient” part of the frontier.

A generalization of my model that includes traded assets, non-traded assets and insurance can be an alternative explanation for the under-diversification puzzle of households. The

interaction of traded assets, non-traded assets and insurable assets, that can be each ambiguous or not, can result in observed behavior that is “under-diversified” from a classical perspective but optimal from a broader perspective. The main message is that deviations from traditional portfolio theory can be explained by expanding the concept of portfolio to include non-traded assets, to allow for preference representations that can explain attitudes towards ambiguity.

A simple extension of my framework that includes a risky asset, an ambiguous asset and risky income can rationalize “seemingly” irrational behavior. Massa and Simonov (2006) found that Swedish investors tend to hold stocks that are positively correlated with their labor income, probably because these stocks are familiar to them, while wealthy individuals have a greater tendency to pick stocks that can help hedging their labor income risk.

Standard explanations consider that wealthy investors are more sophisticated, while the observed positively correlated portfolios are usually explained by “behavioral biases” such as overconfidence, ignorance and familiarity. However, the simple model I propose suggests an alternative explanation. First, people exhibit ambiguity aversion towards less familiar stocks, which give them incentives to tilt their financial portfolio towards more familiar stocks, even if that sometimes means that the chosen portfolio is positively correlated with labor income. Second, an individual with sufficiently high initial wealth can overcome aversion towards ambiguity, just like a risk averter might diminish her aversion towards risk when wealth increases, and change his choices from portfolios tilted towards the risky asset to allocations that put more weight on the ambiguous asset.

In the last section I study the optimality of insurance contracts. First, I show that, in

contrast to the traditional result in Arrow (1971), an insurance contract with a straight deductible may be dominated by an equivalent coinsurance contract if the decision-maker owns a non-traded asset. Second, a modified Borch rule characterizes the optimal contract when I allow for bilateral risk and ambiguity aversion and differences in beliefs. In particular, I provide conditions under which the coinsurance schedule with bilateral risk and ambiguity aversion is higher (or lower) than with bilateral ambiguity neutrality. When there is heterogeneity in beliefs no sharp predictions can be made.

To model individual preferences under ambiguity, I use the smooth ambiguity aversion model axiomatized by Klibanoff, Marinacci and Mukerji (2005), KMM henceforth. They derive a preference representation that allows for a separation of the perception of ambiguity and attitudes towards it. Additionally, I adopt the quadratic approximation of the certainty equivalent of the KMM representation derived by Maccheroni et al. (2011a). This is an extension to the ambiguity domain of the Arrow-Pratt approximation, which allows for the model tractability I mentioned above. However, this may come at the cost of generality, since what is necessary and sufficient in the “small” may only be necessary in a more general setting (e.g., Eeckhoudt and Gollier, 2000, p. 126).

Some recent studies have examined the effect of ambiguity in insurance decisions and portfolio choices. Ju and Miao (2012) and Collard et al. (2009) study a dynamic infinite-horizon portfolio problem that allows for time-varying ambiguity and aversion towards it. They find, numerically, that ambiguity aversion increases the equity premium. However, Gollier (2009) found, in a static portfolio choice problem, that these numerical results rely on the particular calibration of the models. He identifies sufficient conditions

under which more ambiguity aversion increases the demand for the ambiguous asset.

Cherbonnier and Gollier (2011) demonstrate that, in the KMM model, restrictions on risk and ambiguity attitudes are sufficient to guarantee that any uncertain situation that is undesirable at one wealth level is also undesirable at a lower wealth level. Nevertheless, they also show that one has to impose restrictions on both risk and ambiguity attitudes, as well as on the ambiguity structure, to guarantee that an increase in wealth will increase the demand of an ambiguous asset. Finally, Alary et al. (2010) and Snow (2011) study the effect of ambiguity on insurance decisions when they are made in *isolation*.

My tractable approach allows me to study in more depth the interactions between risk and ambiguity attitudes and to provide sharp predictions. Also, I consider a more general framework that models insurance decisions in the presence of other traded and non-traded assets, as well as preference representations that are sensitive both to risk and ambiguity. Sections 2.2 and 2.2 present my approach to model decisions under ambiguity and analyzes the impact of ambiguity and attitudes towards it on the optimal coinsurance demand with and without a non-traded asset. Section 2.4 studies the optimality of an insurance contract with a straight deductible and the Borch rule when ambiguity matters to individuals. The final section concludes. Appendices show proofs of results and well-established results in the insurance literature when SEU is assumed.

## 2.2 Modeling Ambiguity and Attitudes towards

### Ambiguity

Consider a decision maker (DM) with initial wealth  $W_0$  and random *end-of-period* wealth  $W$ . Suppose the agent exhibits aversion towards risk, which is captured by a utility function  $u(\cdot)$ , with  $u'(\cdot) > 0$  and  $u''(\cdot) < 0$ .

The DM perceives  $W$  as ambiguous. This implies that, instead of having a unique probability distribution for  $W$ , the subject behaves as if there is a bounded set of probability distributions,  $\Delta$ , that are reasonably possible to her. Therefore, the DM is not able to commit to only use a particular distribution in this set. On the contrary, for a SEU maximizer, the set  $\Delta$  will be a singleton.

Following KMM, the individual has a subjective probability measure  $\mu$  over  $\Delta$  that captures the ambiguity perceived by her. According to KMM, the preferences of an individual that perceives wealth as ambiguous will have the following preference representation:

$$\int_{\Delta} \phi \left( \int_S u(W) dQ \right) d\mu, \quad (2.1)$$

where  $S$  is the state space of  $W$ ,  $Q \in \Delta$  is a probability measure over  $S$ , and  $\phi$  is a map from reals to reals that captures attitudes towards ambiguity. If the subject perceives her future wealth as ambiguous, and is averse to this situation,  $\phi$  will be concave in the same fashion that the preferences of a risk averse individual are represented by a concave utility function  $u(\cdot)$ . An ambiguity loving (neutral) individual will have a convex (linear)  $\phi$ , exactly parallel to the formal characterization of risk attitudes. I focus on the case of

ambiguity aversion and risk aversion.

In the KMM framework, an increase in ambiguity can be characterized as an increase in the (subjective) variance of expected utility  $\int_S u(W)dQ$ , which is a random variable with subjective probability distribution  $\mu$ . We can define a reduced compound probability distribution according to  $\bar{Q} = \int_{\Delta} Qd\mu(Q)$ . An individual that is ambiguity neutral, which is equivalent to behaving according to the postulates in Savage (1972), will maximize the SEU derived from  $u(\cdot)$  and  $\bar{Q}$ .

To make the models tractable, I use the second-order approximation of the CE of the KMM representation developed by Maccheroni et al. (2011a), MMR hereafter. This is an extension to the ambiguity domain of the Arrow-Pratt approximation, where the risk and ambiguity premiums are characterized by the KMM preference representation. This CE is defined by the following equation:

$$\phi(u(\widetilde{CE})) = \int_{\Delta} \phi \left( \int_S u(W)dQ \right) d\mu \quad (2.2)$$

Define  $W = W_0 + h$ , where  $h$  represents the variable component of the end-of-period wealth. MMR approximate  $\widetilde{CE}$  with the following quadratic smooth ambiguity functional that maps square integrable random variables to the reals:

$$\widetilde{CE} = W_0 + E_{\bar{Q}}[h] - \frac{\theta}{2}\sigma_{\bar{Q}}^2(h) - \frac{\gamma}{2}\sigma_{\mu}^2(E[h]) + R_2(h) \quad (2.3)$$

where

$$\lim_{t \rightarrow 0} \frac{R_2(th)}{t^2} = 0. \quad (2.4)$$

The first two terms correspond to the Arrow-Pratt approximation of the CE of an individual that makes decisions according to the SEU defined by the compound probability distribution  $\bar{Q}$  and utility function  $u(\cdot)$ . The term  $\theta = -\frac{u''(W_0)}{u'(W_0)} > 0$  is the local measure of absolute risk aversion. This reflects the risk aversion of an ambiguity neutral individual that behaves consistently with the SEU theory.

The new, third term is an ambiguity premium. The term  $\gamma = u'(W_0)\{-\frac{\phi''(u(W_0))}{\phi'(u(W_0))}\} > 0$  captures the degree of ambiguity aversion (Maccheroni et al., 2011a, p. 6). The term  $\sigma_\mu^2(E[h])$  is the measure of ambiguity around the variable component of wealth perceived by the individual. In other words, it is the (subjective) variance of the expected value of  $h$  under probability measure  $\mu$ . This expected value varies because the individual allows each probability distribution  $Q$  in  $\Delta$  to be a possible candidate to estimate the expected value of  $h$ . Thus, in the finite case, there are  $n$  possible values that  $E[h]$  can take, one for each probability distribution  $Q_i \in \Delta, \forall i : 1, \dots, n$ . Finally, Maccheroni et al. (2011a) interpret  $\sigma_\mu^2(E[h])$  as “model uncertainty” because it represents the possible deviations perceived by agents from a reference individual that maximizes SEU according to distribution  $\bar{Q}$  and utility  $u(\cdot)$ .

The convergence notion of the reminder of the approximation, as described in equation (4), defines the notion of “small” used here. The intuition is that the random component of wealth must be small enough such that the end-of-period wealth is not very different from

initial wealth.

Maccheroni et al. (2011a, p. 9) decompose ambiguous random variables into three orthogonal components. They claim that for each  $h \in L^2$ , there exist unique  $\bar{h} \in \mathbb{R}$ ,  $h^* \in M$ , with  $E_Q[h^*] = 0$  and  $h^\perp \in M^\perp$ , such that  $h = \bar{h} + h^* + h^\perp$ , where

$$M = \{h \in L^2 : \sigma_\mu^2(E[h]) = 0\}.$$

Define  $\sigma_Q^2(h) = \sigma_Q^2(h^*) + \sigma_Q^2(h^\perp)$  and  $\sigma_\mu^2(E[h]) = \sigma_\mu^2(E[h^\perp])$ . Further,  $\bar{h}$  is the risk-free component of random variable  $h$ ,  $h^*$  is a fair risky gamble (i.e., a gamble with zero expected payoff) and  $h^\perp$  is its residual ambiguous component. Maccheroni et al. (2011a) show three possible configurations of a random variable. First, a gamble represented by random variable  $h$  is risk-free if and only if  $h^* = h^\perp = 0$ . Second, it is risky and unambiguous if and only if  $h = \bar{h} + h^\perp$ , because  $\sigma_Q^2(h) = \sigma_Q^2(h^\perp)$  and  $\sigma_\mu^2(E[h^\perp]) = 0$ . Finally, the gamble is ambiguous if and only if  $h^*$  and  $h^\perp$  different from zero.

There are several advantages of the approach to modelling ambiguity aversion used here. Klibanoff et al. (2005, p. 1868) claim that their representation allows for a separation of ambiguity and attitudes towards it, which provides a theoretical basis for undertaking comparative statics of ambiguity. Additionally, by adopting the MMR approximation, the KMM model becomes tractable and comparable to previous literature using the de Arrow-Pratt approximation, of course at the cost of generality that any approximation is exposed to.

## 2.3 Insurance Choices under Ambiguity

In this section I develop a model of coinsurance demand when the DM perceives the potential loss as ambiguous with and without a non-traded asset. I evaluate the robustness of some standard results in the insurance literature that assumes SEU, namely:<sup>3</sup>

1. Full coverage is optimal if coinsurance is available at a fair price. This is the Mossin Theorem developed in Mossin (1968). Smith (1968) found similar results, thus I refer to this result as the Mossin-Smith Theorem.
2. An increase in the degree of risk aversion will lead to an increase in the optimal demand for insurance at all levels of wealth. This result is a direct consequence of a standard result independently derived by Pratt (1964, p. 136) and Arrow (1963), thus I refer to it as the Pratt-Arrow result. They showed that an increase in absolute risk aversion decreases the demand for a risky asset. This would imply that a risk averse agent will demand more insurance to cover the loss than another individual that is less risk averse.<sup>4</sup>
3. If a subject exhibits decreasing absolute risk aversion (DARA), the optimal coinsurance demand decreases when initial wealth increases because her risk tolerance increases. Similar intuitive arguments can be made about CARA and IARA. However, Schlesinger (2000, p. 136) warns that “each of these conditions [DARA, CARA and IARA] is shown to be sufficient for the comparative-static

---

<sup>3</sup>See Appendices B and C at the end of this chapter for the detailed derivation of the following results.

<sup>4</sup>In the context of insurance, the risky asset would be the retained loss. Thus, more insurance would translate into a lower exposure to the risky asset.

effects[...], though not necessary.”

4. The introduction of a non-traded asset, such as human capital, can significantly change some of the standard results. Mayers and Smith (1983) were the first to emphasize the importance of studying insurance decisions in the presence of traded and non-traded assets. Doherty (1984, p. 209) showed that the Mossin-Smith theorem only holds if the covariance between the non-traded asset and the insurable loss is negative. Following the terminology in Mayers and Smith (1983, p. 308), this covariance represents the individual’s incentives to “*self-insure*”. The sign of this covariance makes the insurance demand lower, equal or higher than in the absence of the non-traded asset. However, I prefer to use the term “*self-hedging*” to avoid confusion with the usage of “self-insurance” in the literature.<sup>5</sup> The intuition is that even if the insurance premium is fair, the DM might still not be willing to fully insure if she can compensate high losses with high realizations of her human capital. However, if the covariance is negative, the individual might want to fully insure if a health shock negatively affects her productivity, which would undermine her human capital. Finally, the introduction of a risky non-traded asset does not affect the Arrow-Pratt result.

I now turn to check the robustness of these results when ambiguity matters to individuals.

---

<sup>5</sup>Schlesinger (2000, p. 139) defines self-insurance as a mechanism that “lowers the financial severity of any loss that occurs”.

### 2.3.1 Model for Coinsurance Demand under Ambiguity

Consider a DM with initial wealth  $W_0$  and exposed to a potential loss  $h$  that she perceives as ambiguous. Assume she is ambiguity averse. Thus, she behaves as if there is a bounded set  $\Delta$  of possible probability distributions of  $h$ , with probability measure  $\mu$  over that set. If the DM was ambiguity neutral, the compound probability distribution  $\bar{Q}$  would be the only element in  $\Delta$ . This is the case of a subject that complies with the postulates in Savage (1972) and behaves as if she maximizes the SEU defined by probability distribution  $\bar{Q}$  and utility function  $u(\cdot)$ . MMR's orthogonal decomposition of ambiguous acts implies that  $h = E_{\bar{Q}}[h] + h^* + h^\perp$ , with  $h^* \in M$  and  $h^\perp \in M^\perp$ .

There is a risk neutral and ambiguity neutral insurer that is willing to offer the DM a coinsurance contract<sup>6</sup> that covers a fraction  $\alpha \in (0, 1]$  of losses and in exchange for a premium  $\pi$  per unit of insurance. I assume that the insurer shares with the DM the same (compound) probability distribution  $\bar{Q}$  of  $h$  and calculates the premium per unit of insurance according to  $\pi = (1 + m)E_{\bar{Q}}[h]$ , where  $m \geq 0$  is the insurance loading that does not include any ambiguity charge. It is restrictive to assume that the insurer is both risk and ambiguity neutral and shares with the DM the compound distribution  $\bar{Q}$ . However, the final section studies deviations from these restrictive assumptions.

If the DM buys insurance, her ambiguous end-of-period wealth is  $W = W_0 - h - \alpha\pi + \alpha h$ .

I assume that the DM's preferences have a KMM representation with concave functions  $u(\cdot)$  and  $\phi(\cdot)$  which, respectively, represent her aversion towards risk and ambiguity.

---

<sup>6</sup>I focus in this section only on coinsurance contracts and deal later with the optimality of this type of contract and aversion towards ambiguity of the insurer.

Consequently, her maximization problem is:

$$\max_{\alpha} \int_{\Delta} \phi \left( \int_S u(W) dQ \right) d\mu \quad (2.5)$$

$$\text{subject to: } W = W_0 - h - \alpha\pi + \alpha h$$

Using the MMR quadratic approximation, this problem can be approximated by:

$$\max_{\alpha} E_{\bar{Q}}[W] - \frac{\theta}{2} \sigma_{\bar{Q}}^2(W) - \frac{\gamma}{2} \sigma_{\mu}^2(E[W]) \quad (2.6)$$

$$\text{subject to: } W = W_0 - h - \alpha\pi + \alpha h$$

where  $\theta$  and  $\gamma$  capture the degree of aversion towards risk and ambiguity, respectively. The definition of the end-of-period wealth, the insurance premium and the tripartite decomposition of  $h$  imply the following:

$$E_{\bar{Q}}[W] = W_0 - E_{\bar{Q}}[h] - \alpha(1+m)E_{\bar{Q}}[h] + \alpha E_{\bar{Q}}[h] = W_0 - (1+\alpha m)E_{\bar{Q}}[h]$$

$$\sigma_{\bar{Q}}^2(W) = (1-\alpha)^2 \sigma_{\bar{Q}}^2(h) = (1-\alpha)^2 [\sigma_{\bar{Q}}^2(h^*) + \sigma_{\bar{Q}}^2(h^{\perp})], \text{ and}$$

$$\sigma_{\bar{Q}}^2(E[W]) = (1-\alpha)^2 \sigma_{\bar{Q}}^2(E[h^{\perp}])$$

Substituting the restriction into the objective function and differentiating with respect to  $\alpha$ , the first order condition is:

$$\begin{aligned} [\alpha] : & -mE_{\bar{Q}}[h] - \frac{\theta}{2} \{2(1-\alpha)(-1)[\sigma_{\bar{Q}}^2(h^*) + \sigma_{\bar{Q}}^2(h^{\perp})]\} \\ & - \frac{\gamma}{2} \{2(1-\alpha)(-1)\sigma_{\mu}^2(E[h^{\perp}])\} = 0 \end{aligned} \quad (2.7)$$

The optimal demand for coinsurance is:<sup>7</sup>

$$\alpha_{amb}^{M1*} = 1 - \frac{mE_{\bar{Q}}[h]}{\theta[\sigma_{\bar{Q}}^2(h^*) + \sigma_{\bar{Q}}^2(h^\perp)] + \gamma\sigma_\mu^2(E[h^\perp])} \quad (2.8)$$

To facilitate comparison, I assume that a DM that faces only risk (i.e.,  $\sigma_\mu^2(E[h^\perp]) = 0$ ), uses an objective distribution that is equal to the compound distribution  $\bar{Q}$  to make choices. Under these conditions, the optimal insurance demand is defined by:

$$\alpha_{risk}^{M1*} = 1 - \frac{mE_{\bar{Q}}[h]}{\theta[\sigma_{\bar{Q}}^2(h^*) + \sigma_{\bar{Q}}^2(h^\perp)]} \quad (2.9)$$

Alternatively, we can also interpret equation (9) in the light of a risk averse but ambiguity neutral individual. In the KMM framework, this agent is equivalent to a DM with the SEU preference representation given by utility  $u(\cdot)$  and reduced compound distribution  $\bar{Q}$ .

### Comparative Statics of Risk and Ambiguity Attitudes

The Mossin-Smith theorem is robust to the introduction of ambiguity, because  $\alpha_{amb}^{M1*} = 1$  if and only if  $m = 0$ . This can easily be seen from the definition of  $\alpha_{amb}^{M1*}$ . Alary et al. (2010, p. 9) found a similar result.

Moreover, in the presence of ambiguity and ambiguity aversion, the direction of the Pratt-Arrow result is not affected, so an increase in risk aversion increases the insurance demand. Nevertheless, as we will see in the next section, this result may not hold in the presence of a non-traded asset. Assuming  $m > 0$  such that  $\alpha_{amb}^{M1*} \in (0, 1)$ , the following

---

<sup>7</sup>The second order condition is satisfied.

derivative proves this statement:

$$\frac{\partial \alpha_{amb}^{M1*}}{\partial \theta} = \frac{mE_{\bar{Q}}[h]}{\{\theta\sigma_{\bar{Q}}^2(h) + \gamma\sigma_{\mu}^2(E[h^{\perp}])\}^2} [\sigma_{\bar{Q}}^2(h)] > 0 \quad (2.10)$$

with  $\sigma_{\bar{Q}}^2(h) = \sigma_{\bar{Q}}^2(h^*) + \sigma_{\bar{Q}}^2(h^{\perp})$ .

However, there is a second order difference with respect to the baseline model with only risk. The response of the insurance demand of an ambiguity averse agent to marginal increases in risk aversion is lower than that of an ambiguity neutral agent (i.e.,

$0 < \frac{\partial \alpha_{amb}^{M1*}}{\partial \theta} < \frac{\partial \alpha_{risk}^{M1*}}{\partial \theta}$ ), *ceteris paribus*. More generally, higher ambiguity aversion decreases the response of the insurance demand to higher risk aversion, i.e.  $\frac{\partial^2 \alpha_{amb}^{M1*}}{\partial \theta \partial \gamma} < 0$ . This implies that ambiguity averse individuals will increase insurance when risk aversion is higher, but at a slower rate than less ambiguity averse agents.

The intuition of this counterintuitive result is as follows. In the first order condition of the DM's maximization problem, the first term is the marginal cost of increasing insurance demand and the other two are the marginal benefits of reducing risk and ambiguity:

$$\underbrace{-mE_{\bar{Q}}[h]}_{\text{Marginal cost}} + \underbrace{\theta\{(1-\alpha)[\sigma_{\bar{Q}}^2(h^*) + \sigma_{\bar{Q}}^2(h^{\perp})]\}}_{\text{Risk reduction marginal benefit}} + \underbrace{\gamma\{(1-\alpha)\sigma_{\mu}^2(E[h^{\perp}])\}}_{\text{Ambiguity reduction marginal benefit}} = 0$$

Higher ambiguity aversion makes a marginal increase in insurance more valuable.

Therefore, when risk aversion increases and more insurance is required to reduce variance, a smaller increase in insurance is needed when ambiguity aversion is higher because the additional coverage provides the double benefit of reducing risk and ambiguity.<sup>8</sup>

<sup>8</sup>This is a prediction that could be tested in the laboratory. If one is able to identify risk and ambiguity attitudes, then one could rank individuals according to risk aversion. The prediction above says that, an observed increase in risk aversion from  $\theta_1$  to  $\theta_2$  should increase more the insurance demand in the group of ambiguity neutral individuals than in the group of ambiguity averse agents.

Moreover, given a level of risk aversion and subjective probability  $\bar{Q}$ , an ambiguity averse agent will demand more (co)insurance at the optimum than an ambiguity neutral individual (i.e.,  $\alpha_{amb}^{M1*} > \alpha_{risk}^{M1*}$ ). Alary et al. (2010, p. 12) showed that this is a general result for two states of nature. My tractable framework allows us to derive a “local” version of this result: *An increase in absolute ambiguity aversion increases the optimal insurance demand.* The following derivative proves this proposition:

$$\frac{\partial \alpha_{amb}^{M1*}}{\partial \gamma} = \frac{m E_{\bar{Q}}[h]}{\{\theta \sigma_{\bar{Q}}^2(h) + \gamma \sigma_{\mu}^2(E[h^{\perp}])\}^2} [\sigma_{\mu}^2(E[h^{\perp}])] > 0 \quad (2.11)$$

### The Effects of Changes in Initial Wealth

It is usually assumed that people become less risk averse as they get wealthier. There are two important definitions of decreasing aversion in the economic literature. The first definition states that, in the portfolio choice problem with one safe and one risky assets, the optimal demand for the risky asset is increasing in wealth (Arrow, 1963; Pratt, 1964). The second definition states that an individual has “decreasing aversion if any risk that is undesirable at some specific wealth level is also undesirable at all smaller wealth levels” (Cherbonnier and Gollier, 2011, p. 1). In the expected utility model both definitions of decreasing aversion are equivalent. A necessary and sufficient condition for this equivalency to hold is that the utility function  $u$  exhibits decreasing absolute risk aversion. One would expect that a similar condition would hold in the presence of ambiguity.

However, Cherbonnier and Gollier (2011, p. 18) show, in a general setting of the KMM model, that the equivalency of the two definitions above does not hold in general. They

demonstrate that restrictions on risk and ambiguity attitudes are sufficient to guarantee that any uncertain situation that is undesirable at one wealth level is also undesirable at a lower wealth level. Nevertheless, they show that one has to impose restrictions on both risk and ambiguity attitudes, *as well as* on the ambiguity structure, to guarantee that an increase in wealth will increase the demand of an ambiguous asset.

Since there is a close relationship between the portfolio choice and the coinsurance choice problems (see Schlesinger, 2000, p. 135), the results in Cherbonnier and Gollier (2011) also apply to the choice of the optimal coinsurance rate.

I show below that in the “small,” in the spirit of Pratt (1964) and as discussed by Maccheroni et al. (2011b), restrictions on risk and ambiguity attitudes are sufficient to guarantee that the optimal coinsurance demand is decreasing in wealth. This discrepancy between the small and the large is not uncommon.<sup>9</sup>

In the expected utility model, individual’s risk preferences are represented by  $u$  and exhibit DARA if and only if  $-u''(x)/u'(x)$  is decreasing in  $x$ . Similarly, ambiguity attitudes in the KMM model exhibit decreasing absolute ambiguity aversion (DAAA) if  $-\phi''(z)/\phi'(z)$  is decreasing in  $z$ . The following definition characterizes this property for any function as decreasing concavity.

**Definition 1 (Cherbonnier and Gollier, 2011, p. 3):** A function  $f : \mathbb{R} \rightarrow \mathbb{R}$  satisfies (weak) Decreasing Concavity (DC) if  $-f'/f''$  is non-increasing.

A fact that I will use in the proposition below is that  $\sigma_Q^2(h) > \sigma_\mu^2(E[h])$  (see Lemma 1 in

---

<sup>9</sup>For instance, in the classical expected utility model, “a necessary and sufficient condition for any pure small background risk to reduce the optimal exposure to other risks... is just necessary if one wants the comparative statics property to hold for any risk” (Eeckhoudt and Gollier, 2000, p. 126).

Appendix A). This means that the subjective variance of  $E[h]$ , the measure of ambiguity in MMR [2010], is always going to be smaller than the subjective variance of  $h$  calculated with the compound distribution  $\bar{Q}$ .<sup>10</sup>

An important component of the KMM model is the function  $v = \phi \circ u$ . In fact, one of the key assumptions in Klibanoff et al. (2005, p. 1855) is that subjects behave according to SEU, derived from  $v$  and  $\mu$ , when they are presented with second order acts.<sup>11</sup> We are now ready to state the first proposition.

**Proposition 1:** Assume  $m > 0$ . An increase in the initial wealth level  $W_0$  reduces the optimal insurance demand if  $u$  and  $v = \phi \circ u$  satisfy DC.

*Proof.* See Appendix A.

Proposition 1 implies that, in the “small,” restrictions on risk and ambiguity attitudes of the KMM model are sufficient to guarantee that the optimal coinsurance demand under ambiguity is decreasing in wealth. No additional restrictions on the ambiguity structure are needed, like the ones imposed by Cherbonnier and Gollier (2011) in a general setting, to obtain the desired comparative static.<sup>12</sup>

---

<sup>10</sup>This is easily seen in a simple example. Suppose  $h \in \{0, 100\}$ , and the probability distribution could be either  $\{.25, .75\}$  or  $\{.75, .25\}$ . Also assume that the subjective weights over these distributions are such that  $\mu_1 = \mu_2 = .5$ . Then,  $\sigma_{\bar{Q}}^2(h) = 2500 > 625 = \sigma_{\mu}^2(E[h])$ .

<sup>11</sup>The Ellsberg thought experiment can be represented by a second order urn that is ambiguous to subjects and that defines the possible configurations of the first order urn used to illustrate the Ellsberg paradox. A bet on the second order urn is a second order act.

<sup>12</sup>This result is potentially important for possible experimental applications attempting to test the predictions of models under ambiguity like the ones in this study. An experimenter trying to test such a model under ambiguity in the “small” will have to identify risk and ambiguity attitudes, as well as the perceived ambiguity, but does not have to control for the structure of ambiguity to test certain theoretical predictions.

### 2.3.2 Model for Coinsurance Demand in the Presence of Ambiguity and a Non-traded Asset

Suppose that the DM faces the same insurance decision as before, except that the individual owns a non-tradable and uninsurable asset (e.g., human capital) with risky return  $H$  that might be correlated with the loss. I argue that it could be reasonable to assume that subjects have a better idea about the uncertainty of their human capital, thus treating it as risk; meanwhile they may have *ambiguous* information about events that can negatively affect their health such as a genetic chronic disease. Although it is a fair question to ask if people perceive human capital as risky or ambiguous, this choice is more for expositional purposes since it allows us to study the potential trade-offs that agents might face in the presence of both risk and ambiguity.

As before, the risk and ambiguity neutral insurer offers coinsurance  $\alpha \in (0, 1]$  and charges an insurance premium  $\pi = (1 + m)E_{\bar{Q}}[h]$ . Consequently, the ambiguous end-of-period wealth is  $W = W_0 + H - h - \alpha\pi + \alpha h$ . The MMR orthogonal decomposition implies that  $h = E_{\bar{Q}}[h] + h^* + h^\perp$  and  $H = E_{\bar{Q}}[H] + H^*$ . Thus the maximization problem is:

$$\max_{\alpha} E_{\bar{Q}}[W] - \frac{\theta}{2}\sigma_{\bar{Q}}^2(W) - \frac{\gamma}{2}\sigma_{\mu}^2(E[W]) \quad (2.12)$$

where

$$\begin{aligned} E_{\bar{Q}}[W] &= W_0 + E_{\bar{Q}}[H] - (1 + \alpha m)E_{\bar{Q}}[h] \\ \sigma_{\bar{Q}}^2(W) &= \sigma_{\bar{Q}}^2(H^*) + (1 - \alpha)^2[\sigma_{\bar{Q}}^2(h^*) + \sigma_{\bar{Q}}^2(h^\perp)] - 2(1 - \alpha)cov_{\bar{Q}}(H^*, h^*) \\ \sigma_{\mu}^2(E[W]) &= (1 - \alpha)^2\sigma_{\mu}^2(E[h^\perp]) \end{aligned}$$

The covariance between  $H$  and  $h$  is defined as  $cov_{\bar{Q}}(H^*, h^*)$ , because the ambiguous component of the loss ( $h^\perp$ ) is orthogonal to the elements of set  $M$  (the set of risky and risk-free gambles). The joint probability distribution  $\bar{Q}$  is the DM's best estimate to model the process driving the movements of  $H$  and  $h$ . The ambiguity of  $h$  implies that the DM behaves as if there were many joint distributions  $Q_i$ . The interpretation of this ambiguous situation is that the individual behaves as if he was sure about the marginal distribution in the direction of  $H$ , but is unsure about the marginal distribution of  $h$ . The distribution  $\bar{Q}$  is the DM's best estimate of the joint distribution that is calculated by taking the average over the reasonably possible joint distributions with respect to a probability measure  $\mu$ . The perceived ambiguity in the end-of-period wealth arises from  $h^\perp$ , and is captured by  $\sigma_\mu^2(E[h^\perp])$ .

Taking the derivative with respect to  $\alpha$ , the first order condition is:

$$\begin{aligned} [\alpha] : & -mE_{\bar{Q}}[h] - \frac{\theta}{2}\{2(1-\alpha)(-1)[\sigma_{\bar{Q}}^2(h^*) + \sigma_{\bar{Q}}^2(h^\perp)] - 2(-1)cov_{\bar{Q}}(H^*, h^*)\} \\ & - \frac{\gamma}{2}\{2(1-\alpha)(-1)\sigma_\mu^2(E[h^\perp])\} = 0 \end{aligned} \quad (2.13)$$

The optimal demand for insurance is:

$$\alpha_{amb}^{M2*} = 1 - \frac{mE_{\bar{Q}}[h]}{\theta\sigma_{\bar{Q}}^2(h) + \gamma\sigma_\mu^2(E[h^\perp])} - \frac{cov_{\bar{Q}}(H^*, h^*)}{\sigma_{\bar{Q}}^2(h) + \frac{\gamma}{\theta}\sigma_\mu^2(E[h^\perp])} \quad (2.14)$$

where  $\sigma_{\bar{Q}}^2(h) = \sigma_{\bar{Q}}^2(h^*) + \sigma_{\bar{Q}}^2(h^\perp)$ .

For comparison purposes, an ambiguity neutral but risk averse DM that has a subjective

distribution  $\bar{Q}$ , will exhibit an optimal insurance demand defined by:

$$\alpha_{risk}^{M2*} = 1 - \frac{mE_{\bar{Q}}[h]}{\theta\sigma_{\bar{Q}}^2(h)} - \frac{cov_{\bar{Q}}(H^*, h^*)}{\sigma_{\bar{Q}}^2(h)} \quad (2.15)$$

### Comparative Statics of Risk and Ambiguity Attitudes

The optimality of full insurance is not qualitatively changed by the introduction of ambiguity and a non-traded asset. As shown by Doherty (1984), the Mossin-Smith theorem holds only if  $cov_{\bar{Q}}(H^*, h^*) \leq 0$ . However, there is a quantitative difference that I explain in the next paragraph.

Given the same level of risk aversion and subjective probability distribution  $\bar{Q}$ , an ambiguity averse agent will demand more (co)insurance at the optimum than an ambiguity neutral individual, because  $\alpha_{amb}^{M2*} > \alpha_{risk}^{M2*}$ . The inclusion of a non-traded asset does not change significantly a similar result shown by Alary et al. (2010, p. 12). They show that, for two states of nature and given risk aversion, an ambiguity averse agent demands more insurance than an ambiguity neutral person when insurance decisions are modeled in isolation. However, depending on the sign and level of covariance, the optimal demand  $\alpha_{amb}^{M2*}$  can be smaller, equal or greater than  $\alpha_{amb}^{M1*}$  and/or  $\alpha_{risk}^{M1*}$ . Finally, as the next proposition shows, my tractable model allows us to derive a “local” version of the comparative static of ambiguity attitudes.

**Proposition 2:** In the presence of ambiguity and aversion towards it, if the incentives to *self-hedge*, represented by  $cov_{\bar{Q}}(H^*, h^*)$ , are low (high) enough, more ambiguity aversion decreases (increases) the optimal demand for insurance.

*Proof.*

$$\begin{aligned} \frac{\partial \alpha_{amb}^{M2*}}{\partial \gamma} &= - \frac{mE_{\bar{Q}}[h][-\sigma_{\bar{Q}}^2(h^\perp)]}{[\theta\sigma_{\bar{Q}}^2(h) + \gamma\sigma_\mu^2(E[h^\perp])]^2} - \left[ - \frac{cov_{\bar{Q}}(H^*, h^*) \left[ \frac{\sigma_\mu^2(E[h^\perp])}{\theta} \right]}{\sigma_{\bar{Q}}^2(h) + \frac{\gamma}{\theta}\sigma_\mu^2(E[h^\perp])^2} \right] \\ &= \frac{[mE_{\bar{Q}}[h] + \theta cov_{\bar{Q}}(H^*, h^*)]\sigma_\mu^2(E[h^\perp])}{[\theta\sigma_{\bar{Q}}^2(h) + \gamma\sigma_\mu^2(E[h^\perp])]^2} \end{aligned}$$

Define a threshold covariance  $\kappa_{H,h;\gamma}^{M2*} = -\frac{mE_{\bar{Q}}[h]}{\theta}$ . Therefore,

$$\frac{\partial \alpha_{amb}^{M2*}}{\partial \gamma} = \begin{cases} < 0, & \text{if } cov_{\bar{Q}}(H^*, h^*) < \kappa_{H,h;\gamma}^{M2*} \\ = 0, & \text{if } cov_{\bar{Q}}(H^*, h^*) = \kappa_{H,h;\gamma}^{M2*} \\ > 0, & \text{if } cov_{\bar{Q}}(H^*, h^*) > \kappa_{H,h;\gamma}^{M2*} \end{cases} \quad (2.16)$$

■

The optimal demand  $\alpha_{amb}^{M2*}$  is always greater than one whenever  $cov_{\bar{Q}}(H^*, h^*) < \kappa_{H,h;\gamma}^{M2*}$ . If the supply of insurance is restricted to  $[0, 1]$ , then we would only observe that  $\frac{\partial \alpha_{amb}^{M2*}}{\partial \gamma} \geq 0$ .

This is consistent with Alary et al. (2010) who found that the demand for insurance is increasing in ambiguity aversion when the insurance decisions are made in isolation.

However, if the agent is allowed to own a non-traded asset, to have an insurance demand greater than one and  $cov_{\bar{Q}}(H^*, h^*) < \kappa_{H,h;\gamma}^{M2*}$ , she will reduce her demand for insurance when ambiguity aversion increases.

Gollier (2009) found that under certain conditions more ambiguity aversion can increase the demand for the ambiguous asset. We can infer that if my model is framed in a portfolio choice context without restrictions on the asset demand, the incentives to “*self-hedge*” may

constitute a rationale different from Gollier (2009) to explain why more ambiguity aversion might increase the demand for the uncertain asset.

An important result is that the introduction of ambiguity and a non-traded asset may contradict the Pratt-Arrow result that insurance demand increases with risk aversion. I state the result in the following proposition and explain its rationale in the next section.

**Proposition 3:** When ambiguity matters to the individual, an increase in risk aversion may or may not increase the demand for insurance depending on the incentives to “*self hedge*” created by  $cov(H^*, h^*)$ .

*Proof:* The derivative of the optimal insurance demand with respect to the absolute risk aversion coefficient is given by:

$$\begin{aligned} \frac{\partial \alpha_{amb}^{M2*}}{\partial \theta} &= - \left[ \frac{mE_{\bar{Q}}[h] \times [\sigma_{\bar{Q}}^2(h^*) + \sigma_{\bar{Q}}^2(h^\perp)]}{[\theta[\sigma_{\bar{Q}}^2(h^*) + \sigma_{\bar{Q}}^2(h^\perp)] + \gamma\sigma_\mu^2(E[h^\perp])]^2} \right] \\ &\quad - \left[ \frac{cov_{\bar{Q}}(H^*, h^*) \times \left[ -\frac{\gamma \times \sigma_\mu^2(E[h^\perp])}{\theta^2} \right]}{[\sigma_{\bar{Q}}^2(h^*) + \sigma_{\bar{Q}}^2(h^\perp)] + \frac{\gamma}{\theta}\sigma_\mu^2(E[h^\perp])]^2} \right] \\ &= \frac{mE_{\bar{Q}}[h] \times [\sigma_{\bar{Q}}^2(h^*) + \sigma_{\bar{Q}}^2(h^\perp)] - \gamma cov_{\bar{Q}}(H^*, h^*)\sigma_\mu^2(E[h^\perp])}{[\theta[\sigma_{\bar{Q}}^2(h^*) + \sigma_{\bar{Q}}^2(h^\perp)] + \gamma\sigma_\mu^2(E[h^\perp])]^2} \end{aligned} \quad (2.17)$$

Define a threshold covariance  $\kappa_{H,h;\theta}^{M2*} = \frac{mE_{\bar{Q}}[h]}{\gamma} \times \frac{\sigma_{\bar{Q}}^2(h^*) + \sigma_{\bar{Q}}^2(h^\perp)}{\sigma_\mu^2(E[h^\perp])}$ . Therefore,

$$\frac{\partial \alpha_{amb}^{M2*}}{\partial \theta} = \begin{cases} > 0, & \text{if } cov_{\bar{Q}}(H^*, h^*) < \kappa_{H,h;\theta}^{M2*} \\ = 0, & \text{if } cov_{\bar{Q}}(H^*, h^*) = \kappa_{H,h;\theta}^{M2*} \\ < 0, & \text{if } cov_{\bar{Q}}(H^*, h^*) > \kappa_{H,h;\theta}^{M2*} \end{cases} \quad (2.18)$$



In the limit, when the agent behaves as a SEU-maximizer, we obtain the Pratt-Arrow result because the optimal insurance demand will be non-decreasing in risk aversion. When the ambiguity averse agent converges to an SEU-maximizer ( $\gamma \rightarrow 0$ ) or ambiguity disappears ( $\sigma_\mu^2(E[h^\perp]) \rightarrow 0$ ), the threshold  $\kappa_{H,h;\theta}^{M2*}$  will never be reached for variables with finite covariance. Thus, we would only observe that  $\frac{\partial \alpha_{amb}^{M2*}}{\partial \theta} > 0$ . However, proposition 3 shows an exception to Pratt-Arrow result that arises when ambiguity and a non-traded asset are present. The rationale of this counterintuitive result, which I explain in the next section, relies on possible trade-offs between risk and ambiguity that the DM must solve.

### **A Trade-off between Risk and Ambiguity: Are Subjects Behaving**

#### **“Inefficiently”?**

The Pratt-Arrow result with only risk, or with ambiguity but with a SEU-maximizer, implies that more risk aversion increases the insurance demand because the lower variance that higher risk aversion demands can always be met with more insurance. In Proposition 3 I showed that if ambiguity matters to the agent and there is a non-traded asset, more risk aversion does not always increase the insurance demand. The reason is that in this environment the lower variance of wealth required by an increase in risk aversion is not necessarily achieved through more insurance if the incentives to *self-hedge* are high enough. The source for this counterintuitive result is that the DM might have to face a potential trade-off between risk and ambiguity that depends on the incentives to self-hedge. This can induce the subject to behave *as if* she was choosing voluntarily a risk allocation that is

“inefficient” from a SEU perspective. However, this allocation might be optimal once ambiguity is taken into account. I explain below the origin of the risk-ambiguity trade-off by expressing the insurance problem in the form of a portfolio problem. This will allow me to draw some possible implications of my results to the theory of portfolio.

The insurance decision problem can be interpreted as a *portfolio problem* (Schlesinger, 2000, p. 135), since a higher (lower) demand for insurance is equivalent to a lower (higher) demand for the risky asset. Therefore, I can define a mean-variance frontier, in the same spirit as the financial literature, by finding the locus of  $\{\sigma_Q^2(W(\alpha)), E_Q[W(\alpha)]\}$  pairs generated by every value  $\alpha \in [0, 1]$ . Define  $\alpha_{MinVar}^* = 1 - \frac{cov(H^*, h^*)}{\sigma_Q^2(h)}$  as the insurance level that provides the wealth with minimum variance.<sup>13</sup> Moreover, suppose that

$\alpha_{MinVar}^* \in (0, 1)$ , which is guaranteed by  $cov(H^*, h^*) \in (0, \sigma_Q^2(h))$ .

The efficient (inefficient) portion of the mean-variance frontier is usually defined as the set of mean-variance allocations that offer the highest (lowest) expected wealth at a given level of risk. In my framework, the efficient (inefficient) part of the frontier is defined as the set of  $\{\sigma_Q^2(W(\alpha)), E_Q[W(\alpha)]\}$  pairs where  $\frac{d\sigma_Q^2(W(\alpha))}{dE_Q[W(\alpha)]} > (<)0$  (See Lemma 2 in the Appendix A).

**Definition 2:** The mean-variance allocation  $\{\sigma_Q^2(W(\alpha)), E_Q[W(\alpha)]\}$  will be located on the SEU-efficient (SEU-inefficient) part of the frontier whenever the insurance demand

$\alpha < \alpha_{MinVar}^*$  ( $\alpha > \alpha_{MinVar}^*$ ).

I refer to the efficient (inefficient) region as the *SEU-efficient* (*SEU-inefficient*) part of the frontier. This emphasizes the fact that a subject, who would appear inefficient in the light of SEU theory, might optimally choose such an allocation to deal both with risk and

---

<sup>13</sup>This is easily found by minimizing  $\sigma_Q^2(W)$  with respect to  $\alpha$ .

ambiguity. I show below that this is the case under certain conditions.

**Proposition 4:** Assume that the loss  $h$  is ambiguous and the non-traded asset  $H$  is risky.

An ambiguity neutral agent will choose an optimal mean-variance allocation that will be located on the *SEU-efficient* region of the frontier.

This proposition is obvious since a subject that does not care about ambiguity and behaves according to Savage (1972), will choose an allocation that maximizes expected wealth given a certain level of variance.

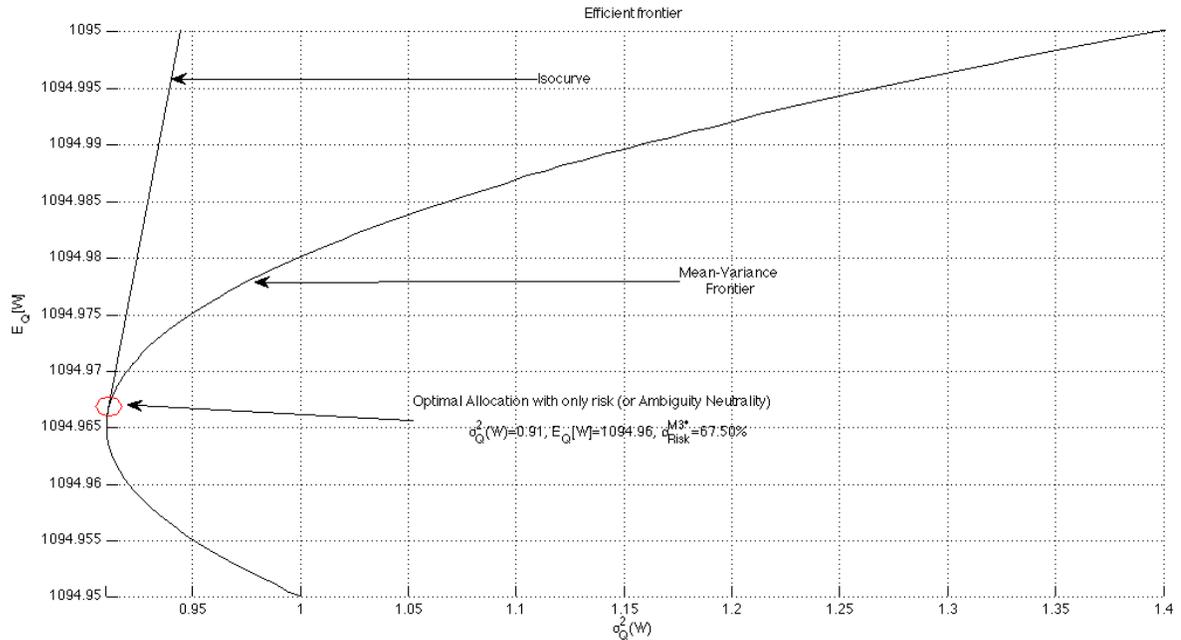
*Proof.* According to definition 2, the optimal allocation  $\{\sigma_{\bar{Q}}^2(W(\alpha_{risk}^{M3*})), E_{\bar{Q}}[W(\alpha_{risk}^{M3*})]\}$  induced by  $\alpha_{risk}^{M2*}$  will be located on the efficient part of the frontier because:

$$\alpha_{risk}^{M2*} = 1 - \frac{mE_{\bar{Q}}[h]}{\theta\sigma_{\bar{Q}}^2(h)} - \frac{cov_{\bar{Q}}(H^*, h^*)}{\sigma_{\bar{Q}}^2(h)} < 1 - \frac{cov(H^*, h^*)}{\sigma_{\bar{Q}}^2(h)} = \alpha_{MinVar}^*$$

■

Figure 2.1 shows the optimal mean variance allocation induced by the optimal insurance demand of a SEU maximizer. In the particular parametrization used to construct the figure, this optimal demand is  $\alpha_{risk}^{M2*} = 67.5\%$  (see the tangency of the isocurve and the mean-variance frontier). Remember that, according to definition 2, a mean-variance allocation will be SEU-efficient if the optimal insurance demand is less than the coinsurance demand that provides minimum variance, which is  $\alpha_{MinVar}^* = 70\%$  in Figure 2.1. Although it is still possible in this example for the DM to choose an insurance coverage above  $\alpha_{MinVar}^*$ , she will not do it so because there is still an attainable allocation (in the SEU-efficient part of the frontier) that provides higher expected return for the same variance. Finally, if  $cov(H^*, h^*) < 0$  then  $\alpha_{MinVar}^* = 100\%$ , provided the insurer only offers

Figure 2.1: Portfolio Problem Interpretation of a Simple Insurance Decision



Parametric assumptions:  $\theta = \gamma = 2$ ,  $\sigma_Q^2(h^*) + \sigma_Q^2(h^\perp) = \sigma_\mu^2(E[h^\perp]) = 1$ ,  $W_0 = 1000$ ,  $cov_{\bar{Q}}(H^*, h^*) = 0.3$ ,  $m = .01$ ,  $E_{\bar{Q}}[H] = 100$  and  $E_{\bar{Q}}[h] = 5$ . The isocurve contains the  $\{\sigma_Q^2(W), E_Q[W]\}$  pairs that make the individual indifferent to the optimal allocation which is determined by the tangency with the mean-variance frontier.

coinsurance  $\alpha \in (0, 1]$ ; as a consequence all mean-variance allocations induced by  $\alpha \in (0, 1]$  will be located in the SEU-efficient region of the frontier.

In the presence of ambiguity, the optimal risk-return allocation will not always be located in the efficient part of the frontier because high incentives to self-hedge (i.e.,

$cov(H^*, h^*) > \kappa_{H,h;\theta}^{M2*}$ ) create a trade-off between the variance and ambiguity of wealth.

Thus, the individual would appear to be “SEU-inefficient”, however such action is optimal in the presence of the ambiguity dimension.

**Proposition 5.** Suppose that an ambiguity and risk averse agent is exposed to an ambiguous loss  $h$  and owns a risky non-traded asset  $H$ . Whenever the incentives to

“self-hedge” are high (low) enough, i.e.,  $cov_{\bar{Q}}(H^*, h^*) > (<) \kappa_{H,h;\theta}^{M2*}$ , the optimal mean-variance allocation  $\{\sigma_{\bar{Q}}^2(W(\alpha^{M2*})), E_{\bar{Q}}[W(\alpha^{M2*})]\}$  induced by  $\alpha^{M2*}$  will be located on the SEU-inefficient (SEU-efficient) part of the frontier. More formally,

$$\frac{d\sigma_{\bar{Q}}^2(W(\alpha))}{dE_{\bar{Q}}[W(\alpha)]}\Big|_{\alpha=\alpha^{M2*}} = \begin{cases} > 0, & \text{if } cov_{\bar{Q}}(H^*, h^*) < \kappa_{H,h;\theta}^{M2*} \\ = 0, & \text{if } cov_{\bar{Q}}(H^*, h^*) = \kappa_{H,h;\theta}^{M2*} \\ < 0, & \text{if } cov_{\bar{Q}}(H^*, h^*) > \kappa_{H,h;\theta}^{M2*} \end{cases} \quad (2.19)$$

where  $\kappa_{H,h;\theta}^{M2*} = \frac{mE_{\bar{Q}}[h]}{\gamma} \times \frac{\sigma_{\bar{Q}}^2(h^*) + \sigma_{\bar{Q}}^2(h^\perp)}{\sigma_{\mu}^2(E[h^\perp])} > 0$ .

*Proof.* See Appendix A.

To prove the proposition I show that the position of the SEU-efficient mean-variance allocation on the frontier depends on the incentives to “self-hedge” by comparing the optimal insurance demand with  $\alpha_{MinVar}^*$ .

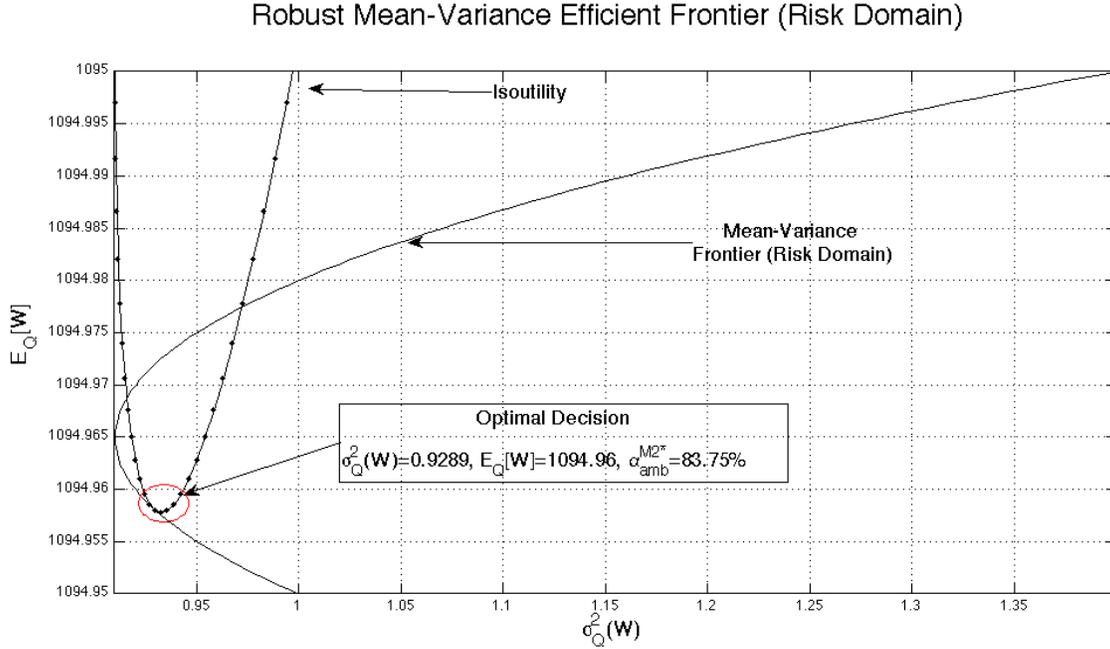
When the incentives to self-hedge are high enough ( $cov(H^*, h^*) > \kappa_{H,h;\theta}^{M2*}$ ), the optimal risk-return allocation of an ambiguity averse individual will be located on the SEU-inefficient part of the frontier, because  $\alpha_{amb}^{M2*} > \alpha_{MinVar}^*$ . The individual would appear to be *SEU-inefficient* because she will buy more insurance than a SEU-maximizer would buy in the same situation. However, in the presence of ambiguity and aversion towards it, this “additional” demand for insurance is optimal.

Figure 2.2 shows an example of an optimal allocation that is seemingly SEU-inefficient but completely optimal when the ambiguity domain is taken into account. According to definition 2, the optimal insurance demand  $\alpha_{amb}^{M2*} = 83.8\%$  (see tangency of isocurve and

mean-variance frontier) would be SEU-inefficient because it is greater than  $\alpha_{MinVar} = 70\%$ .

However, one has to take into account the ambiguity dimension.

Figure 2.2: Optimal Choice in the Risky Domain in the Presence of Ambiguity



Parametric assumptions:  $\theta = \gamma = 2$ ,  $\sigma_Q^2(h^*) + \sigma_Q^2(h^\perp) = \sigma_\mu^2(E[h^\perp]) = 1$ ,  $W_0 = 1000$ ,  $cov_{\bar{Q}}(H^*, h^*) = 0.3$ ,  $m = .01$ ,  $E_{\bar{Q}}[H] = 100$  and  $E_{\bar{Q}}[h] = 5$ .

Figure 2.3 shows the “mean-ambiguity” frontier that a subject faces in the presence of ambiguity as well as the isocurve that contains the optimal mean-ambiguity allocation

$\{\sigma_\mu^2(W(\alpha_{amb}^{M2*}), E_{\bar{Q}}[W(\alpha_{amb}^{M2*})])\}$ . If the individual was an SEU-maximizer, she would exhibit

an insurance demand of  $\alpha_{risk}^{M2*} = 67.5\%$ . However, she would have to bear more ambiguity

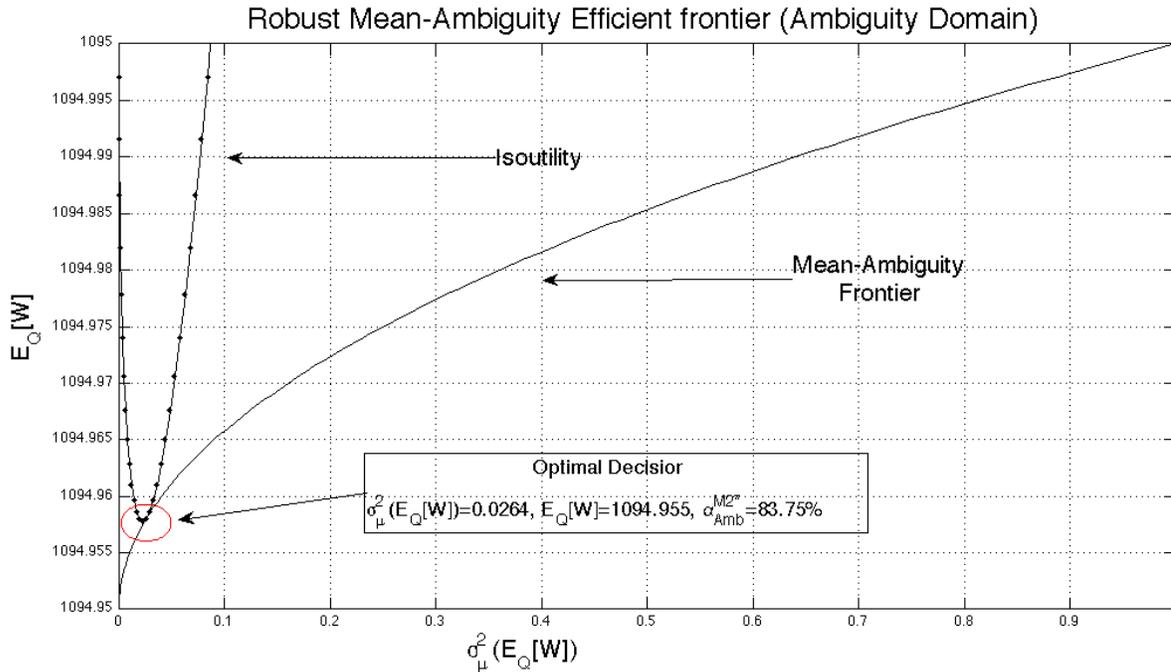
( $\sigma_\mu^2(W(\alpha_{risk}^{M2*})) = 0.11$ ) than she would be willing to accept if she was ambiguity averse (

$\sigma_\mu^2(W(\alpha_{amb}^{M2*})) = 0.03$ ). Thus, when incentives to *self-hedge* are high ( $cov_{\bar{Q}}(H^*, h^*) > \kappa_{H,h;\theta}^{M2*}$ ),

the ambiguity averse individual will demand a level of insurance that seems excessive from

an classical SEU perspective in order to reduced ambiguity to the desired level.

Figure 2.3: Optimal Choice in the Ambiguity Domain



Parametric assumptions:  $\theta = \gamma = 2$ ,  $\sigma_Q^2(h^*) + \sigma_Q^2(h^\perp) = \sigma_\mu^2(E[h^\perp]) = 1$ ,  $W_0 = 1000$ ,  $cov_{\bar{Q}}(H^*, h^*) = 0.3$ ,  $m = .01$ ,  $E_{\bar{Q}}[H] = 100$  and  $E_{\bar{Q}}[h] = 5$ .

The exhibited SEU-inefficient type of behavior that I just described arises from a trade-off between risk and ambiguity that the DM must resolve. The intuition is that under certain conditions, more insurance does not necessarily reduce variance and ambiguity at the same time. Figure 2.4 shows the locus of risk/ambiguity  $\{\sigma_Q^2(W), \sigma_\mu^2(E_Q[W])\}$  pairs that are attainable for any given level of insurance demand  $\alpha \in [0, 1]$ . An increase in the coinsurance demand moves the DM from right to left in the figure. For instance, Figure 2.4 emphasizes at the top right corner the case where the DM does not insure at all ( $\alpha = 0$ ) and at the lower left corner the case when the DM is fully insured ( $\alpha = 1$ ).

The parametrization for the example in the Figure 2.4 was carefully chosen to exemplify the situation in which it is not always possible to reduce variance and ambiguity at the

same time. This is represented by the non-monotonicity of the curve in Figure 2.4. The first part of the curve that is decreasing indicates that more insurance decreases ambiguity but increases variance. The part of the figure that is increasing indicates that buying more insurance decreases both ambiguity and variance in that region. The non-monotonicity is present only when  $cov_{\bar{Q}}(H^*, h^*) > 0$ ;<sup>14</sup> however, its mere presence does not imply that a DM will exhibit a SEU-inefficient type of behavior. We need the stronger condition that  $cov_{\bar{Q}}(H^*, h^*) > \kappa_{H,h;\theta}^{M2*} > 0$ , identified in proposition 5. Therefore, a DM will behave as if she was SEU-inefficient, but optimally if ambiguity is taken into account, when the incentives to self-hedge are sufficiently high.

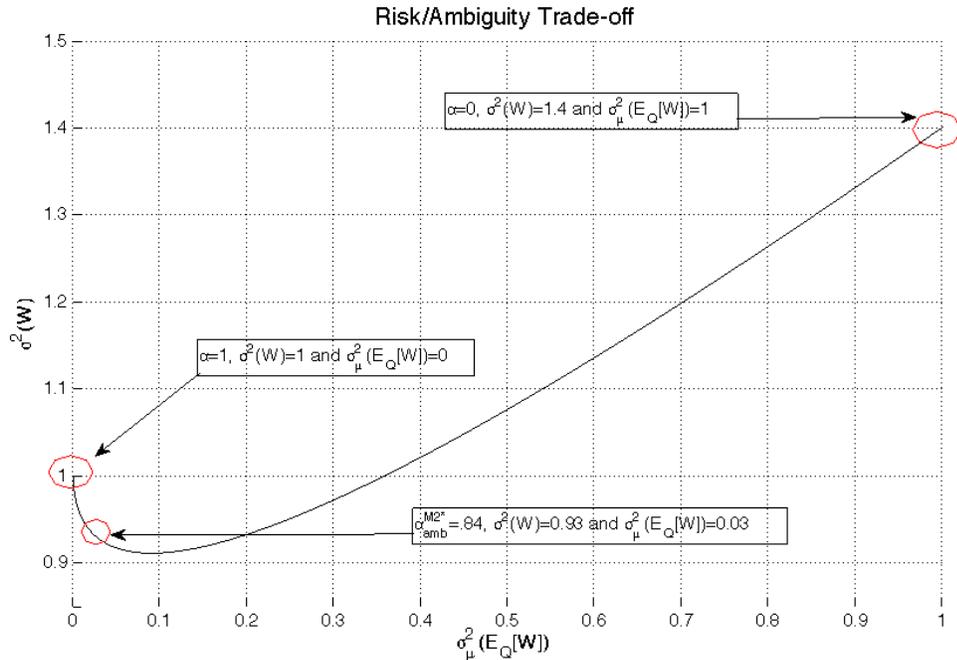
**What is the intuition for  $\frac{\partial \alpha^{M2*}}{\partial \theta} < 0$ ?**

I can finally give a more comprehensive explanation for the intuitive result that more risk aversion might reduce the coinsurance demand in the presence of ambiguity and a non-traded asset. Proposition 3 and 5 show, respectively, that the same condition ( $cov_{\bar{Q}}(H^*, h^*) < \kappa_{H,h;\theta}^{M2*}$ ) is needed for  $\frac{\partial \alpha^{M2*}}{\partial \theta}$  to be positive *and* the optimal mean-variance allocation to be on the SEU-efficient part of the frontier. The intuition is that in the SEU-efficient part of the frontier the lower variance required by higher risk aversion can be achieved by buying more insurance. This is possible because in this region of the frontier

---

<sup>14</sup>It is easily shown that the curve defined by the locus of the  $\{\sigma_{\bar{Q}}^2(W), \sigma_{\mu}^2(E_Q[W])\}$  pairs is non-monotonic if  $cov_{\bar{Q}}(H^*, h^*) > 0$ . This is done by noting that  $\frac{d\sigma_{\bar{Q}}^2(W)}{d\sigma_{\mu}^2(E_Q[W])} = \frac{1}{\sigma_{\mu}^2(E_Q[W])} \left( \sigma_{\bar{Q}}^2(W) - \frac{cov_{\bar{Q}}(H^*, h^*)}{(1-\alpha)} \right)$ . The comparative static  $\frac{d\sigma_{\bar{Q}}^2(W)}{d\sigma_{\mu}^2(E_Q[W])}$  is greater than 0 for any  $\alpha \in [0, 1]$  if  $cov_{\bar{Q}}(H^*, h^*) < 0$ . If this covariance was positive, there is always a given coinsurance demand for which  $\frac{d\sigma_{\bar{Q}}^2(W)}{d\sigma_{\mu}^2(E_Q[W])}$  is negative.

Figure 2.4: Risk/Ambiguity Trade-off and Optimal Choice



Parametric assumptions:  $\theta = \gamma = 2$ ,  $\sigma_Q^2(h^*) + \sigma_Q^2(h^\perp) = \sigma_\mu^2(E[h^\perp]) = 1$ ,  $W_0 = 1000$ ,  $cov_{\bar{Q}}(H^*, h^*) = 0.3$ ,  $m = .01$ ,  $E_{\bar{Q}}[H] = 100$  and  $E_{\bar{Q}}[h] = 5$ . The figure shows the  $\{\sigma_Q^2(W), \sigma_\mu^2(E_Q[W])\}$  pairs that can be achieved for any value of coinsurance demand  $\alpha \in [0, 1]$ . Three cases are emphasized in the figure: (i)  $\alpha = 1$ , (ii)  $\alpha = \alpha_{amb}^{M2*} = .8375$ , and (iii)  $\alpha = 0$ .

the variance of wealth is decreasing in the insurance demand  $\alpha$ .<sup>15</sup>

Nevertheless, when the optimal risk-return allocation is in the SEU-inefficient part of the frontier, more risk aversion will decrease the demand for insurance. The reason is that the reduction in variance required by higher risk aversion cannot be met through more insurance under these conditions. When the individual buys more insurance, she reduces the variance of loss  $h$  she is exposed to, i.e.,  $(1 - \alpha)^2 \sigma_Q^2(h)$ . However, she also affects the

<sup>15</sup>  $\frac{\partial \sigma_Q^2(W)}{\partial \alpha} = -2(1 - \alpha)\sigma_Q^2(h) + 2cov(H^*, h^*) < 0$ , if  $\alpha < \alpha_{MinVar}^* = 1 - \frac{cov(H^*, h^*)}{\sigma_Q^2(h)}$ . By proposition 5,  $\alpha_{amb}^{M2*} < \alpha_{MinVar}^*$  if  $cov_{\bar{Q}}(H^*, h^*) < \kappa_{H,h;\theta}^{M2*}$ . Thus, the variance of wealth is decreasing in the optimal insurance demand (i.e.,  $\frac{\partial \sigma_Q^2(W)}{\partial \alpha} \Big|_{\alpha=\alpha_{amb}^{M2*}} < 0$ ) whenever  $cov_{\bar{Q}}(H^*, h^*) < \kappa_{H,h;\theta}^{M2*}$ .

*self-hedging* possibilities because more insurance reduces the effective covariance she actually faces, i.e.,  $(1 - \alpha)cov(H^*, h^*)$ .

When the optimal insurance demand is sufficiently high, such that  $\alpha_{amb}^{M2*} > \alpha_{MinVar}^*$ , a marginal increase in insurance also increases the variance of wealth because the reduction in self-hedging possibilities outweighs the reduction in the variance of  $h$ . Under these conditions, buying more insurance would increase the variance of wealth. Hence, an increment in risk aversion induces a reduction in the insurance demand. In the example shown in Figure 2.3, increasing the parameter of absolute risk aversion from 2 to 3 will decrease the optimal insurance demand from 83.8% to 81%.

Finally, one could assume that the subject perceives both the non-traded asset and the loss as ambiguous. Suppose the measure  $\mu$  is a subjective joint distribution of beliefs about  $H$  and  $h$  and  $cov_{\mu}(E[H^{\perp}], E[h^{\perp}])$  represents the incentives to *self-hedge* in the ambiguity domain. One can infer that it is possible to have similar versions of proposition 4 and 6 for the ambiguity domain. The bottom line is that that incentives to *self-hedge* are very important to insurance and asset demand both in the risk and ambiguity domain.

### **Trade-offs between Risk and Ambiguity in Portfolio Choice Problems**

Cochrane (2007, p. 49) claims that in the multifactor efficient portfolio model developed by Fama (1996) “typical investors do not hold mean-variance efficient portfolios... [t]hey are willing to give up some mean-variance efficiency in return for a portfolio that hedges the state variable innovations.” My model resembles the most simple case of such a model with one asset and one non-traded asset that plays the role of a factor. Proposition 5 suggests

that there exist conditions under which ambiguity induces the subject to choose a SEU-inefficient mean-variance allocation that even takes into account the covariance of the factor with the other assets.

There is literature documenting the (SEU-)inefficiency in household portfolio and studying its source (e.g., Benartzi, 2001; Goetzmann and Kumar, 2008). Typically, these studies propose a myriad of behavioral explanations for the under-diversification of portfolios like familiarity bias, ignorance, overconfidence, informational frictions, subjective beliefs and alternative preferences representations (e.g. Rank-Dependent utility theory).

However, there is limited research on the presence of non-traded assets as another possible explanation for the “seemingly” observed under-diversified portfolios, probably due to the difficulty of getting detailed data on households. Massa and Simonov (2006) study a unique data set of Swedish investors where they can identify various components of income, wealth and demographic characteristics. Massa and Simonov (2006, p. 661 and 667) find evidence that a wide range of investors do not hedge neither their financial assets nor their labor and entrepreneurial income (i.e., returns of non-traded assets). They conclude that familiarity, the tendency to invest in stocks that are geographically or professionally close to them, or that have been held for a long period, affects investors’ hedging behavior and induces them to behave inefficiently from a classical SEU perspective. In particular, Massa and Simonov (2006) find that people tend to choose portfolio allocations that are positively correlated with their labor income. However, individuals with greater wealth tend to pick asset allocations that are more in the lines of traditional portfolio theory. Another explanation within the SEU model is that the subjective beliefs that investors use to make

portfolio choices are different from the ones used by researchers to define their notion of efficiency. The explanation below also depends on subjective beliefs but departs from the SEU framework.

The issue of familiarity has a long tradition in experimental economics and was originally studied in contexts with ambiguity. Heath and Tversky (1991) and Fox and Tversky (1995) present experimental evidence in which they claim that subjects tend to prefer (or to avoid) ambiguous prospects for which they feel they are more (less) familiar. This familiarity interpretation was introduced as an explanation to typical behavior under ambiguity. However, many of the familiarity interpretations can be formalized in terms of attitudes towards ambiguity. In fact, studies such as Myung (2009) provide evidence to support the claim that ambiguity aversion is related to the equity market home bias paradox and Boyle et al. (2012) model the trade-offs between familiarity (modeled as attitudes towards ambiguity) and diversification.

A generalization of my model that includes traded assets, non-traded assets and insurance can offer an alternative explanation to the under-diversification puzzle of households.

Intuitively, the interaction of traded assets, non-traded assets and insurable assets, that can each be ambiguous or not, can result in observed behavior that is “under-diversified” from a classical SEU perspective but optimal from a broader perspective. The main message is that deviations from the classical portfolio theory that assumes SEU can be explained by expanding the concept of portfolio to non-traded assets, which has been recognized in the financial literature, but more importantly to also allow for preference representations that can explain attitudes towards ambiguity.

I present below a simple model that can explain the following two observations in Massa and Simonov (2006): (i) investors tend to hold stocks that are familiar to them and that are positively correlated with their labor income; and (ii) wealthy individuals have a greater tendency to pick stocks that can help hedging their labor income risk.

Suppose a DM (who could be an individual or a household) with labor income  $H$ , and that has the opportunity to invest part of his initial wealth  $W_0$  into two financial assets,  $X$  and  $Y$ . For exposition purposes, assume that the DM perceives labor income  $H$  and asset  $X$  as risky, while asset  $Y$  is perceived as ambiguous. Labor income could be modeled as an ambiguous process but it is assumed risky for simplicity.<sup>16</sup> In the spirit of Massa and Simonov (2006), asset  $X$  can be understood as the set of stocks with which the DM is very familiar and asset  $Y$  is the set of stocks that are less familiar to the DM. There is a joint probability distribution  $\bar{Q}$  that the DM considers is the best estimate to model the process driving the comovements of  $H$ ,  $X$  and  $Y$ . The ambiguity of  $Y$  implies that the DM behaves as if there were more than one joint distributions, each denoted by  $Q_i$ . The interpretation of this particular case is that the individual behaves as if he was sure about the marginal distributions in the direction of  $H$  and  $X$ , but is unsure about the marginal distribution of  $Y$ . This results in the individual behaving as if there was a myriad of reasonable possible distribution  $Q_i$  with  $i \in I$  and  $\bar{Q}$  his best estimate of the joint distribution. The latter is calculated by taking the average over the many possible joint

---

<sup>16</sup>For example, take the case of a dentist for whom income varies from month to month but she is very familiar with the number of patients that might or might not come each month. In this case, the dentist might perceive her labor income as risky. In contrast, think of a self-employed consultant that is just starting up his business. He might have an idea of what a consultant can earn each month from the experience of others. However, this situation could be perceived as ambiguous because he doesn't have enough information to believe that his labor income is going to follow the path of other colleagues' income. In this case, the consultant might perceive his income as ambiguous.

distributions with respect to a probability measure  $\mu$  that captures the beliefs that any of the reasonable possible joint distributions is the true one.

The MMR orthogonal decomposition implies that labor income and assets can be expressed as  $H = E_{\bar{Q}}[H] + H^*$ ,  $X = E_{\bar{Q}}[X] + X^*$  and  $Y = E_{\bar{Q}}[Y] + Y^* + Y^\perp$ . The subindex  $\bar{Q}$  is used in the expectations operator to remind the reader that the mean of the random variables is taken using the marginals of  $\bar{Q}$ . Define  $\alpha$  and  $\beta$  as the proportions of initial wealth invested in asset  $X$  and  $Y$ , respectively, and there are no restrictions on these proportions. The final wealth can be written as  $W = H + (1 - \alpha - \beta)W_0 + \alpha X + \beta Y$ . Thus the maximization problem is:

$$\max_{\alpha} E_{\bar{Q}}[W] - \frac{\theta}{2}\sigma_{\bar{Q}}^2(W) - \frac{\gamma}{2}\sigma_{\mu}^2(E[W]) \quad (2.20)$$

where

$$\begin{aligned} E_{\bar{Q}}[W] &= E_{\bar{Q}}[H] + (1 - \alpha - \beta)W_0 + \alpha E_{\bar{Q}}[X] + \beta E_{\bar{Q}}[Y]; \\ \sigma_{\bar{Q}}^2(W) &= \sigma_{\bar{Q}}^2(H^*) + (\alpha)^2\sigma_{\bar{Q}}^2(X^*) + (\beta)^2[\sigma_{\bar{Q}}^2(Y^*) + \sigma_{\bar{Q}}^2(Y^\perp)] \\ &+ 2(\alpha\beta)cov_{\bar{Q}}(X^*, Y^*) + 2(\alpha)cov_{\bar{Q}}(X^*, H^*) + 2(\alpha)cov_{\bar{Q}}(Y^*, H^*) \\ &\text{and } \sigma_{\mu}^2(E[W]) = (\beta)^2\sigma_{\mu}^2(E[Y^\perp]) \end{aligned}$$

Taking the derivative with respect to  $\alpha$  and  $\beta$ , the first order conditions are:

$$[\alpha] : (E_{\bar{Q}}[X] - W_0) - \frac{\theta}{2} \left[ 2\alpha\sigma_{\bar{Q}}^2(X^*) + 2\beta cov_{\bar{Q}}(X^*, Y^*) + 2cov_{\bar{Q}}(X^*, H^*) \right] = 0 \quad (2.21)$$

$$\begin{aligned} [\beta] : (E_{\bar{Q}}[Y] - W_0) - \frac{\theta}{2} \left[ 2\beta\sigma_{\bar{Q}}^2(Y^*) + 2\alpha cov_{\bar{Q}}(X^*, Y^*) + 2cov_{\bar{Q}}(Y^*, H^*) \right] \\ - \frac{\gamma}{2} [2\beta\sigma_{\mu}^2(E[Y^\perp])] = 0 \end{aligned} \quad (2.22)$$

For the sake of the argument assume that under probability  $\bar{Q}$  asset  $X$  and  $Y$  only differ in that  $X$  is positively correlated with  $H$ :

$$\text{cov}_{\bar{Q}}(Y^*, H^*) = \text{cov}_{\bar{Q}}(X^*, Y^*) = 0,$$

$$\text{cov}_{\bar{Q}}(X^*, H^*) > 0,$$

$$E_{\bar{Q}}[X^*] = E_{\bar{Q}}[Y^*] = \bar{Z},$$

$$\sigma_{\bar{Q}}^2(X^*) = \sigma_{\bar{Q}}^2(Y^*) = \sigma^2.$$

Therefore, the optimal asset demands are given by:

$$\alpha^* = \frac{\bar{Z} - W_0}{\theta\sigma^2} - \frac{\text{cov}_{\bar{Q}}(X^*, H^*)}{\sigma^2} \quad (2.23)$$

$$\beta^* = \frac{\bar{Z} - W_0}{\theta\sigma^2 + \gamma\sigma_{\bar{Q}}^2(E[Y])} \quad (2.24)$$

An SEU maximizer should choose an asset allocation such that  $\frac{\alpha^*}{\beta^*} < 1$  because both assets have the same mean and variance. However, asset  $Y$  is more suitable to hedge income risk than asset  $X$  is, given that  $\text{cov}_{\bar{Q}}(Y^*, H^*) = 0$  and  $\text{cov}_{\bar{Q}}(X^*, H^*) > 0$ . This can be seen by taking the ratio of both asset demands and setting  $\gamma\sigma_{\bar{Q}}^2(E[Y]) = 0$ , since an SEU maximizer does not care about ambiguity.

However, an individual that is sufficiently averse to ambiguity and/or finds asset  $Y$  very ambiguous, represented by a high value of  $\gamma\sigma_{\bar{Q}}^2(E[Y])$ , may choose a portfolio allocation that puts more weight on asset  $X$  (i.e.,  $\frac{\alpha^*}{\beta^*} > 1$ ) even though it is positively correlated with his income. This rationalizes the first empirical finding in Massa and Simonov (2006).

The second finding in Massa and Simonov (2006), that wealthy individuals behave according to what SEU would predict, can be explained in several ways. One common

explanation is that wealthy investors are more financially sophisticated as suggested by Campbell (2006, p. 1576). Another interpretation is that ambiguity attitudes are affected by the level of wealth. In my highly stylized model, an individual with sufficiently high wealth can overcome an aversion towards ambiguity and choose a portfolio allocation that puts more weight in the asset that helps to hedge income risk (i.e.  $\frac{\alpha^*}{\beta^*} < 1$ ). This can be shown by taking the derivative of the ratio of asset demands with respect to initial wealth:

$$\begin{aligned} \frac{\partial \alpha^*/\beta^*}{\partial W_0} = & \left[ \frac{\partial \gamma}{\partial W_0} \theta - \frac{\partial \theta}{\partial W_0} \gamma \right] \\ & - cov_{\bar{Q}}(X^*, H^*) \left[ \left( \frac{\partial \theta}{\partial W_0} + \frac{\partial \gamma}{\partial W_0} \frac{\sigma_{\bar{Q}}^2(E[Y^\perp])}{\sigma^2} \right) (\bar{Z} - W_0) + \theta + \gamma \frac{\sigma_{\bar{Q}}^2(E[Y^\perp])}{\sigma^2} \right] \end{aligned} \quad (2.25)$$

For simplicity, assume that the DM exhibits CARA which implies that  $\frac{\partial \theta}{\partial W_0} = 0$ . Therefore, we can guarantee that  $\frac{\partial \alpha^*/\beta^*}{\partial W_0} < 0$  by assuming the following: (i)  $\frac{\partial \gamma}{\partial W_0} < 0$  which is implied by  $\phi(\cdot)$  satisfying the decreasing concavity property, that is to say, that absolute ambiguity aversion is decreasing in wealth; and (ii) that the estimated covariance between  $X$  and  $H$  is not so high, i.e.  $cov_{\bar{Q}}(X^*, H^*) < \frac{Z-W_0}{\theta}$ , otherwise strong hedging incentives would be the main explanation for the tilt of the portfolio towards the ambiguous asset. Therefore, an individual that originally chose a portfolio that was tilted towards the risky asset  $X$  and positively correlated to income can revert his choice towards a portfolio that puts more weight on the ambiguous asset  $Y$  if his wealth increases such that his aversion to ambiguity is significantly reduced.

## 2.4 Optimal Insurance Contracts Under Ambiguity

### 2.4.1 Optimality under Ambiguity of Insurance Contracts with a Straight Deductible: A Counterexample

A well-established result in insurance economics states that the optimal insurance contract for a risk averse DM is one with a straight deductible. The latter is due to Arrow (1971, p. 212), who proves the following proposition:

If an insurance company is willing to offer any insurance policy against loss desired by the buyer at a premium which depends only on the policy's actuarial value, then the policy chosen by a risk-averting buyer will take the form of 100 per cent coverage above a deductible minimum.

Alary et al. (2010, p. 12-13) proves that this result is robust to the introduction of ambiguity when insurance decisions are made in isolation. This section shows that under certain conditions this result might not hold. I use a similar argument to Doherty (1984, p. 214), who showed that when a “portfolio contains non-insurable risky assets [e.g., human capital], the preference of the insured between a deductible and a coinsurance policy depends upon the covariance” between the loss and the non-tradable asset.

**Proposition 6:** In the presence of ambiguity and a non-traded asset, an insurance contract with a straight deductible does not always dominate a coinsurance contract.

*Proof.* See Appendix A.

The rationale of this result is as follows. The level of the deductible could be so high that there is a substantial loss in self-hedging possibilities that outweighs the benefits of variance and ambiguity reduction. However, in the presence of ambiguity, the conditions for a coinsurance arrangement to dominate are more stringent than in the case with only

risk. The reason is that the potential benefits of self-hedging of the coinsurance arrangement must compensate both for the reduction in variance and ambiguity created by the contract with a deductible.

Notice that self-hedging is restricted in the present model to the risk domain. However, one can easily construct a model in which the non-traded asset is also ambiguous. This would make the self-hedging possibilities important in the ambiguity domain. The latter would be captured by a covariance term under the second order subjective measure  $\mu$  of the ambiguous non-traded asset and the ambiguous loss. As a consequence, one can think of numerous trade-offs between the risk domain and the ambiguity domain.

### 2.4.2 The Borch Rule Under Ambiguity

An important result in the economic literature, derived by Borch (1962, p. 428), is the optimality of a coinsurance contract when both parties are risk averse. The rule is stated by Arrow (1971, p. 216) as follows:

If the insured and the insurer are both risk averters and there are no costs other than coverage of losses, then any nontrivial Pareto-optimal policy,  $[\alpha(h)]$ , as a function of the loss,  $[h]$ , must have the property,  $0 < [d\alpha/dh] < 1$ .<sup>17</sup>

I show that this result is robust to the introduction of ambiguity aversion. However, the level of coinsurance under ambiguity, relative to the case with only risk, will depend on the relative importance of risk and ambiguity for each party and the differences in beliefs. In other words, it will depend on the ratios of the risk aversion and ambiguity aversion parameters, as well as on the compound distribution and the second order probability

---

<sup>17</sup>I change the original notation in Arrow's quote to match ours. In his book, the loss is denoted by  $X$  and the insurance policy by  $I$ .

measure  $\mu$ . I use similar arguments of the derivation in Arrow (1971, p. 217-219) and extend its logic to the ambiguity domain.

**Proposition 7:** Let  $u(\cdot)$  and  $v(\cdot)$ <sup>18</sup> be the von-Neumann-Morgenstern utility functions that capture, respectively, the risk attitudes of the insured and the insurer. Assume both are risk averse, i.e.  $u''(\cdot) < 0$  and  $v''(\cdot) < 0$ . Moreover, define  $\phi_u(\cdot)$  and  $\phi_v(\cdot)$  as the functions that capture the ambiguity aversion of the insured and the insurer, respectively. Both agents have a KMM representation of preferences under ambiguity. Let  $W_0$  and  $W_1$  be the initial wealth of the insured and the insurer. Additionally, define  $\alpha(\tilde{h})$  as the insurance payment net of the insurance premium when the ambiguous loss takes the value  $\tilde{h}$ . The end-of-period wealth of each agent is:

- Insurer:  $\widetilde{W}_v = W_1 - \alpha(\tilde{h})$
- Insured:  $\widetilde{W}_u = W_0 - h + \alpha(\tilde{h})$

Each party's valuation of any insurance schedule  $\alpha(\tilde{h})$  has a KMM structure:

- Insurer:  $V = \int_{\Delta_v} \phi_v \left( \int_S v(\widetilde{W}_v) dQ_v \right) d\mu_v$
- Insured:  $U = \int_{\Delta_u} \phi_u \left( \int_S u(\widetilde{W}_u) dQ_u \right) d\mu_v$

where the subindices imply the obvious definition for each variable.

Furthermore, assume a finite world where there are  $n$  possible outcomes for  $h$ , and

$\bar{Q} = \{q_1, q_2, \dots, q_i, \dots, q_n\}$ ,  $\theta$ ,  $\gamma$  and  $\mu_u = \{\mu_u^1, \mu_u^2, \dots, \mu_u^k, \dots, \mu_u^K\}$  represent, respectively, the compound probability distribution, risk attitudes, ambiguity attitudes and second order

---

<sup>18</sup>Not to be confused with  $v = \phi \circ u$  as defined before

beliefs of the insured. Similarly, for the insurer these variables are represented by

$\bar{P} = \{p_1, p_2, \dots, p_n\}$ ,  $\tilde{\theta}$ ,  $\tilde{\gamma}$  and  $\mu_v = \{\mu_v^1, \mu_v^2, \dots, \mu_v^k, \dots, \mu_v^K\}$ . Notice that this is a general model that allows for different beliefs ( $\bar{Q} \neq \bar{P}$ ), risk and ambiguity attitudes ( $\theta \neq \tilde{\theta}$  and  $\gamma \neq \tilde{\gamma}$ ) and probability measures over sets  $\Delta_u = \{Q_1, Q_2, \dots, Q_k, \dots, Q_K\}$  and  $\Delta_v = \{P_1, P_2, \dots, P_k, \dots, P_K\}$  ( $\mu_u \neq \mu_v$ ). Elements in  $Q_k$  and  $P_k$  are denoted by  $q_j^k$  and  $p_j^k$ , respectively.

The risk/ambiguity arrangement in each of the possible outcomes  $h_i$  of the ambiguous loss is defined by

$$\frac{d\alpha(h_i)}{dh_i} = \frac{a [\theta q_i(1 - q_i) + \gamma \sigma_{\mu_u}^2(q_i)]}{a [\theta q_i(1 - q_i) + \gamma \sigma_{\mu_u}^2(q_i)] + b [\tilde{\theta} p_i(1 - p_i) + \tilde{\gamma} \sigma_{\mu_v}^2(p_i)]} \quad (2.26)$$

Notice that  $\frac{d\alpha(h_i)}{dh_i} \in (0, 1)$ , since  $\theta > 0$ ,  $\tilde{\theta} > 0$ ,  $\gamma > 0$ ,  $\tilde{\gamma} > 0$ , and  $p_i, q_i \geq 0$ . In other words, the optimal contract is a *coinsurance* schedule.

*Proof.* See Appendix A

This implies that the Borch rule is robust to the introduction of ambiguity and even of heterogeneity of beliefs. However, the rule corrects for differences in beliefs, ambiguity and attitudes towards it. Though not reported here, the introduction of a risky non-traded asset does not affect the coinsurance structure of the optimal program. The intuition is that such an extension of the model will not affect the concavity of the Pareto-optimal frontier of the set of feasible  $\{U, V\}$ , leaving the structure of the maximization program unaffected. For comparison purposes,  $\frac{d\bar{\alpha}(h_i)}{dh_i} = \frac{a\theta}{a\theta + b\tilde{\theta}}$  is the optimal coinsurance schedule with only risk and homogeneous beliefs. With  $\bar{Q} = \bar{P}$  and  $\mu_v = \mu_u = \mu$ . The equivalent coinsurance rule

when ambiguity is introduced is equal to:

$$\frac{a [\theta(1 - q_i) + \gamma\sigma_\mu^2(q_i)/q_i]}{a [\theta(1 - q_i) + \gamma\sigma_\mu^2(q_i)/q_i] + b [\tilde{\theta}(1 - q_i) + \tilde{\gamma}\sigma_\mu^2(q_i)/q_i]} \quad (2.27)$$

For all  $h_i$ , the coinsurance demand is higher (lower) under ambiguity if  $\frac{\tilde{\theta}}{\tilde{\gamma}} > (<) \frac{\theta}{\gamma}$ .

Intuitively, if the insurer is more ambiguity tolerant, relative to its own risk tolerance, than the insured is, the coinsurance demand is higher under ambiguity. However, this sharp comparative static is not possible when there are heterogeneous beliefs.

For any  $h_i$ ,  $\bar{Q} \neq \bar{P}$  and  $\mu_v \neq \mu_u$ , the optimal coinsurance demand is higher (lower) in the presence of ambiguity if  $\frac{\tilde{\theta}}{\tilde{\gamma}} \frac{q_i(1-q_i)}{\sigma_{\mu_u}^2(q_i)} > (<) \frac{\theta}{\gamma} \frac{q_i(1-q_i)}{\sigma_{\mu_u}^2(q_i)}$ . Consequently, when there are differences in beliefs and ambiguity matters to both parties, it cannot be easily predicted under which conditions the coinsurance demand is higher or lower than that under risk.

## 2.5 Conclusions

I have shown that if ambiguity matters, risk and ambiguity attitudes interact in nontrivial ways to determine the change of insurance demand for a given change in wealth. I derive sufficient conditions to guarantee that the optimal coinsurance demand is decreasing in wealth. Moreover, in the presence of a risky non-traded asset, I identify conditions under which more risk or ambiguity aversion decrease the demand for coinsurance.

Additionally, my model predicts behavior that is inconsistent with the classical portfolio theory that assumes Subjective Expected Utility theory, however, it provides hints to a possible solution of the under-diversification puzzle of households. The main message is

that deviations from the traditional portfolio theory can be explained by expanding the concept of portfolio to non-traded assets, which has been recognized in the financial literature, but also by simultaneously allowing for preference representations that can explain attitudes towards ambiguity.

A modified Borch rule remains the optimal contract with bilateral risk/ambiguity aversion and heterogeneity in beliefs. However, an insurance contract with straight deductible might be dominated by a coinsurance schedule in the presence of ambiguity and a non-traded asset.

Several challenging questions remain. Other non-EU models of decision under ambiguity should be explored to corroborate that my results are not a mere artifact of the preference representation that I chose. However, an important challenge in the ambiguity literature is to derive more general results that do not depend heavily on the structure of the preference representation. Also, the optimality of contracts should be studied in the presence of asymmetric information and ambiguity. Finally, it would be desirable to derive a model which does not depend on an approximation and that allows for traded assets, non-traded assets and insurance.

## 2.6 Appendix A. Proofs of Results

**Proof of Lemma 1.**  $\sigma_Q^2(h) > \sigma_\mu^2(E[h])$ . I show below that  $\sigma_\mu^2(E[h]) = \sigma_Q^2(h) - E_\mu[\sigma^2(h)]$ ,

and hence the inequality holds.

$$\begin{aligned}
\sigma_\mu^2(E[h]) &= \int_\Delta (E[h])^2 d\mu - \left( \int_\Delta E[h] d\mu \right)^2 \\
&= \int_\Delta \left( \int_S h dQ \right)^2 d\mu - \left( \int_\Delta \int_S h dQ d\mu \right)^2 \\
&\quad - \int_\Delta \int_S h^2 dQ d\mu + \int_\Delta \int_S h^2 dQ d\mu \\
&= \int_\Delta \int_S h^2 dQ d\mu - \left( \int_\Delta \int_S h dQ d\mu \right)^2 \\
&\quad - \int_\Delta \left[ \int_S h^2 dQ - \int_\Delta \left( \int_S h dQ \right)^2 \right] d\mu \\
&= \sigma_Q^2(h) - E_\mu[\sigma^2(h)]
\end{aligned}$$

Rearranging the last equality we have that  $\sigma_Q^2(h) = E_\mu[\sigma^2(h)] + \sigma_\mu^2(E[h])$ . Since  $E_\mu[\sigma^2(h)] > 0$  because it is an average of variances, the latter equation implies that  $\sigma_Q^2(h) > \sigma_\mu^2(E[h])$ . ■

**Proof of Proposition 1.** Assume  $m > 0$ . The following derivative defines the comparative statics of the optimal insurance demand with respect to changes in initial wealth:

$$\frac{\partial \alpha_{amb}^{M1*}}{\partial W_0} = \frac{mE_{\bar{Q}}[h]}{\{\theta\sigma_Q^2(h) + \gamma\sigma_\mu^2(E[h])\}^2} \sigma_Q^2(h) \left[ \frac{\partial \theta}{\partial W_0} + \frac{\partial \gamma}{\partial W_0} \frac{\sigma_\mu^2(E[h])}{\sigma_Q^2(h)} \right] \quad (2.28)$$

where  $\sigma_Q^2(h) = \sigma_Q^2(h^*) + \sigma_Q^2(h^\perp)$ . Since  $\gamma = \left\{ -\frac{v''(W_0)}{v'(W_0)} \right\} - \left\{ -\frac{u''(W_0)}{u'(W_0)} \right\}$  the partial derivative above can be written as:

$$\frac{\partial \alpha_{amb}^{M1*}}{\partial W_0} = \frac{mE_{\bar{Q}}[h]}{\{\theta\sigma_Q^2(h) + \gamma\sigma_\mu^2(E[h])\}^2} \sigma_Q^2(h) \left[ \left( 1 - \frac{\sigma_\mu^2(E[h])}{\sigma_Q^2(h)} \right) \frac{\partial \theta}{\partial W_0} + \frac{\sigma_\mu^2(E[h])}{\sigma_Q^2(h)} \frac{\partial \theta_v}{\partial W_0} \right] \quad (2.29)$$

where  $\theta_v = -\frac{v''(W_0)}{v'(W_0)}$ .

Notice that the sign of this partial derivative depends on the sign of the term in the square brackets. Since both  $u$  and  $v$  satisfy DC, which implies that  $\theta_v$  and  $\theta$  are decreasing in  $W_0$ , and  $\sigma_Q^2(h) > \sigma_\mu^2(E[h]) > 0$ , the optimal insurance demand under ambiguity is decreasing in  $W_0$ , *i.e.*  $\frac{\partial \alpha_{amb}^{M1*}}{\partial W_0} < 0$ . ■

**Proof of Lemma 2.** The efficient (inefficient) part of the frontier is defined as the set of  $\{\sigma_{\bar{Q}}^2(W(\alpha)), E_{\bar{Q}}[W(\alpha)]\}$  pairs where  $\frac{d\sigma_{\bar{Q}}^2(W(\alpha))}{dE_{\bar{Q}}[W(\alpha)]} > (<)0$ .

This relationship can be found by taking the total derivative of the variance and expected value of wealth and dividing one over the other to obtain the following:

$$\frac{d\sigma_{\bar{Q}}^2(W(\alpha))}{dE_{\bar{Q}}[W(\alpha)]} = 2 \frac{(1-\alpha)\sigma_{\bar{Q}}^2(h) - cov(H^*, h^*)}{mE_{\bar{Q}}[h]}$$

The sign of this derivative depends on the level of insurance demand:

$$\frac{d\sigma_{\bar{Q}}^2(W(\alpha))}{dE_{\bar{Q}}[W(\alpha)]} = \begin{cases} > 0, & \text{if } \alpha < \alpha_{MinVar}^* \\ = 0, & \text{if } \alpha = \alpha_{MinVar}^* \\ < 0, & \text{if } \alpha > \alpha_{MinVar}^* \end{cases}$$

■

**Proof of Proposition 5.** According to definition 2, the optimal allocation  $\{\sigma_{\bar{Q}}^2(W(\alpha_{amb}^{M2*})), E_{\bar{Q}}[W(\alpha_{amb}^{M2*})]\}$  will be located on the SEU-inefficient part of the frontier whenever

$$\alpha_{amb}^{M2*} = 1 - \frac{mE_{\bar{Q}}[h]}{\theta\sigma_{\bar{Q}}^2(h) + \gamma\sigma_{\mu}^2(E[h^{\perp}])} - \frac{cov_{\bar{Q}}(H^*, h^*)}{\sigma_{\bar{Q}}^2(h) + \frac{\gamma}{\theta}\sigma_{\mu}^2(E[h^{\perp}])} > 1 - \frac{cov_{\bar{Q}}(H^*, h^*)}{\sigma_{\bar{Q}}^2(h)} = \alpha_{MinVar}^*$$

Thus,

$$\alpha_{amb}^{M2*} > \alpha_{MinVar}^* \text{ if } cov_{\bar{Q}}(H^*, h^*) > \frac{mE_{\bar{Q}}[h]}{\gamma} \times \frac{\sigma_{\bar{Q}}^2(h^*) + \sigma_{\bar{Q}}^2(h^{\perp})}{\sigma_{\mu}^2(E[h^{\perp}])} = \kappa_{H,h;\theta}^{M2*}$$

Similarly,  $\{\sigma_{\bar{Q}}^2(W(\alpha_{amb}^{M2*})), E_{\bar{Q}}[W(\alpha_{amb}^{M2*})]\}$  will be located on the SEU-efficient part of the frontier if  $cov_{\bar{Q}}(H^*, h^*) < \kappa_{H,h;\theta}^{M2*}$ .

■

**Proof of Proposition 6.** Suppose there is a DM that is both risk and ambiguity averse, owns a risky non-traded asset  $H$  and is exposed to an ambiguous loss  $h$ . Furthermore, suppose there is an insurance contract that covers 100% of any realization of the ambiguous loss above  $\delta$ , a deductible amount chosen by DM. However, if  $h < \delta$ , the DM assumes the loss in its entirety. Moreover, assume that there is an alternative coinsurance contract that pays zero if no loss is realized and  $\alpha h$  if a loss of amount  $h$  is realized, where  $\alpha \in [0, 1]$ . The alternative coinsurance contract is carefully chosen such that a risk and ambiguity neutral insurer charges the same premium it does in the contract with deductible  $\delta$ . Thus,  $\alpha$  is defined by the following condition:  $\alpha(1+m)E_{\bar{Q}}[h] = (1+m)E_{\bar{Q}}[\max\{0, h - \delta\}] = \bar{\pi}$ . The strategy is to show that, under certain conditions, the CE implied by a coinsurance contract ( $CE_C$ , henceforth) can dominate the one implied by an actuarially equivalent contract with straight deductible ( $CE_D$ ).

A coinsurance contract dominates one with a deductible if the following condition is satisfied:

$$\begin{aligned} CE_C &\approx E_{\bar{Q}}[W_C] - \frac{\theta}{2}\sigma_{\bar{Q}}^2(W_C) - \frac{\gamma}{2}\sigma_{\mu}^2(E[W_C]) > \\ E_{\bar{Q}}[W_D] - \frac{\theta}{2}\sigma_{\bar{Q}}^2(W_D) - \frac{\gamma}{2}\sigma_{\mu}^2(E[W_D]) &\approx CE_D \end{aligned} \quad (2.30)$$

where

$$W_C = H - h - \bar{\pi} + \alpha h,$$

$$W_D = H - h - \bar{\pi} + \max\{0, h - \delta\},$$

$$\sigma_{\bar{Q}}^2(W_C) = \sigma_{\bar{Q}}^2(H) + (1 - \alpha)^2\sigma_{\bar{Q}}^2(h) - (1 - \alpha)\text{cov}_{\bar{Q}}(H, h),$$

$$\sigma_{\bar{Q}}^2(W_D) = \sigma_{\bar{Q}}^2(H) + \sigma_{\bar{Q}}^2(\max\{h, \delta\}) - \text{cov}_{\bar{Q}}(H, \max\{h, \delta\})$$

$W_C$  and  $W_D$  denote, respectively, the end-of-period wealth under the coinsurance contract and under the contract with straight deductible.

Given that the same premium  $\bar{\pi}$  is charged, the expected value of wealth is the same under both contracts (i.e.,  $E_{\bar{Q}}[W_C] = E_{\bar{Q}}[W_D]$ ). Thus, we just need to focus on the variance-covariance structure and the perceived ambiguity generated by each contract. Vajda [1962] showed that the deductible offers a greater reduction in the variance of retained losses given an amount of insurance premium. This implies that:

$$\frac{(1 - \alpha)^2\sigma_{\bar{Q}}^2(h)}{[(1 - \alpha)E_{\bar{Q}}[h]]^2} > \frac{\sigma_{\bar{Q}}^2(\max\{h, \delta\})}{[E_{\bar{Q}}[\max\{h, \delta\}]]^2}$$

A similar argument can be made about the ambiguity perceived by the DM, since in my framework it is measured as the (subjective) variance of  $E[h]$ . Therefore, Vajda's argument also implies that an insurance contract with a straight deductible will induce lower perceived ambiguity than a coinsurance arrangement, given an expected value of retained losses. Therefore:

$$\frac{(1 - \alpha)^2\sigma_{\mu}^2(E[h])}{[(1 - \alpha)E_{\bar{Q}}[h]]^2} > \frac{\sigma_{\mu}^2(E[\max\{h, \delta\}])}{[E_{\bar{Q}}[\max\{h, \delta\}]]^2}$$

In the absence of the non-traded asset, these two simple facts show why Gollier's result holds. Intuitively, for a given level of insurance premium, such an insurance contract with straight deductible provides the DM with lower variance and ambiguity than the coinsurance arrangement does. However, as shown by Doherty (1984, p.213-216) in a context with only risk, the covariance between the non-traded asset and the retained losses plays a crucial role in the robustness of Arrow's result. The same logic can be extended to the ambiguity domain.

A coinsurance arrangement dominates a contract with straight deductible if  $CE_C > CE_D$ . Using the MMR approximation of each CE, the latter condition is satisfied whenever

$$\begin{aligned} & \frac{\gamma}{\theta} \left[ (1 - \alpha)^2 \sigma_\mu^2(E[h]) - \sigma_\mu^2(E[\max\{h, \delta\}]) \right] + \left[ (1 - \alpha)^2 \sigma_Q^2(h) - \sigma_Q^2(\max\{h, \delta\}) \right] \\ & < (1 - \alpha) \text{cov}(H, h) - \text{cov}(H, \max\{h, \delta\}) \end{aligned} \quad (2.31)$$

It is plausible that, even though  $\text{cov}(H, h) = 0$ , the covariance induced by the deductible,  $\text{cov}(H, \max\{h, \delta\})$ , is low enough such that the above condition is satisfied. In this case the coinsurance arrangement dominates the contract with a deductible. ■

**Proof of Proposition 7.** Let  $u(\cdot)$  and  $v(\cdot)$ <sup>19</sup> be the von-Neumann-Morgenstern utility functions that capture, respectively, the risk attitudes of the insured and the insurer. Assume both are risk averse, i.e.  $u''(\cdot) < 0$  and  $v''(\cdot) < 0$ . Moreover, define  $\phi_u(\cdot)$  and  $\phi_v(\cdot)$  as the functions that capture the ambiguity aversion of the insured and the insurer, respectively. Both agents have a KMM representation of preferences under ambiguity. Let  $W_0$  and  $W_1$  be the initial wealth of the insured and the insurer. Additionally, define  $\alpha(\tilde{h})$  as the insurance payment net of the insurance premium when the ambiguous loss takes the value  $\tilde{h}$ . The end-of-period wealth of each agent is:

- Insurer:  $\tilde{W}_v = W_1 - \alpha(\tilde{h})$
- Insured:  $\tilde{W}_u = W_0 - h + \alpha(\tilde{h})$

Each party's valuation of any insurance schedule  $\alpha(\tilde{h})$  has a KMM structure:

- Insurer:  $V = \int_{\Delta_v} \phi_v \left( \int_S v(\tilde{W}_v) dQ_v \right) d\mu_v$
- Insured:  $U = \int_{\Delta_u} \phi_u \left( \int_S u(\tilde{W}_u) dQ_u \right) d\mu_u$

where the subindices imply the obvious definition for each variable.

The set of all  $\{U, V\}$  pairs has a boundary that is convex to the northeast. To show this, suppose any two insurance policies  $\alpha_1(\tilde{h})$  and  $\alpha_2(\tilde{h})$  that induce allocations  $\{U_1, V_1\}$  and  $\{U_2, V_2\}$ , respectively. Define a third insurance schedule  $\alpha_3(\tilde{h}) = .5\alpha_1(\tilde{h}) + .5\alpha_2(\tilde{h})$  for each  $\tilde{h}$ . The three insurance schedules induce the following end-of-period wealth  $W_{i1}$ ,  $W_{i2}$  and  $W_{i3} = .5W_{i1} + .5W_{i2}$  for  $i = u, v$ . Given that both parties are risk averse, the following statements are true:

$$\begin{aligned} u(W_{u3}) &> .5u(W_{u1}) + .5u(W_{u2}) \\ v(W_{v3}) &> .5v(W_{v1}) + .5v(W_{v2}) \end{aligned}$$

---

<sup>19</sup>Not to be confused with  $v = \phi \circ u$  as defined above.

Since both conditions hold for each  $\tilde{h}$ , it also holds when we take expectations. Without ambiguity, the latter argument will imply that the set of  $\{U, V\}$  pairs induced by all the attainable insurance schedules will have a frontier that is convex to the northeast.

Therefore, any Pareto-optimal insurance schedule can be obtained by maximizing a linear function  $aE[u(\tilde{W}_u)] + bE[v(\tilde{W}_v)]$  for  $a, b \geq 0$  and at least one positive. I show that this is also the case when ambiguity is introduced.

Suppose the sets of reasonably possible probabilities  $\Delta_u$  and  $\Delta_v$  are finite, and elements in each set are denoted by  $Q_{uk}$  and  $Q_{vk}$ , respectively, which are indexed by  $k$ . The second order belief  $\mu_{uk}$  is the subjective probability weight that the insured puts over the probability distribution  $Q_{uk}$ ;  $\mu_{vk}$  is similarly defined for the insurer. The argument in the previous paragraph and the concavity of  $\phi_u(\cdot)$  and  $\phi_v(\cdot)$  imply that the following statements are true:

$$\sum_k \mu_{uk} \times \phi_u(E_{Q_{uk}}[u(W_{u3})]) > .5 \times \sum_k \mu_{uk} \times \phi_u(E_{Q_{uk}}[u(W_{u1})]) + .5 \times \sum_k \mu_{uk} \times \phi_u(E_{Q_{uk}}[u(W_{u2})])$$

or

$$U_3 > .5U_1 + .5U_2$$

and

$$\sum_k \mu_{vk} \times \phi_v(E_{Q_{vk}}[v(W_{v3})]) > .5 \times \sum_k \mu_{vk} \times \phi_v(E_{Q_{vk}}[v(W_{v1})]) + .5 \times \sum_k \mu_{vk} \times \phi_v(E_{Q_{vk}}[v(W_{v2})])$$

or

$$V_3 > .5V_1 + .5V_2$$

Since these statements hold for every pair of points  $\{U_1, V_1\}$  and  $\{U_2, V_2\}$  in the set of allocations defined by the possible insurance schedules, the northeast boundary of this set is convex to the northeast. Therefore, any Pareto-Optimal point, i.e. any point on the northeast boundary, can be obtained by maximizing a linear function  $aU + bV$ , for any  $a, b \geq 0$ , and at least one of them positive. Assume, that both agents have some bargaining power, thus  $a, b > 0$ . Moreover, I approximate the CE of  $U$  and  $V$  with the MMR approximation. Consequently, the maximization problem above is approximated by maximizing  $a\bar{C}\bar{E}_{insured} + b\tilde{C}\tilde{E}_{insurer}$ . Assuming that the state space  $S$  is finite, the optimal insurance schedule with bilateral risk and ambiguity aversion is the solution to:

$$\begin{aligned} \max_{\{\alpha(\tilde{h})\}} a & \left( E_{\bar{Q}}[W_0 - \tilde{h} + \alpha(\tilde{h})] - \frac{\theta}{2} \sigma_{\bar{Q}}^2(\alpha(\tilde{h}) - \tilde{h}) - \frac{\gamma}{2} \sigma_{\mu_u}^2(E[\alpha(\tilde{h}) - \tilde{h}]) \right) \\ & + b \left( E_{\bar{P}}[W_1 - \alpha(\tilde{h})] - \frac{\tilde{\theta}}{2} \sigma_{\bar{P}}^2(\alpha(\tilde{h})) - \frac{\tilde{\gamma}}{2} \sigma_{\mu_v}^2(E[\alpha(\tilde{h})]) \right) \end{aligned} \quad (2.32)$$

where  $\bar{Q} = \{q_1, q_2, \dots, q_i, \dots, q_n\}$ ,  $\theta$ ,  $\gamma$  and  $\mu_u = \{\mu_u^1, \mu_u^2, \dots, \mu_u^k, \dots, \mu_u^K\}$  represent, respectively, the compound probability distribution, risk attitudes, ambiguity attitudes and second order beliefs of the insured. Similarly, for the insurer these variables are represented by  $\bar{P} = \{p_1, p_2, \dots, p_n\}$ ,  $\tilde{\theta}$ ,  $\tilde{\gamma}$  and  $\mu_v = \{\mu_v^1, \mu_v^2, \dots, \mu_v^k, \dots, \mu_v^K\}$ . Notice that this is a general

model that allows for different beliefs ( $\bar{Q} \neq \bar{P}$ ), risk and ambiguity attitudes ( $\theta \neq \tilde{\theta}$  and  $\gamma \neq \tilde{\gamma}$ ) and probability measures over sets  $\Delta_u = \{Q_1, Q_2, \dots, Q_k, \dots, Q_K\}$  and  $\Delta_v = \{P_1, P_2, \dots, P_k, \dots, P_K\}$  ( $\mu_u \neq \mu_v$ ). Elements in  $Q_k$  and  $P_k$  are denoted by  $q_j^k$  and  $p_j^k$ , respectively. Define the following:

$$\begin{aligned} E_{\bar{P}}[W_0 - \tilde{h} + \alpha(\tilde{h})] &= W_1 - \sum_j p_j \alpha(h_j), \\ E_{\bar{Q}}[W_0 - \tilde{h} + \alpha(\tilde{h})] &= W_0 - \sum_j q_j h_j + \sum_j q_j \alpha(h_j), \\ \sigma_{\bar{P}}^2(\alpha(\tilde{h})) &= \sum_j p_j \alpha(h_j)^2 - (\sum_j p_j \alpha(h_j))^2, \\ \sigma_{\bar{Q}}^2(\alpha(\tilde{h}) - \tilde{h}) &= \sum_j q_j (\alpha(h_j) - h_j)^2 - (\sum_j q_j (\alpha(h_j) - h_j))^2, \\ \sigma_{\mu_v}^2(E[\alpha(\tilde{h})]) &= \sum_k \mu_v^k [\sum_j p_j^k \alpha(h_j)]^2 - [\sum_k \mu_v^k \sum_j p_j^k \alpha(h_j)]^2 \\ \sigma_{\mu_u}^2(E[\alpha(\tilde{h}) - \tilde{h}]) &= \sum_k \mu_u^k [\sum_j p_j^k (\alpha(h_j) - h_j)]^2 - [\sum_k \mu_u^k \sum_j p_j^k (\alpha(h_j) - h_j)]^2 \end{aligned}$$

The first order condition with respect to  $\alpha(\tilde{h}_i)$  is:

$$\begin{aligned} & b \left[ -p_i - \tilde{\theta} p_i \left\{ \alpha(h_i) - \sum_j p_j \alpha(h_j) \right\} - \tilde{\gamma} \left\{ \sum_k \mu_v^k p_i^k (\sum_j p_j^k \alpha(h_j)) - p_i \sum_j p_j \alpha(h_j) \right\} \right] \\ & + a \left[ -q_i - \theta q_i \left\{ (\alpha(h_i) - h_i) - \sum_j q_j (\alpha(h_j) - h_j) \right\} \right] \\ & + a \left[ -\gamma \left\{ \sum_k \mu_u^k q_i^k (\sum_j q_j^k (\alpha(h_j) - h_j)) - q_i \sum_j q_j (\alpha(h_j) - h_j) \right\} \right] = 0 \end{aligned} \quad (2.33)$$

Now differentiate the first order condition with respect to  $h_i$ :

$$-b \left[ \tilde{\theta} p_i (1 - p_i) + \tilde{\gamma} \sigma_{\mu_v}^2(p_i) \right] \left( \frac{d\alpha(h_i)}{dh_i} \right) - a \left[ \theta q_i (1 - q_i) + \gamma \sigma_{\mu_u}^2(q_i) \right] \left( \frac{d\alpha(h_i)}{dh_i} - 1 \right) = 0$$

or equivalently

$$\frac{d\alpha(h_i)}{dh_i} = \frac{a \left[ \theta q_i (1 - q_i) + \gamma \sigma_{\mu_u}^2(q_i) \right]}{a \left[ \theta q_i (1 - q_i) + \gamma \sigma_{\mu_u}^2(q_i) \right] + b \left[ \tilde{\theta} p_i (1 - p_i) + \tilde{\gamma} \sigma_{\mu_v}^2(p_i) \right]} \quad (2.34)$$

Notice that  $\frac{d\alpha(h_i)}{dh_i} \in (0, 1)$ , since  $\theta > 0$ ,  $\tilde{\theta} > 0$ ,  $\gamma > 0$ ,  $\tilde{\gamma} > 0$ , and  $p_i, q_i \geq 0$ . In other words, the optimal contract is a *coinsurance* schedule. ■

## 2.7 Appendix B. Baseline Model: Insurance Demand under Risk

Consider a risk averse decision maker (DM) with a von Neumann-Morgenstern utility function  $u(\cdot)$ , with  $u'(\cdot) > 0$  and  $u''(\cdot) < 0$ , and initial wealth  $W_0$ . She is exposed to a risky loss  $h$  that has mean  $E_P[h]$  and variance  $\sigma_P^2(h)$  under probability distribution  $P$ . There is a risk neutral insurer that is willing to offer the individual any desired amount  $\alpha \in (0, 1]$  of a coinsurance contract.<sup>20</sup> The insurer charges a premium  $\pi = (1 + m)E_P[h]$  per unit of insurance, where  $m \geq 0$  is the loading factor. The insurance contract pays zero if no loss is realized and  $\alpha h$  if it is realized. If the individual decides to purchase insurance, her random final wealth is  $W = W_0 - h - \alpha\pi + \alpha h$ . Thus, the maximization problem of the individual is:

$$\max_{\alpha} E_P[u(W)] = \max_{\alpha} E_P[u(W_0 - h - \alpha(1 + m)E_P[h] + \alpha h)] \quad (2.35)$$

I simplify the problem by using the Arrow-Pratt approximation of the certainty equivalent (CE) of the expected utility wealth.<sup>21</sup> Therefore, the problem in equation (1) can be approximated by:

$$\max_{\alpha} E_P[W] - \frac{\theta}{2}\sigma_P^2(W) \quad (2.36)$$

$$\text{subject to: } W = W_0 - h - \alpha\pi + \alpha h$$

where

$$E_P[W] = E_P[W_0 - h - \alpha(1 + m)E_P[h] + \alpha h] = W_0 - (1 + \alpha m)E_P[h],$$

$$\sigma_P^2(W) = \sigma_P^2(W_0 - h - \alpha(1 + m)E_P[h] + \alpha h) = (1 - \alpha)^2\sigma_P^2(h)$$

and  $\theta = -\frac{u''(W_0)}{u'(W_0)} > 0$  is the Arrow-Pratt degree of absolute risk aversion.

The first and second order condition with respect to  $\alpha$  are:

$$[F.O.C] : -mE_P[h] - \frac{\theta}{2}[2(1 - \alpha)(-1)\sigma_P^2(h)] = 0 \quad (2.37)$$

$$[S.O.C] : -\theta\sigma_P^2(h) < 0 \quad (2.38)$$

Solving for  $\alpha$ , we obtain the following optimal insurance demand :

$$\alpha_{risk}^{M1*} = 1 - \frac{mE_P[h]}{\theta\sigma_P^2(h)} \quad (2.39)$$

Four standard results in the literature of insurance demand follow immediately:

<sup>20</sup>I focus in this section only on coinsurance contracts and deal later with the optimality of this type of contract.

<sup>21</sup>See Gollier (2001, p. 21-24) for details of the approximation.

1. Full coverage is optimal ( $\alpha_{risk}^{M1*} = 1$ ), if coinsurance is available at a fair price ( $m = 0$ ). This is the “Mossin-Smith Theorem” developed in Mossin (1968) and Smith (1968).
2. Given  $m > 0$  such that  $0 < \alpha_{risk}^{1*} < 1$ , an increase in the degree of risk aversion will lead to an increase in the optimal demand for insurance at all levels of wealth, *ceteris paribus*. Schlesinger (2000, p. 138)Schlesinger [2000; p. 138] proves this statement. In the present framework, this is easily seen from the following derivative:

$$\frac{\partial \alpha_{risk}^{M1*}}{\partial \theta} = \frac{mE_P[h]}{\theta^2 \sigma_P^2(h)} > 0 \quad (2.40)$$

This result is a direct consequence of a standard result independently derived by Pratt (1964, p. 136) and Arrow (1971, p. 102). They showed that an increase in absolute risk aversion decreases the demand for a risky asset. This would imply that a risk averse agent will demand more insurance to cover the loss than another individual that is less risk averse.<sup>22</sup>

A change in risk,  $\sigma_P^2(h)$ , affects coinsurance demand in the same direction that risk aversion does because:

$$\frac{\partial \alpha_{risk}^{M1*}}{\partial \sigma_P^2(h)} = \frac{mE_P[h]}{\theta[\sigma_P^2(h)]^2} > 0 \quad (2.41)$$

3. Recall that if the local measure of risk aversion,  $\theta = -\frac{u''(W_0)}{u'(W_0)}$ , is decreasing (increasing) in  $W_0$ , individual’s preferences are said to exhibit DARA (IARA). Moreover, if  $\theta$  is independent of  $W_0$ , preferences exhibit CARA.

**Proposition B.1 (Schlesinger,2000, p.136):** Assume that  $m > 0$  but is not too large such that  $0 < \alpha_{risk}^{M1*} < 1$ . Then, for an increase in the initial wealth  $W_0$ ,

- (i)  $\alpha_{risk}^{M1*}$  will decrease under decreasing absolute risk aversion (DARA).
- (ii)  $\alpha_{risk}^{M1*}$  will be invariant under constant absolute risk aversion (CARA).
- (iii)  $\alpha_{risk}^{M1*}$  will be increasing under increasing absolute risk aversion (IARA).

*Proof.* Take the derivative of  $\alpha_{risk}^{M1*}$  with respect to  $W_0$ :

$$\frac{\partial \alpha_{risk}^{M1*}}{\partial W_0} = \frac{mE_P[h]}{\theta^2 \sigma_P^2(h)} \frac{\partial \theta}{\partial W_0} \quad (2.42)$$

The sign of this derivative depends on whether preferences exhibit IARA, CARA or DARA:

$$\frac{\partial \alpha_{risk}^{M1*}}{\partial W_0} = \begin{cases} > 0, & \text{if } \frac{\partial \theta}{\partial W_0} > 0 \text{ (IARA)} \\ = 0, & \text{if } \frac{\partial \theta}{\partial W_0} = 0 \text{ (CARA)} \\ < 0, & \text{if } \frac{\partial \theta}{\partial W_0} < 0 \text{ (DARA)} \end{cases} \quad (2.43)$$

---

<sup>22</sup>In the context of insurance, the risky asset would be the retained loss. Thus, more insurance would translate into a lower exposure to the risky asset.

■

This proposition states that, if a subject exhibits DARA preferences, the optimal coinsurance demand decreases when initial wealth increases because her risk tolerance increases with it, that is to say, it becomes less risk averse. Similar intuitive arguments can be made about CARA and IARA. However, Schlesinger (2000, p. 136) warns that “each of these conditions [DARA, CARA and IARA] is shown to be sufficient for the comparative-static effects in Proposition [1], though not necessary.”

4. **Definition B.1:**  $\eta = -\frac{u'''(W_0)}{u''(W_0)} > 0$  is the index of absolute prudence. An individual is locally *prudent* at  $W_0$  if  $u'''(W_0) > 0$ , which is equivalent to  $u'(W_0)$  being locally convex ( See Gollier, 2001, p. 237).

Prudent individuals will increase savings if uncertainty affecting future income is introduced. In other words, the degree of prudence captures the sensitivity of savings to changes in risk. This is referred to as the *precautionary motive of saving*. The following proposition, derived by (Gollier, 2001, p. 238), links absolute risk aversion to the concept of prudence.

**Proposition B.2:** Prudence is a *necessary* condition for preferences to exhibit DARA.

*Proof.*

$$\begin{aligned} \frac{\partial \theta}{\partial W_0} &= - \left[ \frac{u'''(W_0)u'(W_0) - [u''(W_0)]^2}{[u'(W_0)]^2} \right] \\ &= \left[ -\frac{u''(W_0)}{u'(W_0)} \right] \left\{ \left[ -\frac{u''(W_0)}{u'(W_0)} \right] - \left[ -\frac{u'''(W_0)}{u''(W_0)} \right] \right\} = \theta [\theta - \eta] \end{aligned} \quad (2.44)$$

Given  $\theta > 0$ ,  $\eta > 0$  is a necessary condition for  $\frac{\partial \theta}{\partial W_0} < 0$ .

■

**Corollary B.1:** Strong enough prudence (i.e.,  $\eta > \theta$ ) is a sufficient condition for  $\frac{\partial \alpha_{risk}^{M1*}}{\partial W_0} < 0$ . *Proof.* See Proposition 1 and 2.

## 2.8 Appendix C. Insurance Demand in the Presence of a Non-traded Asset under Risk

Suppose that the DM faces the same situation as in the baseline model, except that the individual now has a non-tradable and uninsurable asset (e.g., human capital) with risky return  $H$  that might be correlated with a risky loss  $h$ . Thus the risky end-of-period wealth is  $W = W_0 + H - h - \alpha\pi + \alpha h$ . Assume the DM maximizes expected utility defined by utility function  $u(\cdot)$  and  $\bar{Q}$ , the joint distribution of  $H$  and  $h$ . Therefore the maximization problem is:

$$\max_{\alpha} E_{\bar{Q}}[W] - \frac{\theta}{2} \sigma_{\bar{Q}}^2(W) \quad (2.45)$$

where

$$\begin{aligned} E_{\bar{Q}}[W] &= E_{\bar{Q}}[W_0 + H - h - \alpha(1+m)E[h] + \alpha h] = W_0 + E_{\bar{Q}}[H] - (1+\alpha m)E_{\bar{Q}}[h] \\ \sigma_{\bar{Q}}^2(W) &= \sigma_{\bar{Q}}^2(H) + (1-\alpha)^2 \sigma_{\bar{Q}}^2(h) - 2(1-\alpha) \text{cov}_{\bar{Q}}(H, h) \end{aligned}$$

The first order condition of the maximization problem is:

$$[\alpha] : -mE_{\bar{Q}}[h] - \frac{\theta}{2} \left\{ 2(1-\alpha)(-1)\sigma_{\bar{Q}}^2(h) - 2(-1)\text{cov}_{\bar{Q}}(H, h) \right\} = 0 \quad (2.46)$$

The optimal demand for insurance is:<sup>23</sup>

$$\alpha_{risk}^{M3*} = 1 - \frac{mE_{\bar{Q}}[h]}{\theta\sigma_{\bar{Q}}^2(h)} - \frac{\text{cov}_{\bar{Q}}(H, h)}{\sigma_{\bar{Q}}^2(h)} \quad (2.47)$$

The Mossin-Smith theorem only holds if  $\text{cov}_{\bar{Q}}(H, h) \leq 0$ . Doherty (1984, p. 209) derived a related result, and Mayers and Smith (1983) were the first to emphasize the interdependency between insurance demand and traded and non-traded assets.

Following the terminology in Mayers and Smith (1983, p. 308),  $\text{cov}_{\bar{Q}}(H, h)$  represents the individual's incentive to “*self-insure*.” The sign of this covariance makes the insurance demand lower, equal or higher than in the absence of the non-traded asset (i.e.,  $\alpha_{risk}^{M3*} \begin{cases} \leq \\ \geq \end{cases} \alpha_{risk}^{M1*}$ ). However, I prefer to use the term “*self-hedging*” to avoid confusion with the usage of “*self-insurance*” in the literature. The intuition is that even if the insurance premium is fair (i.e.,  $m = 0$ ), the DM might still not be willing to fully insure if she can compensate high losses with high realizations of her human capital. However, if  $\text{cov}_{\bar{Q}}(H, h) < 0$ , the individual might want to fully insure if a health shock ( $h$ ) affects negatively her productivity, which would undermine her human capital.

Moreover, the introduction of a risky non-traded asset does not affect the Arrow-Pratt result. As a result, higher risk aversion increases the demand for insurance:

$$\frac{\partial \alpha_{risk}^{M3*}}{\partial \theta} = \frac{mE_{\bar{Q}}[h]}{\theta^2 \sigma_{\bar{Q}}^2(h)} > 0 \quad (2.48)$$

---

<sup>23</sup>The second order condition is satisfied.

Finally, Proposition B.1 ( the sufficiency of CARA, DARA or IARA for the insurance demand to be independent of, decreasing or increasing in  $W_0$ ) and Proposition B.2 (the necessity of prudence to exhibit DARA) are robust to the presence of a non-traded asset. The optimal demands  $\alpha_{risk}^{M1*}$  and  $\alpha_{risk}^{M3*}$ , differ only in the term that depends on  $\frac{cov_{\bar{Q}}(H,h)}{\sigma_{\bar{Q}}^2(h)}$ , which is not affected by initial wealth. Thus, the proofs of these two propositions remain basically unchanged.

## Chapter 3

# Reduction of Compound Lotteries with Objective Probabilities: Theory and Evidence

*by Glenn W. Harrison, Jimmy Martínez-Correa and Todd J. Swarthout*

### **Abstract**

The reduction of compound lotteries (ROCL) has assumed a central role in the evaluation of behavior towards risk and uncertainty. We present experimental evidence on its validity in the domain of objective probabilities. Our experiment explicitly recognizes the impact that the random lottery incentive mechanism payment procedure may have on preferences, and so we collect data using both “1-in-1” and “1-in-K” payment procedures, where  $K > 1$ . We do not find violations of ROCL when subjects are presented with only one choice that is played for money. However, when individuals are presented with many choices and random lottery incentive mechanism is used to select one choice for payoff, we do find violations of ROCL. These results are supported by both non-parametric analysis of choice patterns, as well as structural estimation of latent preferences. We find evidence that the model that best describes behavior when subjects make only one choice is the Rank-Dependent Utility model. When subjects face many choices, their behavior is better characterized by our sourcedependent version of the Rank-Dependent Utility model which can account for violations of ROCL. We conclude that payment

protocols can create distortions in experimental tests of basic axioms of decision theory.

## 3.1 Introduction

The reduction of compound lotteries has assumed a central role in the evaluation of behavior towards risk, uncertainty and ambiguity. We present experimental evidence on its validity in domains defined over objective probabilities, as a prelude to evaluating it over subjective probabilities<sup>1</sup>

Because of the attention paid to violations of the Independence Axiom, it is noteworthy that early formal concerns with the possibility of a “utility or disutility for gambling” centered around the Reduction of Compound Lotteries (ROCL) axiom.<sup>2</sup> von Neumann and Morgenstern (1953, p. 28) commented on the possibility of allowing for a (dis)utility of gambling component in their preference representation:

Do not our postulates introduce, in some oblique way, the hypotheses which bring in the mathematical expectation [of utility]? More specifically: May there not exist in an individual a (positive or negative) utility of the mere act of ‘taking a chance,’ of gambling, which the use of the mathematical expectation obliterates? How did our axioms (3:A)- (3:C) get around this possibility? As far as we can see, our postulates (3:A)-(3:C) do not attempt to avoid it. Even the one that gets closest to excluding the ‘utility of gambling’ - (3:C:b)- seems to be plausible and legitimate - unless a much more refined system of

---

<sup>1</sup>We explain in Appendix A of this chapter the theoretical, methodological and experimental issues of testing ROCL under subjective probabilities.

<sup>2</sup>The issue of the (dis)utility of gambling goes back at least as far as Pascal, who argued in his *Pensées* that “people distinguish between the pleasure or displeasure of chance (uncertainty) and the objective evaluation of the worth of the gamble from the perspective of its consequences” (see Luce and Marley (2000, p. 102). Referring to the ability of bets to elicit beliefs, Ramsey (1926) claims that [t]his method I regard as fundamentally sound; but it suffers from being insufficiently general, and from being necessarily inexact. It is inexact partly [...] because the person may have a special eagerness or reluctance to bet, because he either enjoys or dislikes excitement or for any other reason, e.g. to make a book. The difficulty is like that of separating two different cooperating forces” (from the reprint in Kyburg and Smokler (1964, p. 73)).

psychology is used than the one now available for the purposes of economics [...]. Since (3:A)-(3:C) secure that the necessary construction [of utility] can be carried out, concepts like a specific utility of gambling cannot be formulated free of contradiction on this level.

On the very last page of their *magnus opus*, von Neumann and Morgenstern (1953, p. 632) propose that if their postulate (3:C:b), which is the ROCL, is relaxed, one could indeed allow for a specific utility for the act of gambling:

It seems probable, that the really critical group of axioms is (3:C) - or, more specifically, the axiom (3:C:b). This axiom expresses the combination rule for multiple chance alternatives, and it is plausible, that a specific utility or disutility of gambling can only exist if this simple combination rule is abandoned. Some change of the system [of axioms] (3:A)-(3:B), at any rate involving the abandonment or at least a radical modification of (3:C:b), may perhaps lead to a mathematically complete and satisfactory calculus of utilities which allows for the possibility of a specific utility or disutility of gambling. It is hoped that a way will be found to achieve this, but the mathematical difficulties seem to be considerable.

Thus, the relaxation of the ROCL axiom opens the door to the possibility of having a distinct (dis)utility for the act of gambling with objective probabilities.<sup>3</sup> Fellner (1961, 1963) and Smith (1969) used similar reasoning to offer an explanation for several of the Ellsberg (1961) paradoxes.

This argument rests on the hypothesis that subjects potentially view simple and compound random processes differently. If this hypothesis is true, it could explain why people prefer risky over ambiguous gambles in the thought experiments of Ellsberg (1961). Fellner (1961, 1963) and Smith (1969) believed that if a subject could exhibit utility or disutility of gambling she may also use different utility functions to make decisions under different

---

<sup>3</sup>Of course, it is of some comfort to the egos of modern theorists that no less than von Neumann and Morganstern at least viewed it as a serious mathematical challenge.

processes. Smith (1969) went further and explicitly conjectured that a compound lottery defined over *objective* probabilities, and its actuarially-equivalent lottery over objective probabilities, might be viewed by decision makers as two different random processes. In fact, he proposed a preference representation that allowed people to have different utility functions for different random processes. We use this conjectured preference representation to test for violations of ROCL.

One fundamental methodological problem with tests of the ROCL assumption, whether or not the context is objective or subjective probabilities, is that one cannot use incentives for decision makers that rely on the validity of ROCL. This means, in effect, that experiments must be conducted in which a subject has one, and only one, choice.<sup>4</sup> Apart from the expense and time of collecting data at such a pace, this also means that all evaluations have to be on a between-subjects basis, implying the necessity of modeling assumptions about heterogeneity in behavior.

In sections 3.2 and 2 we define the theory and experimental tasks used to examine ROCL in the context of objective probabilities. In section 3 and 4 we present evidence from our experiment. We find no violations of ROCL when subjects are presented with one and only one choice, and that their behavior is better characterized by the Rank-Dependent Utility model (RDU) rather than Expected Utility Theory (EUT). However, we do find violations of ROCL when many choices are given to each subject and the random lottery incentive mechanism (RLIM) is used as the payment protocol. Under RLIM, behavior is better

---

<sup>4</sup>One alternative is to present the decision maker with several tasks at once and evaluate the portfolio chosen, or to present the decision maker with several tasks in sequence and account for wealth effects. Neither is attractive, since they each raise a number of (fascinating) theoretical confounds to the interpretation of observed behavior. One uninteresting alternative is not to pay the decision maker for the outcomes of the task

characterized by our source-dependent version of RDU that can account for violations of ROCL. Section 5 draws conclusions for modeling, experimental design, and inference about decision making.

## 3.2 Theory

Following Segal (1988a, 1990, 1992), we distinguish between three axioms. In words, the **Reduction of Compound Lotteries** axiom states that a decision-maker is indifferent between a compound lottery and the actuarially-equivalent simple lottery in which the probabilities of the two stages of the compound lottery have been multiplied out. To use the language of Samuelson (1952, p. 671), the former generates a *compound income-probability-situation*, and the latter defines an *associated income-probability-situation*, and that “...only algebra, not human behavior, is involved in this definition.”

To state this more explicitly, with notation to be used to state all axioms, let  $X$ ,  $Y$  and  $Z$  denote simple lotteries,  $A$  and  $B$  denote compound lotteries,  $\succ$  express strict preference, and  $\sim$  express indifference. Then the ROCL axiom says that  $A \sim X$  if the probabilities and prizes in  $X$  are the actuarially-equivalent probabilities and prizes from  $A$ . Thus if  $A$  is the compound lottery that pays “double or nothing” from the outcome of the lottery that pays \$10 if a coin flip is a head and \$2 if the coin flip is a tail, then  $X$  would be the lottery that pays \$20 with probability  $0.5 \times 0.5 = 0.25$ , \$4 with probability  $0.5 \times 0.5 = 0.25$ , and nothing with probability 0.5. From an observational perspective, one would have to see choices

between compound lotteries and the actuarially-equivalent simple lottery to test ROCL.

The **Compound Independence Axiom** (CIA) states that a compound lottery formed from two simple lotteries by adding a positive common lottery with the same probability to each of the simple lotteries will exhibit the same preference ordering as the simple lotteries. So this is a statement that the ordering of the two constructed compound lotteries will be the same as the ordering of the different simple lotteries that distinguish the compound lotteries, provided that the common prize in the compound lotteries is the same and has the same (compound lottery) probability. It says nothing about how the compound lotteries are to be evaluated, and in particular it does not assume ROCL. It only restricts the preference ordering of the two constructed compound lotteries to match the preference ordering of the original simple lotteries.

The CIA says that if A is the compound lottery giving the simple lottery X with probability  $\alpha$  and the simple lottery Z with probability  $(1 - \alpha)$ , and B is the compound lottery giving the simple lottery Y with probability  $\alpha$  and the simple lottery Z with probability  $(1 - \alpha)$ , then  $A \succ B$  iff  $X \succ Y \forall \alpha \in (0, 1)$ . So the construction of the two compound lotteries A and B has the “independence axiom” cadence of the common prize Z with a common probability  $(1 - \alpha)$ , but the implication is only that the ordering of the compound and constituent simple lotteries are the same.<sup>5</sup>

Finally, the **Mixture Independence Axiom** (MIA) says that the preference ordering of

---

<sup>5</sup>For example, Segal (1992, p. 170) defines the CIA by assuming that the second-stage lotteries are replaced by their certainty-equivalent, throwing away information about the second-stage probabilities before one examines the first-stage probabilities at all. Hence one cannot then define the actuarially-equivalent simple lottery, by construction, since the informational bridge to that calculation has been burnt. The certainty-equivalent could have been generated by any model of decision making under risk, such as RDU or Prospect Theory.

two simple lotteries must be the same as the actuarially-equivalent simple lottery formed by adding a common outcome in a compound lottery of each of the simple lotteries, where the common outcome has the same value and the same (compound lottery) probability. So stated, it is clear that the MIA strengthens the CIA by making a definite statement that the constructed compound lotteries are to be evaluated in a way that is ROCL-consistent. Construction of the compound lottery in the MIA is actually implicit: the axiom only makes observable statements about two pairs of simple lotteries. To restate Samuelson's point about the definition of ROCL, the experimenter testing the MIA could have constructed the associated income-probability-situation without knowing the risk preferences of the individual (although the experimenter would need to know how to multiply).

The MIA says that  $X \succ Y$  iff the actuarially-equivalent simple lottery of  $\alpha X + (1 - \alpha)Z$  is strictly preferred to the actuarially-equivalent simple lottery of  $\alpha Y + (1 - \alpha)Z$ ,  $\forall \alpha \in (0, 1)$ . The verbose language used to state the axiom makes it clear that MIA embeds ROCL into the usual independence axiom construction with a common prize  $Z$  and a common probability  $(1 - \alpha)$  for that prize.

The reason these three axioms are important is that the failure of MIA does not imply the failure of CIA and ROCL. It does imply the failure of one or the other, but it is far from obvious which one. Indeed, one could imagine some individuals or task domains where only CIA might fail, only ROCL might fail, or both might fail. Because specific types of failures of ROCL lie at the heart of many important models of decision-making under uncertainty and ambiguity, it is critical to keep the axioms distinct as a theoretical *and* experimental

matter.

### 3.2.1 Experimental Payment Protocols

Turning now to experimental procedures, as a matter of theory the most popular payment protocol assumes the validity of MIA. This payment protocol is called the Random Lottery Incentive Mechanism (RLIM). It entails the subject undertaking  $K > 1$  tasks and then one of the  $K$  choices being selected at random to be played out. Typically, and without loss of generality, assume that the selection of the  $k$ -th task to be played out uses a uniform distribution over the  $K$  tasks. Since the other  $K-1$  tasks will generate a payoff of zero, the payment protocol can be seen as a compound lottery that assigns probability  $\alpha = 1/K$  to the selected task and  $(1-\alpha) = (1-(1/K))$  to the other  $K-1$  tasks as a whole. If the task consists of binary choices between simple lotteries  $X$  and  $Y$ , then the RLIM can be immediately seen to entail an application of MIA, where  $Z = U(\$0)$  and  $(1-\alpha) = (1-(1/K))$ , for the utility function  $U(\cdot)$ . Hence, under MIA, the preference ordering of  $X$  and  $Y$  is independent of all of the choices in the other tasks (Holt, 1986).

The need to assume the MIA can be avoided by setting  $K=1$ , and asking each subject to answer one binary choice task for payment. Unfortunately, this comes at the cost of another assumption if one wants to compare choice patterns over two simple lottery pairs, as in most of the popular tests of EUT such as the Allais Paradox and Common Ratio test: the assumption that risk preferences across subjects are the same. This is a strong assumption, obviously, and one that leads to inferential tradeoffs in terms of the “power” of tests of EUT relying on randomization that will vary with sample size. Sadly, plausible

estimates of the degree of heterogeneity in the typical population imply massive sample sizes for reasonable power, well beyond those of most experiments.

The assumption of homogeneous preferences can be diluted, however, by changing it to a conditional form: that risk preferences are homogeneous conditional on a finite set of observable characteristics.<sup>6</sup> Although this sounds like an econometric assumption, and it certainly has statistical implications, it is as much a matter of (operationally meaningful) theory as formal statements of the CIA, ROCL and MIA.

## 3.3 Experiment

### 3.3.1 Lottery Parameters

We designed our battery of lotteries to allow for specific types of comparisons needed for testing ROCL. Beginning with a given *simple* (S) lottery and *compound* (C) lottery, we next create an *actuarially-equivalent* (AE) lottery from the C lottery, and then we construct three pairs of lotteries: a S-C pair, a S-AE pair, and an AE-C pair. By repeating this process 15 times, we create a battery of lotteries consisting of 15 S-C pairs shown in Table 3.11, 15 S-AE pairs shown in Table 3.13, and 10 AE-C pairs<sup>7</sup> shown in Table 3.11.

See section 3.10 for additional information regarding the creation of these lotteries.

Figure 3.1 displays the coverage of lotteries in the Marschak-Machina triangle, covering all

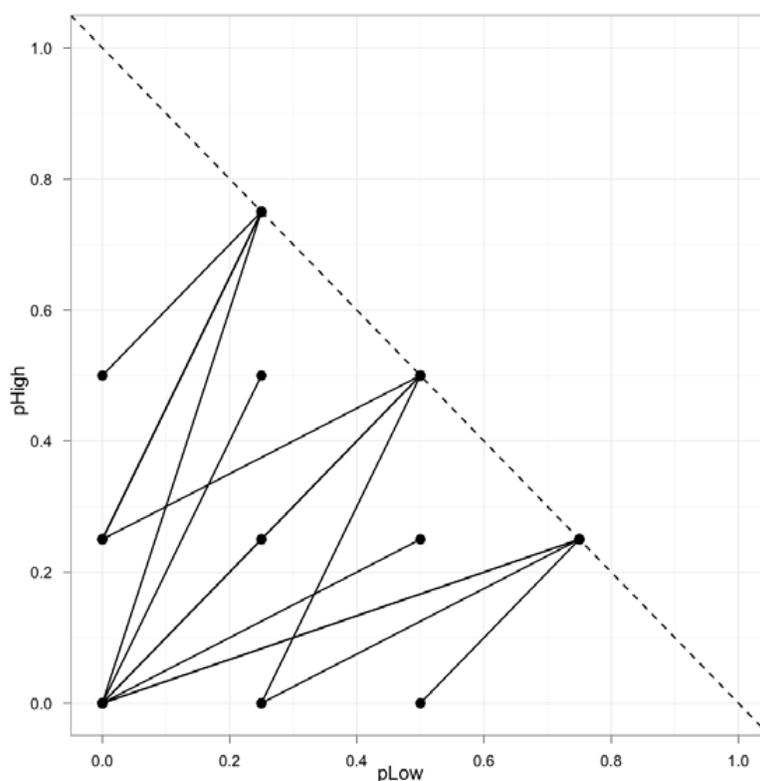
---

<sup>6</sup>Another way of diluting the assumption is to posit some (flexible) parametric form for the distribution of risk attitudes in the population, and use econometric methods that allow one to estimate the extent of that unobserved heterogeneity across individuals. Tools for this “random coefficients” approach to estimating non-linear preference functionals are developed in Andersen et al. (2012).

<sup>7</sup>The lottery battery contains only 10 AE-C lottery pairs because some of the 15 S-C lottery pairs shared the same compound lottery.

of the contexts used.<sup>8</sup> Probabilities were drawn from 0, 0.25, 0.50, 0.75 and 1, and the final prizes from \$0, \$10, \$20, \$35 and \$70. We use the familiar “Double or Nothing” (DON) procedure for creating compound lotteries. So, the first-stage prizes displayed in a compound lottery were drawn from \$5, \$10, \$17.50 and \$35, and then the second-stage DON procedure yields the set of final prizes given above.

Figure 3.1: Battery of 40 Lotteries Pairs Probability Coverage

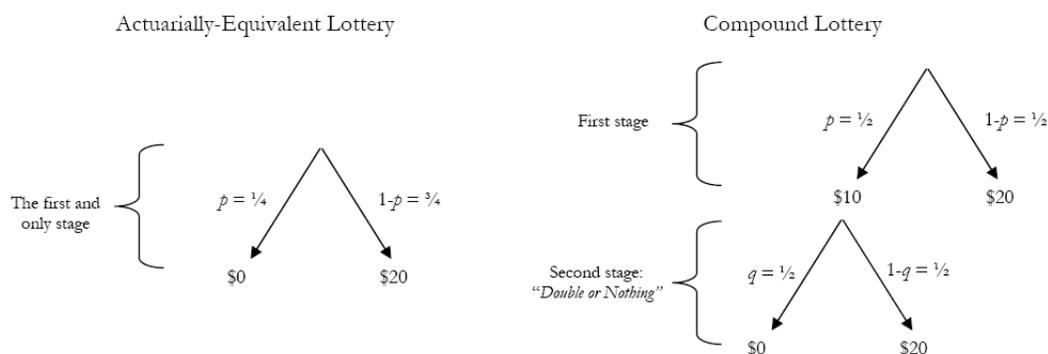


The majority of our compound lotteries use a conditional version of DON in the sense that the initial lottery will trigger the double or nothing option that the subject will face only if a particular outcome is realized in the initial lottery. For example, consider the compound lottery formed by an initial lottery that pays \$10 and \$20 with equal probability and the

<sup>8</sup>Decision screens were presented to subjects in color. Black borders were added to each pie slice in Figures 3.3, 3.4 and 3.5 to facilitate black-and-white viewing.

option of playing DON if the outcome of the initial lottery is \$10, implying a payoff of \$20 or \$0 with equal chance *if the DON stage is reached*. If the initial outcome is \$20, there is no DON option beyond that. The right panel of Figure 3.2 shows a tree representation of the latter compound lottery where the initial lottery is depicted in the first stage and the DON lottery is depicted in the second stage of the compound lottery if reached. The left panel of Figure 3.2 shows the corresponding actuarially-equivalent simple lottery which offers \$20 with probability 0.75 and \$0 with probability 0.25.

Figure 3.2: Tree Representation of a Compound Lottery and its Corresponding Actuarially-Equivalent Simple Lottery



The conditional DON lottery allows us to obtain good coverage in terms of prizes and probabilities and to maintain a simple random processes for the initial lottery and the DON option. One can construct a myriad of compound lotteries with only two components: (1) initial lotteries that pay two outcomes with 50:50 odds or pay a given stake with certainty; and (2) a conditional DON which pays double a predetermined amount with 50% probability or nothing with equal chance. Using only the unconditional DON option would impose an *a priori* restriction on the coverage within the Marschak-Machina triangle.

### 3.3.2 Experimental Procedures

We implement two between-subjects treatments. We call one treatment “Pay 1-in-1” (1-in-1) and the other “Pay 1-in-40” (1-in-40). Table 3.1 summarizes our experimental design and the sample size of subjects and choices in each treatment.

Table 3.1: Default Simple Lotteries

Treatment	Subjects	Choices
1. Pay-1-in-1	133	133
2. Pay-1-in-40	62	2480

In the 1-in-1 treatment, each subject faces a single choice over two lotteries. The lottery pair presented to each subject is randomly selected from the battery of 40 lottery pairs. The lottery chosen by the subject is then played out and the subject receives the realized monetary outcome. There are no other salient tasks, before or after a subject’s binary choice, that affect the outcome. Further, there is no other activity that may contribute to learning about decision making in this context.

In the 1-in-40 treatment, each subject faces choices over all 40 lottery pairs, with the order of the pairs randomly shuffled for each subject. After all choices have been made, one choice is randomly selected for payment using the RLIM, with each choice having a 1-in-40 chance of being selected. The selected choice is then played out and the subject receives the realized monetary outcome, again with no other salient tasks. This treatment is potentially different from the 1-in-1 treatment in the absence of ROCL, since the RLIM induces a compound lottery consisting of a 1-in-40 chance for each of the 40 chosen lotteries to be selected for payment.

The general procedures during an experiment session were as follows. Upon arrival at the laboratory, each subject drew a number from a box which determined random seating position within the laboratory. After being seated and signing the informed consent document, subjects were given printed instructions and allowed sufficient time to read these instructions.<sup>9</sup> Once subjects had finished reading the instructions, an experimenter at the front of the room read aloud the instructions, word for word. Then the randomizing devices<sup>10</sup> were explained and projected onto the front screen and three large flatscreen monitors spread throughout the laboratory. The subjects were then presented with lottery choices, followed by a non-salient demographic questionnaire that did not affect final payoffs. Next, each subject was approached by an experimenter who would provide dice so for the subject to roll and determine her own payoff. If a DON stage was reached, a subject would flip a U.S. quarter dollar coin to determine the final outcome of the lottery. Finally, subjects then left the laboratory and were privately paid their earnings: a \$7.50 participation payment in addition to the monetary outcome of the realized lottery.

We used software created in Visual Basic .NET to present lotteries to subjects and record their choices. Figure 3.3 shows an example of the subject display of an AE-C lottery pair. The first and second stages of the compound lottery, like the one depicted in Figure 3.2, are presented as an initial lottery, represented by the pie on the right of Figure 3.3 that has a DON option identified by text. The pie chart on the left of Figure 3.3 shows the AE lottery of the paired C lottery on the right. Figure 3.4 shows an example of the subject

---

<sup>9</sup>Section 3.8 provides complete subject instructions.

<sup>10</sup>Only physical randomizing devices were used, and these devices were demonstrated prior to any decisions. In the 1-in-40 treatment, two 10-sided dice were rolled by each subject until a number between 1 and 40 came up to select the relevant choice for payment. Subjects in both treatments would roll the two 10-sided dice (a second roll in the case of the 1-in-40 treatment) to determine the outcome of the chosen lottery.

display of a S-C lottery pair, and Figure 3.5 shows an example of the subject display of a S-AE lottery pair.

Figure 3.3: Choices Over Compound and Actuarially-Equivalent Lotteries



Figure 3.4: Choices Over Simple and Compound Lotteries

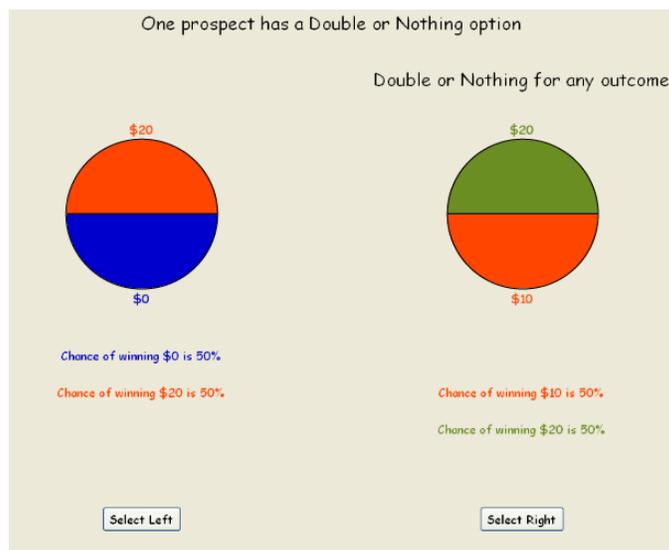


Figure 3.5: Choices Over Simple and Actuarially-Equivalent Lotteries



### 3.3.3 Evaluation of Hypotheses

If the subjects in both treatments have the same risk preferences and behavior is consistent with ROCL, we should see the same pattern of decisions for comparable lottery pairs across the two treatments. The same pattern should also be observed as one characterizes heterogeneity of individual preferences towards risk, although these inferences depend on the validity of the manner in which heterogeneity is modeled.

Nothing here assumes that behavior is characterized by EUT. The validity of EUT requires both ROCL and CIA, and the validity of ROCL does not imply the validity of CIA. So when we say that risk preferences should be the same in the two treatments under ROCL, these are simply statements about the Arrow-Pratt risk premium, and not about how that is decomposed into explanations that rely on diminishing marginal utility or probability weighting. We later analyze the decomposition of the risk premium as well as the nature of any violation of ROCL.

Our method of evaluation is twofold. First, we use non-parametric tests to evaluate the choice patterns of subjects. Our experimental design allows us to evaluate ROCL using choice patterns in two ways: (1) directly examine choice patterns in AE-C lottery pairs where ROCL predicts indifference; and (2) examine the choice patterns across the linked S-C and S-AE lottery pairs. We have 15 tests, one for each linked pair of lottery pairs, as well as a pooled test over all 15 pairs of pairs. We are agnostic as to the choice pattern itself: if subjects have a clear preference for S over C in a given lottery pair, then under ROCL we should see the same preference for the identical S over the AE in the linked lottery pair.

For our second method of evaluation of ROCL, we estimate structural models of risk preferences and test if the risk preference parameters depend on whether a C or an AE lottery is being evaluated. This method does not assume EUT, and indeed we allow non-EUT specifications. We specify a source dependent form of utility and probability weighting function and test for violations of ROCL by determining if the subjects evaluate simple and compound lotteries differently.

In both of our methods of evaluation of ROCL, we use data from the 1-in-1 treatment and the 1- in-40 treatment which uses RLIM as the payment protocol. Of course, analysis of the data from the 1-in- 40 treatment requires us to assume incentive compatibility with respect to the experiment payment protocol. However, by also analyzing choices from the 1-in-1 treatment we can test if the RLIM itself creates distortions that could be confounded with violations of ROCL. We conclude with discussion of the relative advantages and disadvantages of the econometric tests and the choice pattern tests.

## 3.4 Non-Parametric Analysis of Choice Patterns

### 3.4.1 Choice Patterns Where ROCL Predicts Indifference

The basic prediction of ROCL is that subjects who satisfy the axiom are indifferent between a compound lottery and its actuarially-equivalent lottery. We analyze the observed responses from subjects who were presented with any of the 10 pairs that contained both a C lottery and its AE lottery.<sup>11</sup> First, we study the responses from the 32 subjects who were presented with an AE-C pair in 1-in-1 treatment. Then, we study the 620 responses from the 62 subjects who each were presented with all of the 10 AE-C pairs in the 1-in-40 treatment.

We analyze the data separately because, in contrast to the 1-in-40 treatment, any conclusion drawn from the 1-in-1 treatment do not depend on the incentive compatibility of the RLIM. We want to control for the possibility that the observed choice patterns in the 1-in-40 treatment are affected by this payment protocol.<sup>12</sup> By analyzing data from the 1-in-1 treatment only, we avoid any possible confounds created by the RLIM.

Our null hypothesis is that subjects behave according to ROCL. ROCL predicts that a subject is indifferent between a C lottery and its paired AE lottery, and therefore we should observe equiprobable response proportions between C and AE lotteries in our 10 AE-C pairs. ROCL is violated if, for a given AE-C lottery pair, we observe that the proportion C lottery choices is significantly different from the proportion of AE lottery choices.

---

<sup>11</sup>These are pairs 31 through 40 of Table 3.13.

<sup>12</sup>An additional consideration is that our interface did not allow expression of indifference, so we test for equal proportions of expressions of strict preference. Even if we had allowed direct expression of indifference, we have no way of knowing if subjects were in fact indifferent but preferred to use their own randomizing device (in their heads). The same issue confronts tests of mixed strategies in strategic games.

We do not find statistical evidence to reject the basic ROCL prediction of indifference in the 1-in-1 treatment, although we do find statistical evidence to support violations of ROCL in the 1-in-40 treatment. Thus, giving many lottery pairs to individuals and using the RLIM to select one choice at random for payoff create distortions in the individual decision-making process that can be confounded with violations of ROCL.

### **Analysis of Data from the 1-in-1 Treatment**

We use a generalized version of the Fisher Exact test to jointly test the null hypothesis that the proportion of subjects who chose the C lottery over the AE lottery in each of the AE-C lottery pairs are the same, as well as the Binomial Probability test to evaluate our null hypothesis of equiprobable choice in each of the AE-C lottery pairs.

We do not observe statistically significant violations of the ROCL indifference prediction in the 1-in-1 treatment. Table 3.2 presents the generalized Fisher Exact test for all AE-C lottery pair choices, and the test's  $p$ -value of 0.342 provides support for the null hypothesis. We see from this test that the proportions are the same across pairs. We now use a series of Binomial Probability tests to see if the proportions are different from 50%. Table 3.3 shows the Binomial Probability test applied individually to each of the AE-C lottery pairs for which we have observations. We see no evidence to reject the null hypothesis that subjects chose the C and the AE lotteries in equal proportions, as all  $p$ -values are insignificant at any reasonable level of confidence. The results of both of these tests suggest that ROCL is satisfied in the 1-in-1 treatment.

Table 3.2: Generalized Fisher Exact Test on the Actuarially-Equivalent Lottery vs. Compound Lottery Pairs

Treatment: 1-in-1

Fisher Exact  $p$ -value = 0.342

AE-C Lottery Pair	Observed # of choices of AE lotteries	Observed # of choices of C lotteries	Total
31	0	1	1
32	0	3	3
33	2	5	7
36	4	1	5
37	1	1	2
38	2	1	3
39	1	4	5
40	3	3	6
Total	13	19	32

Note: due to the randomization assignment of lottery pairs to subjects, there were no observations for pairs 34 and 35.

### Analysis of Data from the 1-in-40 Treatment

The strategy to test the ROCL prediction of indifference in this treatment is different from the one used in the 1-in-1 treatment, given the repeated measures we have for each subject in the 1-in-40 treatment. We now use the Cochran Q test to evaluate whether the proportion of subjects who choose the C lottery is the same in each of the 10 AE-C lottery pairs.<sup>13</sup> A significant difference of proportions identified by this test is sufficient to reject the null prediction of indifference.<sup>14</sup> Of course, an insignificant difference of proportions

<sup>13</sup>The Binomial Probability test is inappropriate in this setting, as it assumes independent observations. Obviously, observations are not independent when each subject makes 40 choices in this treatment.

<sup>14</sup>For example, suppose there were only 2 AE-C lottery pairs. If the Cochran Q test finds a significant difference, we conclude that the proportion of subjects choosing the C lottery is not the same in the two lottery pairs. Therefore, even if the proportion for one of the pairs was truly equal to 50%, the test result

Table 3.3: Binomial Probability Tests on Actuarially-Equivalent Lottery vs. Compound Lottery Pairs

Treatment: 1-in-1

AE-C Lottery Pair	Total # of observations	Observed # of choices of C lotteries	Observed proportion of choices of C lotteries ( $p$ )	$p$ -value for $H_0: p = 0.5$
32	3	3	1	0.25
33	7	5	0.714	0.453
36	5	1	0.2	0.375
37	2	1	0.5	1
38	3	1	0.333	1
39	5	4	0.8	0.375
40	6	3	0.5	1

Note: due to the randomization assignment of lottery pairs to subjects there were no observations for pairs 34 and 35 and only 1 observation for pair 31.

would require us to additionally verify that the common proportion across pairs the pairs is indeed 50% before we fail to reject the null hypothesis of indifference.

We observe an overall violation of the ROCL indifference prediction in the 1-in-40 treatment. Table 3.4 reports the results of the Cochran Q test, as well as summary statistics of the information used to conduct the test. The Cochran Q test yields a  $p$ -value of less than 0.0001, which strongly suggests rejection of the null hypothesis of equiprobable proportions. We conclude, for at least for one of the AEC lottery pairs, that the proportion of subjects who chose the C lottery is not equal to 50%. This result is a violation of ROCL and we cannot claim that subjects satisfy ROCL and choose at random in all of the 10 AE-C lottery pairs in the 1-in-40 treatment.

would imply that the other proportion is not statistically equal to 50%, and thus indifference fails.

Table 3.4: Cochran Q Test on the Actuarially-Equivalent Lottery vs. Compound Lottery Pairs

Treatment: 1-in-40

Cochran's  $\chi^2$  statistic (9 d.f) = 86.090

$p$ -value < 0.0001

Data	
AE-C Lottery Pair	Observed # of choices of C lotteries (out of 62 observations)
31	48
32	46
33	34
34	26
35	24
36	18
37	34
38	35
39	49
40	28

### 3.4.2 Choice Patterns Where ROCL Predicts Consistent Choices

Suppose a subject is presented with a given S-C lottery pair, and further assume that she prefers the C lottery over the S lottery. If the subject satisfies ROCL and is separately presented with a second pair of lotteries consisting of the same S lottery and the AE lottery of the previously-presented C lottery, then she would prefer and should choose the AE lottery. Similarly, of course, if she instead prefers the S lottery when presented separately with a given S-C lottery pair, then she should choose the S lottery when presented with the corresponding S-AE lottery pair.

Recall that each of the 15 S-C lottery pairs in Table 3.11 has a corresponding S-AE pair in

Table 3.12. Therefore, we can construct 15 comparisons of lottery pairs that constitute 15 consistency tests of ROCL. In the 1-in-40 treatment we again must assume that the RLIM is incentive compatible, and we again use data from the 1-in-1 treatment to control for possible confounds created by the RLIM. We must now assume homogeneity in risk preferences for the analysis of behavior in the 1-in-1 treatment, since we are making across-subject comparisons. However, in the next section we will present econometric analysis which allows for heterogeneity in risk preferences and test if a violation of the homogeneity assumption is confounded with a violation of ROCL.

Our hypothesis is that a given subject chooses the S lottery when presented with the S-C lottery pair if and only if the same subject also chooses the S lottery when presented with the corresponding SAE lottery pair.<sup>15</sup> Therefore, ROCL is satisfied if we observe that the proportion of subjects who choose the S lottery when presented with a S-C pair is equal to the proportion of subjects who choose the S lottery when presented with the corresponding S-AE pair. Conversely, ROCL is violated if we observe unequal proportions of choosing the S lottery across a S-C pair and linked S-AE pair.

We do not find evidence to reject the consistency in patterns implied by ROCL in the 1-in-1 treatment, while we do find evidence of violations of ROCL in the 1-in-40 treatment. As in the case of the ROCL indifference prediction, we conclude that giving many lottery pairs to individuals and using the RLIM to select one choice at random for payoff create distortions in the individual choice making process that can be confounded with violations of ROCL.

---

<sup>15</sup>Notice that this is equivalent to stating the null hypothesis using the C and AE lotteries. We chose to work with the S lottery for simplicity.

## Analysis of Data from the 1-in-1 Treatment

We use the Cochran-Mantel-Haenszel (CMH) test to test the joint hypothesis that in all of the 15 paired comparisons, subjects choose in the same proportion the S lottery when presented with the S-C lottery pair and its linked S-AE lottery pair.<sup>16</sup> If the CMH test rejects the null hypothesis, then we interpret this as evidence of overall ROCL-inconsistent observed behavior. We also use the Fisher Exact test to evaluate individually the consistency predicted by ROCL in each of the 15 linked comparisons of S-C pairs and S-AE pairs for which we have enough data to conduct the test.

We do not reject the ROCL consistency prediction. The CMH test does not reject the joint null hypothesis that the proportion of subjects chose the S lottery when they were presented with any given SC pair is equal to the proportion of subjects that chose the S lottery when they were presented with the corresponding S-AE pair. The  $\chi^2$ -statistic for the CMH test with the continuity correction<sup>17</sup> is equal to 2.393 with a corresponding  $p$ -value of 0.122. Similarly, the Fisher Exact tests presented in Table 5 show only in one comparison the  $p$ -value is less than 0.05. These results suggest that the ROCL consistency prediction holds in the 1-in-1 treatment. However, as we mentioned previously, this conclusion relies on the assumption of homogeneity in preferences.

---

<sup>16</sup>The proportion of subjects who choose the S lottery when presented with a S-C pair, or its paired S-AE lottery pair, has to be equal within each paired comparison, but can differ across comparisons. More formally, the CMH test evaluates the null hypothesis that the odds ratio of each of the 15 contingency tables constructed from the 15 paired comparisons are jointly equal to 1.

<sup>17</sup>We follow Li et al. (1979) and use the continuity correction to avoid possible misleading conclusions from the test in small samples.

Table 3.5: Fisher Exact Test on Matched Simple-Compound and Simple-Actuarially-Equivalent Pairs

Treatment: 1-in-1

Comparison	Total # of subjects in		Proportion of subjects that chose the S lottery in the S-AE pair ( $\pi_1$ )	Proportion of subjects that chose the S lottery in the S-C pair ( $\pi_2$ )	$p$ -value for $H_0: \pi_1 = \pi_2$
	S-AE Pair	S-C Pair			
Pair 1 vs. Pair 16	3	2	1	0.5	0.4
Pair 3 vs. Pair 18	6	2	0.5	0	0.464
Pair 5 vs. Pair 20	1	2	0	1	0.333
Pair 6 vs. Pair 21	3	4	0.67	0.5	1
Pair 7 vs. Pair 22	4	9	1	0.56	0.228
Pair 8 vs. Pair 23	3	4	0.33	0.5	1
Pair 9 vs. Pair 24	3	6	0	0.83	0.048
Pair 11 vs. Pair 26	5	9	0.6	0.56	1
Pair 12 vs. Pair 27	5	2	0.8	1	1
Pair 13 vs. Pair 28	4	1	0.5	1	1
Pair 15 vs. Pair 30	3	1	1	0	0.250

Note: due to the randomization assignment of lottery pairs to subjects, the table only shows the Fisher Exact test for 11 S-AE/S-C comparisons for which there are sufficient data to conduct the test.

### Analysis of Data from the 1-in-40 Treatment

We use the Cochran Q test coupled with the Bonferroni-Dunn (B-D) correction

procedure<sup>18</sup> to test the hypothesis that subjects choose the S lottery in the same

<sup>18</sup>The B-D method is a post-hoc procedure that is conducted after calculating the Cochran Q test. The first step is to conduct the Cochran Q test to evaluate the null hypothesis that the proportions of individuals who choose the S lottery is the same in all 15 S-C and 15 S-AE linked lottery pairs. If this null is rejected the B-D method involves calculating a critical value  $d$  that takes into account all the information of the 30 lottery pairs. The B-D method allows us to test the statistical significance of the observed difference between proportions of subjects who choose the S lottery in any given paired comparison. Define  $p_1$  as the proportion of subjects who choose the S lottery when presented with a given S-AE lottery pair. Similarly, define  $p_2$  as the proportion of subjects who chose the S lottery in the paired S-C lottery pair. The B-D method rejects the null hypothesis that  $p_1 = p_2$  if  $|p_1 - p_2| > d$ . In this case we would conclude that the observed difference is statistically significant. This is a more powerful test than conducting individual tests for each paired comparison because the critical value  $d$  takes into account the information of all 15 comparisons. See Sheskin (2004, p. 871) for further details of the B-D method.

proportion when presented with linked S-C and S-AE lottery pairs. The B-D procedure takes into account repeated comparisons and allows us to maintain a familywise error rate across the 15 paired comparisons of S-C and S-AE lottery pairs.

We find evidence to reject the ROCL consistency prediction. Table 3.6 shows the results of the BD method<sup>19</sup> for each of the 15 paired comparisons. Table 3.6 provides evidence that with a 5% familywise error rate, subjects choose the S lottery in different proportions across linked S-C lottery pairs and S-AE lottery pairs in two comparisons: Pair 1 vs. Pair 16 and Pair 3 vs. Pair 18. This implies that the ROCL prediction of consistency is rejected in 2 of our 15 consistency comparisons.

We are also interested in studying the patterns of violations of ROCL. A pattern inconsistent with ROCL would be subjects choosing the S lottery when presented with a given S-C lottery pair, but switching to prefer the AE lottery when presented with the matched S-AE pair. We construct  $2 \times 2$  contingency tables that show the number of subjects in any given matched pair who exhibit each of the four possible choice patterns: (i) always choosing the S lottery; (ii) choosing the S lottery when presented with a S-C pair and switching to prefer the AE lottery when presented with the matched S-AE pair; (iii) choosing the C lottery when presented with a S-C pair and switching to prefer the S lottery when presented with the matched S-AE; and (iv) choosing the C lottery when presented with the S-C lottery and preferring the AE lottery when presented with the matched S-AE. Since we have paired observations, we use the McNemar test to evaluate the null hypothesis of equiprobable occurrences of discordant choice patterns (ii) and (iii) within

---

<sup>19</sup>The Cochran Q test rejected its statistical null hypothesis  $\chi^2$  statistic 448.55, 29 degrees of freedom and  $p$ -value $<0.0001$ .

Table 3.6: Bonferroni-Dunn Method on Matched Simple-Compound and Simple-Actuarially-Equivalent Pairs

Treatment: 1-in-40

Matching	Proportion of subjects that chose the S lottery in the S-AE pair ( $p_1$ )	Proportion of subjects that chose the S lottery in the S-C pair ( $p_2$ )	$ p_1 - p_2 $
Pair 1 vs. Pair 16	0.871	0.387	0.484
Pair 2 vs. Pair 17	0.984	0.952	0.032
Pair 3 vs. Pair 18	0.887	0.629	0.258
Pair 4 vs. Pair 19	0.226	0.210	0.016
Pair 5 vs. Pair 20	0.403	0.290	0.113
Pair 6 vs. Pair 21	0.742	0.661	0.081
Pair 7 vs. Pair 22	0.677	0.548	0.129
Pair 8 vs. Pair 23	0.548	0.548	0.000
Pair 9 vs. Pair 24	0.258	0.306	0.048
Pair 10 vs. Pair 25	0.919	0.774	0.145
Pair 11 vs. Pair 26	0.581	0.613	0.032
Pair 12 vs. Pair 27	0.565	0.645	0.081
Pair 13 vs. Pair 28	0.871	0.726	0.145
Pair 14 vs. Pair 29	0.742	0.677	0.065
Pair 15 vs. Pair 30	0.387	0.419	0.032

Note: the test rejects the null hypothesis of  $p_1 = p_2$  if  $|p_1 - p_2| > d$ . The calculation of the critical value  $d$  requires that one first define ex ante a familywise Type I error rate ( $\alpha_{FW}$ ). For  $\alpha_{FW} = 10\%$  the corresponding critical value is 0.133, and for  $\alpha_{FW} = 5\%$  the critical value is 0.159.

each set of matched pairs. We find a statistically significant difference in the number of (ii) and (iii) choice patterns within 4 of the 15 matched pairs. Table 3.7 reports the exact  $p$ -values for the McNemar test. The McNemar test results in  $p$ -values less than 0.05 in four comparisons: Pair 1 vs. Pair 16, Pair 3 vs. Pair 18, Pair 10 vs. Pair 25 and Pair 13 vs. Pair 28.<sup>20</sup> Moreover, the odds ratios of the McNemar tests suggest that the predominant switching pattern is choice pattern (iii): subjects tend to switch from the S lottery in the

<sup>20</sup>These violations of ROCL are also supported by the B-D procedure if the familywise error rate is set to 10%.

S-AE pair to the C lottery in the S-C pair. The detailed contingency tables for these 4 matched pairs show that the number of choices consistent with pattern (iii) is considerably greater than the number of choices consistent with (ii).

Table 3.7: McNemar Test on Matched Simple-Compound and Simple-Actuarially-Equivalent Pairs

Treatment: 1-in-40

Matching	Exact $p$ -value	Odds Ratio
Pair 1 vs. Pair 16	<0.0001	0.0625
Pair 2 vs. Pair 17	0.625	0.3333
Pair 3 vs. Pair 18	0.0001	0.0588
Pair 4 vs. Pair 19	1.000	0.8571
Pair 5 vs. Pair 20	0.1671	0.4615
Pair 6 vs. Pair 21	0.3323	0.5454
Pair 7 vs. Pair 22	0.1516	0.5000
Pair 8 vs. Pair 23	1.000	1
Pair 9 vs. Pair 24	0.6072	0.6667
Pair 10 vs. Pair 25	0.0352	0.2500
Pair 11 vs. Pair 26	0.8238	1.222
Pair 12 vs. Pair 27	0.4049	1.555
Pair 13 vs. Pair 28	0.0117	0.100
Pair 14 vs. Pair 29	0.5034	0.6667
Pair 15 vs. Pair 30	0.8388	1.1818

### 3.5 Estimated Preferences from Observed Choices

We now estimate preferences from observed choices, and evaluate whether behavior is consistent with ROCL. Additionally, we test for a treatment effect to determine the impact of RLIM on preferences.

### 3.5.1 Econometric Specification

Assume that utility of income is defined by

$$U(x) = x^{(1-r)}/(1-r) \quad (3.1)$$

where  $x$  is the lottery prize and  $r \neq 1$  is a parameter to be estimated. For  $r = 1$  assume  $U(x) = \ln(x)$  if needed. Thus  $r$  is the coefficient of CRRA:  $r = 0$  corresponds to risk neutrality,  $r < 0$  to risk loving, and  $r > 0$  to risk aversion. Let there be  $J$  possible outcomes in a lottery, and denote outcome  $j \in J$  as  $x_j$ . Under EUT the probabilities for each outcome  $x_j$ ,  $p(x_j)$ , are those that are induced by the experimenter, so expected utility is simply the probability weighted utility of each outcome in each lottery  $i$ :

$$EU_i = \sum_{i=1,J} [p(x_i) \times U(x_i)] \quad (3.2)$$

The EU for each lottery pair is calculated for a candidate estimate of  $r$ , and the index

$$\nabla EU = EU_R - EU_L \quad (3.3)$$

is calculated, where  $EU_L$  is the “left” lottery and  $EU_R$  is the “right” lottery as presented to subjects. This latent index, based on latent preferences, is then linked to observed choices using a standard cumulative normal distribution function  $\Phi(\nabla EU)$ . This “probit” function takes any argument between  $\pm\infty$  and transforms it into a number between 0 and

1. Thus we have the probit link function,

$$\text{prob}(\text{choose lottery R}) = \Phi(\nabla EU) \quad (3.4)$$

Even though this “link function” is common in econometrics texts, it forms the critical statistical link between observed binary choices, the latent structure generating the index  $\nabla EU$ , and the probability of that index being observed. The index defined by equation 3.3 is linked to the observed choices by specifying that the R lottery is chosen when  $\Phi(\nabla EU) > 0.5$ , which is implied by equation 3.4.

The likelihood of the observed responses, conditional on the EUT and CRRA specifications being true, depends on the estimates of  $r$  given the above statistical specification and the observed choices. The “statistical specification” here includes assuming some functional form for the cumulative density function (CDF). The conditional log-likelihood is then

$$\ln L(r; y, X) = \sum [(\ln \Phi(\nabla EU) \times \mathbf{I}(y_i = 1)) + (\ln(1 - \Phi(\nabla EU)) \times \mathbf{I}(y_i = -1))] \quad (3.5)$$

where  $\mathbf{I}(\cdot)$  is the indicator function,  $y_i = 1(-1)$  denotes the choice of the right (left) lottery in risk aversion task  $i$ , and  $X$  is a vector of individual characteristics reflecting age, sex, race, and so on.

Harrison and Rutström (2008, Appendix F) review procedures that can be used to estimate structural models of this kind, as well as more complex non-EUT models, with the goal of illustrating how to write explicit maximum likelihood (ML) routines that are specific to different structural choice models. It is a simple matter to correct for multiple responses

from the same subject (“clustering”), if needed.

It is also a simple matter to generalize this ML analysis to allow the core parameter  $r$  to be a linear function of observable characteristics of the individual or task. We extend the model to be  $r = r_0 + R \times X$ , where  $r_0$  is a fixed parameter and  $R$  is a vector of effects associated with each characteristic in the variable vector  $X$ . In effect, the unconditional model assumes  $r = r_0$  and estimates  $r_0$ . This extension significantly enhances the attraction of structural ML estimation, particularly for responses pooled over different subjects and treatments, since one can condition estimates on observable characteristics of the task or subject.

In our case we also extend the structural parameter to take on different values for the lotteries presented as compound lotteries. That is, equation 3.1 applies to the evaluation of utility for all simple lotteries and a different CRRA risk aversion coefficient  $r + rc$  applies to compound lotteries, where  $rc$  captures the additive effect of evaluating a compound lottery. Hence, for compound lotteries, the decision maker employs the utility function

$$U(x|\text{compound lottery}) = x^{(1-r-rc)}/(1-r-rc) \quad (3.6)$$

instead of equation 3.1, and we would restate it as

$$U(x|\text{simple lottery}) = x^{(1-r)}/(1-r) \quad (3.7)$$

for completeness. Specifying preferences in this manner provide us with a structural test for ROCL. If  $rc = 0$  then this implies that compound lotteries are evaluated identically to

simple lotteries, which is consistent with ROCL. However, if  $rc \neq 0$ , as conjectured by Smith (1969) for objective and subjective compound lotteries, then, decision-makers violate ROCL in a certain source-dependent manner, where the “source” here is whether the lottery is simple or compound.<sup>21</sup> As stressed by Smith (1969),  $rc \neq 0$  for *subjective* lotteries provides a direct explanation for the Ellsberg Paradox, but is much more readily tested on the domain of objective lotteries. Of course, the linear specification  $r + rc$  is a parametric convenience, but the obvious one to examine initially.

An important extension of the core model is to allow for subjects to make some *behavioral* errors. The notion of error is one that has already been encountered in the form of the statistical assumption that the probability of choosing a lottery is not 1 when the EU of that lottery exceeds the EU of the other lottery. This assumption is clear in the use of a non-degenerate link function between the latent index  $\nabla EU$  and the probability of picking a specific lottery as given in equation 3.4. If there were no errors from the perspective of EUT, this function would be a step function: zero for all values of  $\nabla EU < 0$ , anywhere between 0 and 1 for  $\nabla EU = 0$ , and 1 for all values of  $\nabla EU > 0$ . We employ the error specification originally due to Fechner and popularized by Hey and Orme (1994). This

---

<sup>21</sup>Abdellaoui et al. (2011) conclude that different probability weighting functions are used when subjects face risky processes with known probabilities and uncertain processes with subjective processes. They call this “source dependence,” where the notion of a source is relatively easy to identify in the context of an artefactual laboratory experiment, and hence provides the tightest test of this proposition. Harrison (2011c) shows that their conclusions are an artefact of estimation procedures that do not take account of sampling errors. A correct statistical analysis that does account for sampling errors provides no evidence for source dependence using their data. Of course, failure to reject a null hypothesis could just be due to samples that are too small.

error specification posits the latent index

$$\nabla EU = (EU_R - EU_L)/\mu \quad (3.8)$$

instead of equation 3.3, where  $\mu$  is a structural “noise parameter” used to allow some errors from the perspective of the deterministic EUT model. This is just one of several different types of error story that could be used, and Wilcox (2008) provides a masterful review of the implications of the alternatives.<sup>22</sup> As  $\mu \rightarrow 0$  this specification collapses to the deterministic choice EUT model, where the choice is strictly determined by the EU of the two lotteries; but as  $\mu$  gets larger and larger the choice essentially becomes random. When  $\mu = 1$  this specification collapses to equation 3.3, where the probability of picking one lottery is given by the ratio of the EU of one lottery to the sum of the EU of both lotteries. Thus  $\mu$  can be viewed as a parameter that flattens out the link functions as it gets larger. An important contribution to the characterization of behavioral errors is the “contextual error” specification proposed by Wilcox (2011). It is designed to allow robust inferences about the primitive “more stochastically risk averse than,” and posits the latent index

$$\nabla EU = (EU_R - EU_L)/\mu/\nu \quad (3.9)$$

instead of equation 3.8, where  $\nu$  is a new, normalizing term for each lottery pair L and R.

The normalizing term  $\nu$  is defined as the maximum utility over all prizes in this lottery

---

<sup>22</sup>Some specifications place the error at the final choice between one lottery or after the subject has decided which one has the higher expected utility; some place the error earlier, on the comparison of preferences leading to the choice; and some place the error even earlier, on the determination of the expected utility of each lottery.

pair minus the minimum utility over all prizes in this lottery pair. The value of  $\nu$  varies, in principle, from lottery choice pair to lottery choice pair: hence it is said to be “contextual.” For the Fechner specification, dividing by  $\nu$  ensures that the *normalized* EU difference  $[(EU_R - EU_L)/\nu]$  remains in the unit interval for each lottery pair. The term  $\nu$  does not need to be estimated in addition to the utility function parameters and the parameter for the behavioral error term, since it is given by the data and the assumed values of those estimated parameters.

The specification employed here is the source-dependent CRRA utility function from equations 3.6 and 3.7, the Fechner error specification using contextual utility from equation 3.9, and the link function using the normal CDF from 3.4. The log-likelihood is then

$$\ln L(r, rc, \mu; y, X) = \sum [( \ln \Phi(\nabla EU) \times \mathbf{I}(y_i = 1) ) + ( \ln(1 - \Phi(\nabla EU)) \times \mathbf{I}(y_i = -1) )] \quad (3.10)$$

and the parameters to be estimated are  $r$ ,  $rc$  and  $\mu$  given observed data on the binary choices  $y$  and the lottery parameters in  $X$ . It is possible to consider more flexible utility functions than the CRRA specification in equation 3.1, but that is not essential for present purposes. We do, however, consider extensions of the EUT model to allow for rank-dependent decision-making under Rank-Dependent Utility (RDU) models.

The RDU model extends the EUT model by allowing for decision weights on lottery outcomes. The specification of the utility function is the same parametric specification in 3.6 and 3.6 considered for source-dependent EUT. To calculate decision weights under

RDU one replaces expected utility defined by equation 3.2 with RDU

$$RDU_i = \sum_{j=1, J} [w(p(x_j)) \times U(x_j)] = \sum_{j=1, J} [w_j \times U(x_j)] \quad (3.11)$$

where

$$w_i = \omega(p_j + \dots + p_J) - \omega(p_{j+1} + \dots + p_J) \quad (3.12)$$

for  $j = 1, \dots, J - 1$ , and

$$w_j = \omega(p_j) \quad (3.13)$$

for  $j = J$ , with the subscript  $j$  ranking outcomes from worst to best, and  $\omega(\cdot)$  is some probability weighting function.

We adopt the simple “power” probability weighting function proposed by Quiggin (1982), with curvature parameter  $\gamma$ :

$$\omega(p) = p^\gamma \quad (3.14)$$

So  $\gamma \neq 1$  is consistent with a deviation from the conventional EUT representation.

Convexity of the probability weighting function is said to reflect “pessimism” and generates, if one assumes for simplicity a linear utility function, a risk premium since  $\omega(p) < p \forall p$  and hence the “RDU EV” weighted by  $\omega(p)$  instead of  $p$  has to be less than the EV weighted by  $p$ . The rest of the ML specification for the RDU model is identical to the specification for the EUT model, but with different parameters to estimate.

It is obvious that one can extend the probability weighting specification to be

source-dependent, just as we did for the utility function. Hence we extend the probability weighting function to be

$$\omega(p|\text{compound lottery}) = p^{\gamma+\gamma c} \quad (3.15)$$

instead of equation 3.1, and we would restate it as

$$\omega(p|\text{simple lottery}) = p^\gamma \quad (3.16)$$

for simple lotteries. The hypothesis of source-independence, which is consistent with ROCL, in this case is that  $\gamma c = 0$  and  $rc = 0$ .

### 3.5.2 Estimates

#### Analysis of Data from the 1-in-1 Treatment

We focus first on the estimates obtained in the 1-in-1 treatment, since this controls for the potentially contaminating effects of the RLIM on our inferences about ROCL. Of course, this requires us to account for subject heterogeneity, and so we control for heterogeneity in risk preferences. We include the effects of allowing for a series of binary demographic variables: **female** is 1 for women, and 0 otherwise; **senior** is 1 for whether that was the current stage of undergraduate education, and 0 otherwise; **white** is 1 based on self-reported ethnic status; and **gpaHI** is 1 for those reporting a cumulative GPA between 3.25 and 4.0 (at least half A's and B's), and 0 otherwise. The econometric strategy is to estimate our source-dependent version of EUT and RDU separately and compare the

model estimates using the tests developed by Vuong (1989) and Clarke (2003, 2007) for non-nested, nested and overlapping models.<sup>23</sup> This strategy allows us first to choose the model that best describes the data between the two competing models, and then test the chosen model for violations of ROCL.

Controlling for heterogeneity we find that the data are best described by the source-dependent RDU, and conditional on this model there is no evidence of violations of ROCL. Both the Vuong test and the Clarke test provide statistical evidence that our source-dependent version of RDU is the best model to explain the data in the 1-in-1 treatment.<sup>24</sup> Panel A of Table 3.8 shows the estimates for the source-dependent RDU. A joint test of the coefficient estimates for the covariates and the constant in the equation for  $rc$  results in a  $p$ -value of 0.59 and a similar test for parameter  $\gamma c$  results in a  $p$ -value of 0.80. Moreover, a joint test of all the covariates and constants in the equations of  $rc$  and  $\gamma c$  results in a  $p$ -value equal to 0.72. If we had assumed that subjects behave according to the source-dependent EUT we would have incorrectly concluded that there is evidence of violations of ROCL, from the joint tests of the effect of all covariates in the  $rc$  equation which has a  $p$ -value less than 0.001. This highlights the importance of choosing the preference representation that best characterizes observed choice behavior.

A joint test of all covariates and constant terms, both in the equations for  $r$  and  $\gamma$ , results in a  $p$ -value less than 0.01. Figure 3.6 shows the distributions for estimates of the utility

---

<sup>23</sup>The Vuong test is parametric in the sense that it assumes normality to derive the hypothesis test statistic. We also apply the Clarke test which is a distribution-free test.

<sup>24</sup>When we control for heterogeneity, the Vuong test statistic is -1.38 in favor of the sourcedependent RDU, with a  $p$ -value of .083. Further, the Clarke test also gives evidence in favor of the sourcedependent RDU with a test statistic equal to 56.

parameter  $r$  and the probability weighting parameter  $\gamma$ ,<sup>25</sup> which have average values 0.79 and 0.33, respectively. This would imply that the typical subject exhibits diminishing marginal returns in the utility function and probability optimism.<sup>26</sup> Figure 3.6 also shows the distributions for the point estimates for  $r + rc$  and  $\gamma + \gamma c$ .<sup>27</sup>

To summarize, behavior in the 1-in-1 treatment is better characterized by RDU instead of EUT, and we do not find evidence of violations of ROCL with the RDU preference representation. We reach a similar conclusion if preference homogeneity is assumed.

### Analysis of Data from the 1-in-40 Treatment

Controlling for heterogeneity, we again find that the data are best described by our source-dependent version of RDU, and conditional on this model we find evidence of violations of ROCL. Both the Vuong and Clarke tests provide support for the source-dependent RDU as the best model to explain the data in the 1-in-40 treatment.<sup>28</sup>

Panel A of Table 3.9 shows the estimates for this model. A statistical test for the joint null hypothesis that all covariates in the equations for  $rc$  and  $\gamma c$  are jointly equal to zero results in a  $p$ -value less than 0.001, which provides evidence of violations of ROCL. Similarly, the

---

<sup>25</sup>The unobserved parameters  $r$  and  $\gamma$  are predicted for each subject by using the vector of individual characteristics and the vector estimated parameters that capture the effect of each covariate.

<sup>26</sup>These are only descriptive statistics that may not describe in general our subjects' behavior since there is uncertainty around the predicted values of parameters  $r$  and  $\gamma$ . However, a series of tests which test, for each subject, the null hypotheses of linear utility ( $r = 0$ ) and linearity in probabilities ( $\gamma = 1$ ) result, for all subjects, in  $p$ -values less than 0.01 and less than 0.05, respectively. These tests are constructed using the standard errors around the covariates' coefficients in the equations for parameters  $r$  and  $\gamma$ .

<sup>27</sup>Any comparison between the distributions of  $r + rc$  and  $r$ , but also between  $\gamma + \gamma c$  and  $\gamma$ , has to take into account the uncertainty around the distribution fitting process and the significance of the parameter point estimates.

<sup>28</sup>The Vuong test statistic is -5.45 in favor of the source-dependent RDU, with a  $p$ -value less than .001. Further, the Clarke test also gives evidence in favor of the source-dependent RDU with a test statistic equal to 993

hypothesis that all the covariates in the equations for parameters  $r$  and  $\gamma$  are jointly equal to zero also results in a  $p$ -value less than 0.001. Figure 3.7 shows the fitted distributions for the point estimates of the utility and probability weighting parameters across subjects in the 1-in-40 treatment. The average predicted values for  $r$ ,  $r + rc$ ,  $\gamma$  and  $\gamma + \gamma c$  are 0.63, 0.71, 0.95 and 0.62, respectively. This would imply that a typical subject displays diminishing marginal returns when evaluating simple and compound lotteries and exhibits more probability optimism when evaluating compound lotteries.<sup>29</sup>

If we would have assumed that subjects behave according to the source-dependent version of EUT, we would have incorrectly concluded no violation of ROCL. This conclusion derives from a joint test of the effect of all covariates in the  $rc$  equation which result in a  $p$ -value of 0.67. Panel B of Table 3.9 shows the estimates for the source-dependent EUT model. This highlights, yet again, the importance of choosing an appropriate preference representation that best describes observed choice behavior.

To summarize, behavior in the 1-in-40 treatment is best characterized by the source-dependent RDU model, and we find evidence of violations of ROCL. We reach the same conclusion if preference homogeneity is assumed.

---

<sup>29</sup>Again, these are only descriptive statistics that are meant to characterize typical behavior. A series of tests for the null hypotheses of  $r = 0$  and  $rc = 0$  result in  $p$ -values less than 0.001 for all subjects. Similar tests for the null hypothesis of  $\gamma = 1$  result in  $p$ -values greater than 0.05 for 51 out of 62 subjects. Further, tests for the null hypothesis of  $\gamma + \gamma c = 1$  result in  $p$ -values less than 0.05 for 37 out of 62 subjects.

## 3.6 Conclusions

Our primary goal is to test the Reduction of Compound Lotteries axiom under objective probabilities. Our conclusions are influenced by the experiment payment protocols used and the assumptions about how to characterize risk attitudes.

We do not find violations of ROCL when subjects are presented with one and only one choice that is played for money. However, when individuals are presented with many choices, and the Random Lottery Incentive Mechanism is used to select one choice for payoff, we do find violations of ROCL. These results are obtained whether one uses non-parametric statistics to analyze choice patterns or structural econometrics to estimate preferences.

The econometric analysis provides more information about the structure of individual decision making process. In the context where individuals face only one choice for payoff and no violations of ROCL are found, the preference representation that best characterizes behavior is the Rank-Dependent Utility model. Similarly, when subjects face many choices, behavior is better characterized by our sourcedependent version of the RDU model that also accounts for violations of ROCL.

An important methodological conclusion is that the payment protocol used to pay subjects might create distortions of behavior in experimental settings. This is especially important for our purposes since one of the most popular payment protocols assumes ROCL itself.

This issue has been studied and documented by Harrison and Swarthout (2012) and Cox et al. (2011). Our results provide further evidence that payment protocols can create

confounds and therefore affect hypothesis testing about decision making under risk.

Table 3.8: Estimates of Source-Dependent RDU and EUT Models Allowing for Heterogeneity

Data from the 1-in-1 treatment (N=133). Estimates of the Fechner error parameter omitted.

Parameter	Covariate	Point Estimate	Standard Error	p-value	95% Confidence Interval	
<i>A. Source-Dependent RDU (LL= -73.21)</i>						
$\alpha$	female	0.301	0.077	<0.001	0.150	0.451
	senior	-0.044	0.060	0.459	-0.161	0.073
	gpaHI	0.034	0.056	0.552	-0.077	0.144
	white	-0.141	0.095	0.136	-0.326	0.044
	constant	0.636	0.066	<0.001	0.507	0.764
$\alpha c$	female	-0.142	0.089	0.109	-0.316	0.032
	senior	-0.007	0.024	0.784	-0.054	0.041
	gpaHI	-0.002	0.013	0.856	-0.027	0.023
	white	-0.073	0.085	0.391	-0.240	0.094
	constant	0.142	0.091	0.118	-0.036	0.319
$\gamma$	female	-0.231	0.199	0.244	-0.621	0.158
	senior	-0.011	0.129	0.932	-0.264	0.242
	gpaHI	0.019	0.132	0.888	-0.239	0.276
	white	0.059	0.191	0.757	-0.315	0.433
	constant	0.472	0.130	<0.001	0.218	0.726
$\gamma c$	female	-0.066	0.158	0.675	-0.375	0.243
	senior	0.052	0.157	0.741	-0.256	0.360
	white	0.061	0.182	0.736	-0.296	0.419
	constant	-0.082	0.119	0.492	-0.314	0.151
<i>B. Source-Dependent EUT (LL=-78.07)</i>						
$\alpha$	female	0.468	0.185	0.011	0.106	0.830
	senior	-0.151	0.125	0.228	-0.396	0.094
	gpaHI	0.185	0.090	0.039	0.009	0.361
	white	0.068	0.088	0.442	-0.105	0.241
	constant	0.224	0.188	0.233	-0.144	0.593
$\alpha c$	female	-0.555	0.208	0.008	-0.963	-0.147
	senior	0.148	0.142	0.298	-0.131	0.427
	gpaHI	-0.217	0.104	0.036	-0.420	-0.014
	white	-0.097	0.115	0.400	-0.323	0.129
	constant	0.733	0.212	0.001	0.317	1.149

Figure 3.6: Distribution of Parameter Estimates from the RDU Specification in the 1-in-1 Treatment Assuming Heterogeneity in Preferences

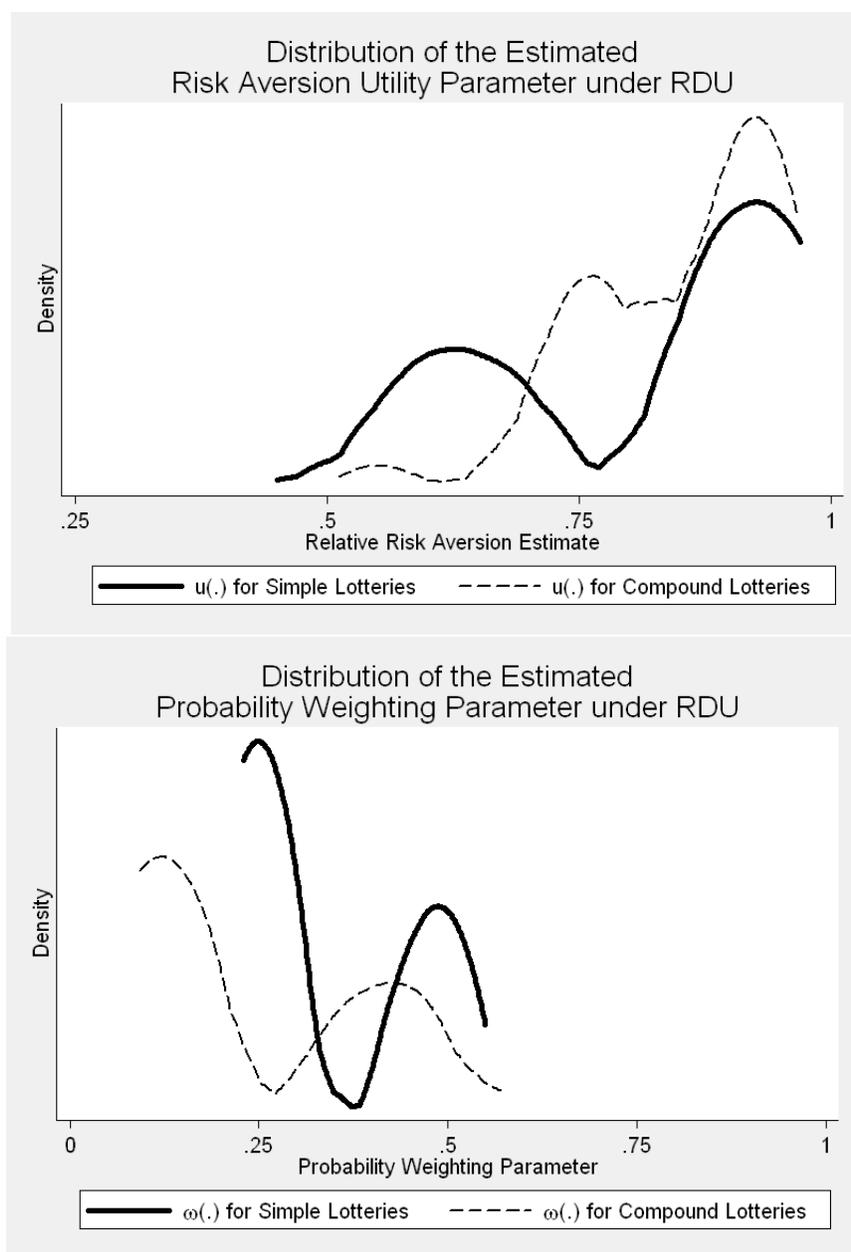
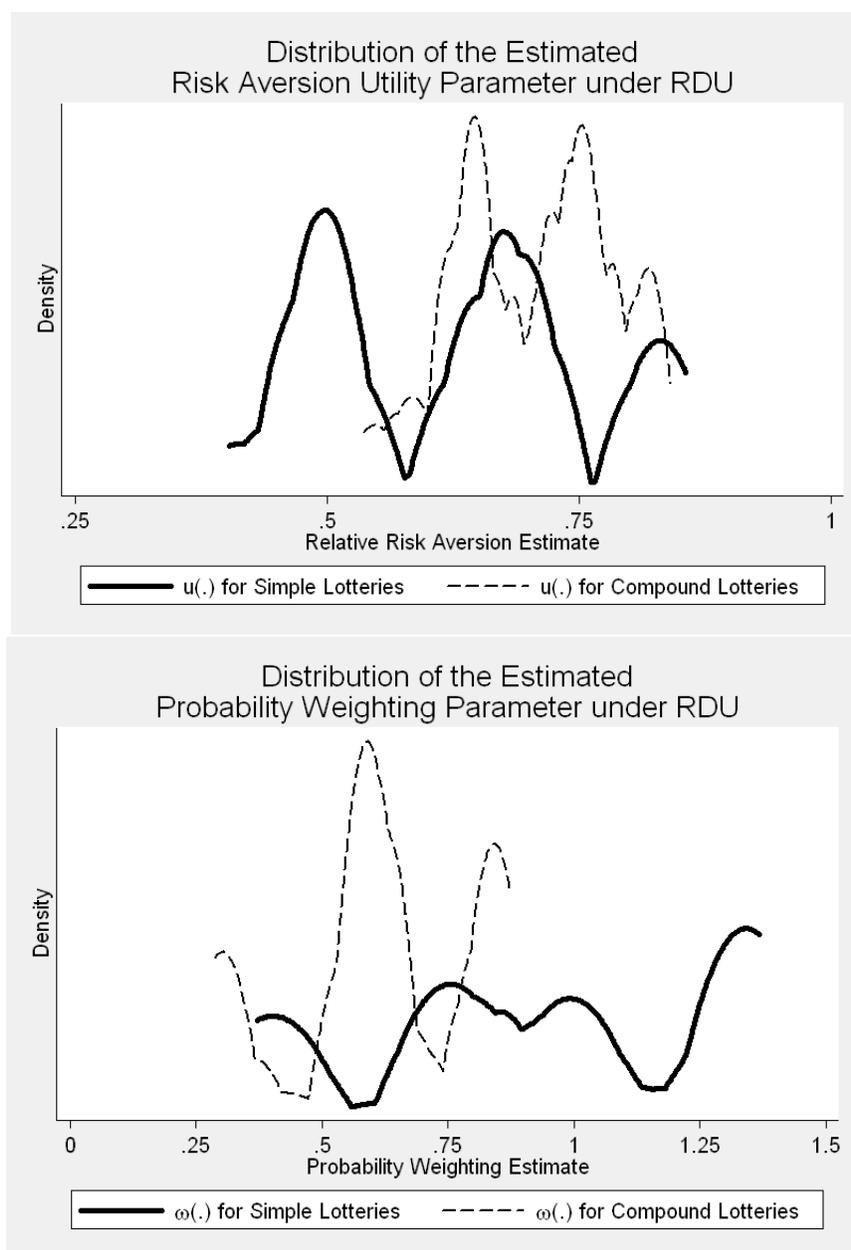


Table 3.9: Estimates of Source-Dependent RDU and EUT Models Allowing for Heterogeneity

Data from the 1-in-40 treatment (N=2480). Estimates of the Fechner error parameter omitted.

Parameter	Covariate	Point Estimate	Standard Error	p-value	95% Confidence Interval	
<i>A. Source-Dependent RDU (LL= -1441.785)</i>						
$\alpha$	female	-0.185	0.066	0.005	-0.314	-0.055
	senior	0.035	0.078	0.658	-0.119	0.188
	gpaHI	0.149	0.096	0.119	-0.039	0.337
	white	-0.233	0.084	0.005	-0.398	-0.069
	constant	0.672	0.072	<0.001	0.530	0.814
$\alpha c$	female	0.084	0.043	0.052	-0.001	0.170
	senior	-0.004	0.049	0.928	-0.101	0.092
	gpaHI	-0.071	0.041	0.085	-0.152	0.010
	white	0.060	0.038	0.115	-0.015	0.135
	constant	0.061	0.051	0.232	-0.039	0.161
$\gamma$	female	0.575	0.198	0.004	0.187	0.962
	senior	-0.042	0.221	0.848	-0.476	0.391
	gpaHI	-0.380	0.225	0.091	-0.821	0.061
	white	0.312	0.255	0.221	-0.187	0.811
	constant	0.794	0.214	<0.001	0.374	1.214
$\gamma c$	female	-0.311	0.122	0.011	-0.551	-0.071
	senior	0.097	0.126	0.443	-0.150	0.343
	gpaHI	0.113	0.134	0.401	-0.150	0.375
	white	-0.230	0.166	0.167	-0.557	0.096
	constant	-0.238	0.130	0.066	-0.493	0.016
<i>B. Source-Dependent EUT (LL= -1504.136)</i>						
$\alpha$	female	0.053	0.088	0.547	-0.119	0.225
	senior	-0.014	0.079	0.857	-0.168	0.140
	gpaHI	-0.041	0.083	0.622	-0.203	0.122
	white	-0.098	0.159	0.538	-0.409	0.213
	constant	0.594	0.104	<0.001	0.390	0.797
$\alpha c$	female	0.007	0.041	0.865	-0.074	0.088
	senior	0.019	0.046	0.676	-0.071	0.110
	gpaHI	-0.025	0.047	0.603	-0.117	0.068
	white	0.035	0.037	0.344	-0.038	0.109
	constant	-0.014	0.041	0.727	-0.095	0.066

Figure 3.7: Distribution of Parameter Estimates from the RDU Specification in the 1-in-40 Treatment Assuming Heterogeneity in Preferences



### 3.7 Appendix A. Reduction of Compound Lotteries with Subjective Probabilities: Theoretical, Experimental and Methodological Issues

The reduction of compound lotteries has assumed a central role in the evaluation of behavior towards risk, uncertainty and ambiguity. We discuss theoretical, experimental and methodological issues relating to the possibility of testing its validity in domains over subjective probabilities.

One fundamental methodological problem with tests of the reduction of compound lotteries (ROCL) assumption, whether or not the context is objective or subjective probabilities, is that one cannot use incentives that rely on it being valid. This means, in effect, that one has to conduct experiments in which the subject has one, and only one, task.<sup>30</sup> Apart from the expense of collecting data at such a pace, this also means that all evaluations have to be on a between-subjects.

If one finds evidence in support of ROCL with subjective probabilities, then one of the main assumptions of the Subjective Expected Utility (SEU) approach to modeling decision making towards subjective risk proposed by Savage (1971) is supported. That approach allows the decision maker to have a non-degenerate subjective probability distribution over some event, but assumes that the decision maker behaves as if that distribution is “boiled down” to one subjective probability, the weighted average of the distribution.

A test of ROCL is of immediate relevance to preference representations that are sensitive to ambiguity. Segal (1987, 1990) proposes that ambiguous situations can be characterize as a compound lottery where the first stage is over the possible probabilities of the states of nature. In his framework ambiguity sensitive preferences do not satisfy ROCL under subjective probabilities. In fact, many models of decisions under ambiguity (Gilboa and Schmeidler, 1989; Klibanoff et al., 2005; Ghirardato et al., 2004) share the same feature: under uncertainty people may have multiple priors, and even if subjects are able to define a nondegenerate subjective probability distribution over those priors, they will not behave as if they “boil down” the distribution to the weighted average probability.

---

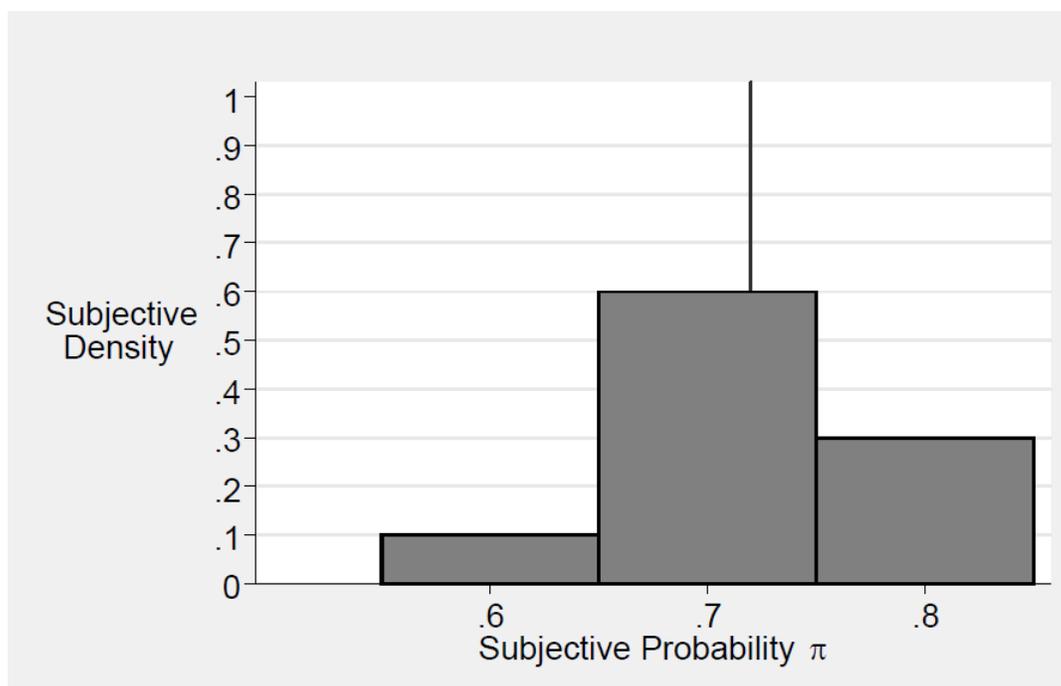
<sup>30</sup>One alternative is to present the decision maker with several tasks at once and evaluate the portfolio chosen, or to present the decision maker with several tasks in sequence and account for wealth effects. Neither is immediately attractive, since they each raise possible confounds in behavior and theoretical inference. One uninteresting alternative is not to pay the decision maker for the outcomes of the task.

### 3.7.1 Theoretical Issues

#### Subjective Belief Distributions and Subjective Probabilities: Their Equivalence under ROCL

Consider a decision-maker with a non-degenerate subjective probability distribution over some event. For example, let the subjective density on the probability  $\pi = 0.6$  be 0.1, the subjective density on the probability  $\pi = 0.7$  be 0.6, and the subjective density on the probability  $\pi = 0.8$  be 0.3. Figure 3.8 illustrates. The weighted average of this distribution is  $\pi = 0.72$ .

Figure 3.8: Illustrative Subjective Beliefs



Now consider a Savage-style bet in which you receive  $\$x > 0$  if event occurs, or  $\$0$  otherwise. The structure of the subjective beliefs assumed above induce a subjective compound lottery that pays lottery  $(0.6, \$x; 0.4, \$0)$  with probability 0.1, lottery  $(0.7, \$x; 0.3, \$0)$  with probability 0.6 and lottery  $(0.8, \$x; 0.2, \$0)$  with probability 0.3. We can write the subjective expected utility (SEU) of this bet as

$$SEU = 0.1 \times 0.6 \times U(x) + 0.1 \times 0.4 \times U(0) + 0.6 \times 0.7 \times U(x) + 0.6 \times 0.3 \times U(0) + 0.3 \times 0.8 \times U(x) + 0.3 \times 0.2 \times U(0) \quad (3.17)$$

where  $U(\cdot)$  is the utility function. Under ROCL, an individual should be indifferent between this subjective compound lottery and the actuarially-equivalent lottery  $(0.72, \$x; 0.28, \$0)$  which has subjective expected utility equal to

$$SEU = (0.1 \times 0.6 + 0.6 \times 0.7 + 0.3 \times 0.8) \times U(x) + (0.1 \times 0.4 + 0.6 \times 0.3 + 0.3 \times 0.2) \times U(0) \quad (3.18)$$

or  $SEU = 0.72 \times U(x) + 0.28 \times U(0)$ . Thus, under ROCL, the decision maker with the original non-degenerate subjective probability distribution would behave as if he had a degenerate subjective probability distribution at  $\pi = 0.72$ . In other words, these two subjective probability distributions, and an infinite number of non-degenerate distributions that have the same weighted average 0.72, are observationally equivalent under ROCL. A test of ROCL requires the elicitation of a subjective non-degenerate probability distribution and a degenerate probability distribution. If ROCL is satisfied the average probability implied by the non-degenerate distribution should be equal to the degenerate distribution. We explain below the theory and the operational procedures we use to elicit the distributions we need for testing ROCL.

### Scoring Rules to Elicit Degenerate Probability Distributions

A binary scoring rule is defined over some binary event, which is either true or false. A binary scoring rule asks the subject to make some report  $\theta$ , and then defines how an elicitor pays a subject depending on their report and the outcome of the event. This framework for eliciting subjective probabilities can be formally viewed from the perspective of a trading game between two agents: you give me a report, and I agree to pay you  $\$X$  if one outcome occurs and  $\$Y$  if the other outcome occurs. The scoring rule defines the terms of the exchange quantitatively, explaining how the elicitor converts the report from the subject into a lottery defined over the subjective probability of the subject. We use the terminology “report” because we want to view this formally as a mechanism, and want to emphasize the idea that the report may or may not be the subjective probability  $\pi$  of the subject. When the report is equal to subjective probability of the individual, we say that the scoring rule is a direct mechanism, following standard methodology.

The popular Quadratic Scoring Rule (QSR) for binary events is defined in terms of two positive parameters,  $\alpha$  and  $\beta$ , that determine a fixed reward the subject gets and a penalty for error. Assume that the possible outcomes are A or B, where B is the complement of A, that  $\theta$  is the reported probability for A, and that  $\Theta$  is the true binary-valued outcome for A. Hence  $\Theta = 1$  if A occurs, and  $\Theta = 0$  if it does not occur (and thus B occurs instead). The subject is paid  $S(\theta|A \text{ occurs}) = \alpha - \beta(\Theta - \theta)^2 = \alpha - \beta(1 - \theta)^2$  if event A occurs and  $S(\theta|B \text{ occurs}) = \alpha - \beta(\Theta - \theta)^2 = \alpha - \beta(0 - \theta)^2$  if B occurs. The score or payment penalizes the subject by the squared deviation of the report from the true binary-valued outcome,  $\Theta$ , which is 1 and 0 respectively for A and B occurring. An omniscient seer would obviously set  $\theta = \Theta$ . The fixed reward is a convenience<sup>31</sup> to ensure that subjects are willing to play this trading game, and the penalty function accentuates the penalty from not responding what the subject thinks an omniscient seer would respond. It can be shown that a risk neutral decision maker will report his subjective probability truthfully. For example, assume  $\alpha = 1$  and  $\beta = 1$  so that the subject could earn up to 1 or as little as 0. If they reported 1 they earned 1 if event A occurred or 0 if event B occurred; if they reported 0.75 they earned 0.9375 or 0.4375; and if they reported 0.5 they earned 0.75 no matter what event occurred.

<sup>31</sup>In the language of mechanism design, it can be chosen to satisfy the participation constraint. This requires that  $\theta > \beta$ .

## Scoring Rules to Elicit Non-Degenerate Probability Distributions

We can test ROCL by considering behavior by the decision maker with respect to a scoring rule designed to elicit his subjective belief distribution. Let this decision maker be asked to report his subjective beliefs in a discrete version of a QSR for continuous distributions, following Mathieson and Winkler (1976). Partition the domain into  $I$  intervals, and denote as  $r_i$  the report of the density of  $r$  in interval  $i = 1, 2, \dots, I$ . These reports could be experimental tokens that sum, for example, to 100; for simplicity below we normalize so that the sum of all tokens is 1. For expositional simplicity assume for the moment that the decision maker is risk neutral. If  $r_j$  is the interval that the actual value lies in, then the payoff score from Mathieson and Winkler (1976, p. 1088, equation (6)) is:

$$S = (2 \times r_j) - \sum_{i=1, J} r_i^2 \quad (3.19)$$

So the reward in the score is a doubling of the number of tokens allocated to the true interval, and the penalty depends on how these tokens are distributed. The subject is rewarded for accuracy, but if that accuracy misses the true interval the punishment is severe. The punishment includes all possible reports, including the correct report. Consider some examples, assuming  $I = 4$ . What if the subject has very tight subjective beliefs and puts all of the tokens in the correct interval? Then the score is

$$S = (2 \times 1) - (1^2 + 0^2 + 0^2 + 0^2) = 2 - 1 = 1 > 0.$$

But if the subject has a tight subjective belief that is wrong, the score is

$$S = (2 \times 0) - (1^2 + 0^2 + 0^2 + 0^2) = 0 - 1 = -1 < 0.$$

So we see that this score would have to include some additional “endowment” to ensure that net earnings are positive.<sup>32</sup> Assuming that the subject has a very diffuse subjective belief and allocates 25% of the tokens to each interval, the score is

$$S = (20.25) - (0.25^2 + 0.25^2 + 0.25^2 + 0.25^2) = 0.50 - 0.25 = 0.25 < 1.$$

## Risk Attitudes and Subjective Beliefs

It is intuitively obvious, and also well known in the literature (e.g., Winkler and Murphy, 1970; Kadane and Winkler, 1988), that risk attitudes will affect the incentive to report one’s subjective probability “truthfully” in each of these scoring rules.

Consider the binary scoring rule first. A sufficiently risk averse agent is clearly going to be drawn to a report of 0.5, and varying degrees of risk aversion will cause varying distortions in reports from subjective probabilities. If we knew the form of the (well-behaved) utility function of the subjects, and their degree of risk aversion, we could infer back from any report what subjective probability they must have had. The need to do this jointly is in

---

<sup>32</sup>This is a point of practical behavioral significance, but is not important for the immediate theoretical point.

fact central to the operational definition of subjective probability provided by Savage (1972): under certain postulates, he showed that there existed a *subjective probability and a utility function* that could rationalize observable choices. Andersen et al. (2010) illustrate how joint estimation of risk attitudes and subjective probability, using structural maximum likelihood, can be used to make the necessary calibration to recover the latent subjective probability.

An alternative approach is to use scoring rules combined with the Binary Lottery Procedure (BLP) to induce linear utility in subjects. The theoretical prediction is that, under certain conditions, this approach allows the researcher to directly elicit the subjective probability without further statistical corrections for risk attitudes. Harrison et al. (2012a) found evidence that the BLP mitigates the distortion in reports introduced by risk aversion in a belief elicitation task for binary events. This implies that the BLP provides incentives to subjects to directly report the true latent subjective probability, at least for belief elicitation tasks that use the QSR to elicit probabilities with only two outcomes. It is yet to be studied if the validity of this result extends to generalized scoring rules that elicit continuous distributions; however, in the following discussion we assume that the BLP induces risk neutrality in this type of general belief elicitation procedures. Next section explains theoretically how the BLP can generate incentives to subjects to directly report their non-degenerate subjective probability distributions.

### Belief Elicitation with Generalized Scoring Rules and the Binary Lottery Procedure

Assume for simplicity that  $I = 10$ , so there are only 10 possible events. For instance, in a Bingo Cage with only red (R) or white (W) balls, the proportion of red balls could be any of the following proportions: 5%, 15%, 25%, 35%, 45%, 55%, 65%, 75%, 85% and 95%. The following arguments apply for any distribution with finite discrete support. A given subject could display a subjective non-degenerate prior  $P_i = (\pi_{5\%}, \pi_{15\%}, \pi_{25\%}, \pi_{35\%}, \pi_{45\%}, \pi_{55\%}, \pi_{65\%}, \pi_{75\%}, \pi_{85\%}, \pi_{95\%})$ , where  $\pi_{a\%}$  is the subjective probability that the true proportion of red balls in the urn is equal to  $a\%$ . Suppose that  $\Pi = (.25, .225, .175, .1, .0417, .0417, .0417, .0417, .0417, .0417)$ . This prior implies that on average the proportion of red balls in the urn is equal to 30% ( $= .05 \times .25 + .15 \times .225 + .25 \times .175 + .35 \times .1 + .45 \times .0417 + .55 \times .0417 + .65 \times .0417 + .75 \times .0417 + .85 \times .0417 + .95 \times .0417$ ). We want to elicit the non-degenerate prior  $\Pi$  by using the generalized scoring rule given by the following function:

$$S_j = \alpha + \beta(2 \times r_j) - \sum_{i=1,10} r_i^2$$

where  $r_j$  is the subject's report about event  $j$  and  $S_j$  is the number of points the subject wins if event  $j$  is realized indeed. Assume that  $\alpha = \beta = 50$ . Thus  $r_1$  is the report for the proportion of red balls in the urn being equal to 5%,  $r_2$  is the report for the proportion of red balls in the urn being equal to 15%, ...,  $r_9$  is the report for the proportion of red balls in the urn being equal to 85%, and  $r_{10}$  is the report for the proportion of red balls in the urn being equal to 95%. In contrast to the binary case, a report now consists of 10 numbers

that can be written in vector notation as  $r = (r_1, r_2, r_3, r_4, r_5, r_6, r_7, r_8, r_9, r_{10})$ . This report gives the subject the following payoffs in points that are translated into objective probabilities of getting, for instance, a high \$100 prize of the binary lottery. In particular, by reporting  $r$ , the subject is going to obtain a vector of points  $S = (S_1(r|5\%), S_2(r|15\%), S_3(r|25\%), S_4(r|35\%), S_5(r|45\%), S_6(r|55\%), S_7(r|65\%), S_8(r|75\%), S_9(r|85\%), S_{10}(r|95\%))$ , where each position in the vector is defined as follows:

$$\begin{aligned} S_1(r|5\%) &= 50 + 50(2r_1) - r_1^2 - r_2^2 - r_3^2 - r_4^2 - r_5^2 - r_6^2 - r_7^2 - r_8^2 - r_9^2 - r_{10}^2 \\ S_2(r|15\%) &= 50 + 50(2r_2) - r_1^2 - r_2^2 - r_3^2 - r_4^2 - r_5^2 - r_6^2 - r_7^2 - r_8^2 - r_9^2 - r_{10}^2 \\ &\cdot \\ &\cdot \\ S_9(r|85\%) &= 50 + 50(2r_9) - r_1^2 - r_2^2 - r_3^2 - r_4^2 - r_5^2 - r_6^2 - r_7^2 - r_8^2 - r_9^2 - r_{10}^2 \\ S_{10}(r|95\%) &= 50 + 50(2r_{10}) - r_1^2 - r_2^2 - r_3^2 - r_4^2 - r_5^2 - r_6^2 - r_7^2 - r_8^2 - r_9^2 - r_{10}^2 \end{aligned}$$

Define now the following 10 objective probabilities of winning the monetary prize \$100, induced by a report  $r$ :

$$\begin{aligned} p_{5\%}(r) &= S_1(r|5\%)/100, p_{15\%}(r) = S_2(r|15\%)/100, \dots, \\ p_{85\%}(r) &= S_9(r|85\%)/100, p_{95\%}(r) = S_{10}(r|95\%)/100 \end{aligned}$$

The task that a subject faces is to choose one among many subjective compound lotteries induced by report  $r$ . Each induced subjective compound lottery has the following structure:

SCL( $r$ ): pays simple lottery ( $p_{5\%}(r)$ , \$100;  $1-p_{5\%}(r)$ , \$0) with subjective probability  $\pi_{5\%}$ ; pays simple lottery ( $p_{15\%}(r)$ , \$100;  $1-p_{15\%}(r)$ , \$0) with subjective probability  $\pi_{15\%}$ ;.....; pays simple lottery ( $p_{85\%}(r)$ , \$100;  $1-p_{85\%}(r)$ , \$0) with subjective probability  $\pi_{85\%}$ ; pays simple lottery ( $p_{95\%}(r)$ , \$100;  $1-p_{95\%}(r)$ , \$0) with subjective probability  $\pi_{95\%}$ .

Figure 3.9 shows graphically the subjective compound lottery induced by report  $r$  as well as its actuarially-equivalent simple lottery. A subjective expected utility maximizers valuation of each report  $r$  is equal to:

$$\begin{aligned} SEU(r) &= \pi_{5\%} \times p_{5\%}(r) \times U(\$100) + (1 - p_{5\%}(r)) \times U(\$0) \\ &\quad + \pi_{15\%} \times p_{15\%}(r) \times U(\$100) + (1 - p_{15\%}(r)) \times U(\$0) \\ &\quad + \dots \\ &\quad + \dots \\ &\quad + \pi_{85\%} \times p_{85\%}(r) \times U(\$100) + (1 - p_{85\%}(r)) \times U(\$0) \\ &\quad + \pi_{95\%} \times p_{95\%}(r) \times U(\$100) + (1 - p_{95\%}(r)) \times U(\$0) \end{aligned}$$

Normalizing  $U(\$100) = 1$  and  $U(\$0) = 0$ ,  $SEU(r)$  is reduced to:

$$SEU(r) = \pi_{5\%} \times p_{5\%}(r) + \pi_{15\%} \times p_{15\%}(r) + \dots + \pi_{85\%} \times p_{85\%}(r) + \pi_{95\%} \times p_{95\%}(r) = P(r)$$

We rename  $SEU(r)$  as  $P(r)$  to emphasize that the subject's valuation of the lottery induced by  $r$  can be interpreted as the subjective average probability  $P$  of winning the high \$100 amount in the binary lottery. For the sake of the argument assume for a moment that the state space of the report for each event  $j$  is a continuum such that  $r_j \in [0, 100]$  and  $\sum_{i=1,10} r_i = 100$ . Therefore a SEU maximizer would make a report  $r^*$  that maximizes the subjective expected probability  $P(\cdot)$  of winning the binary lottery. The solution to this problem is given by a multivariate maximization where the first order conditions are given by

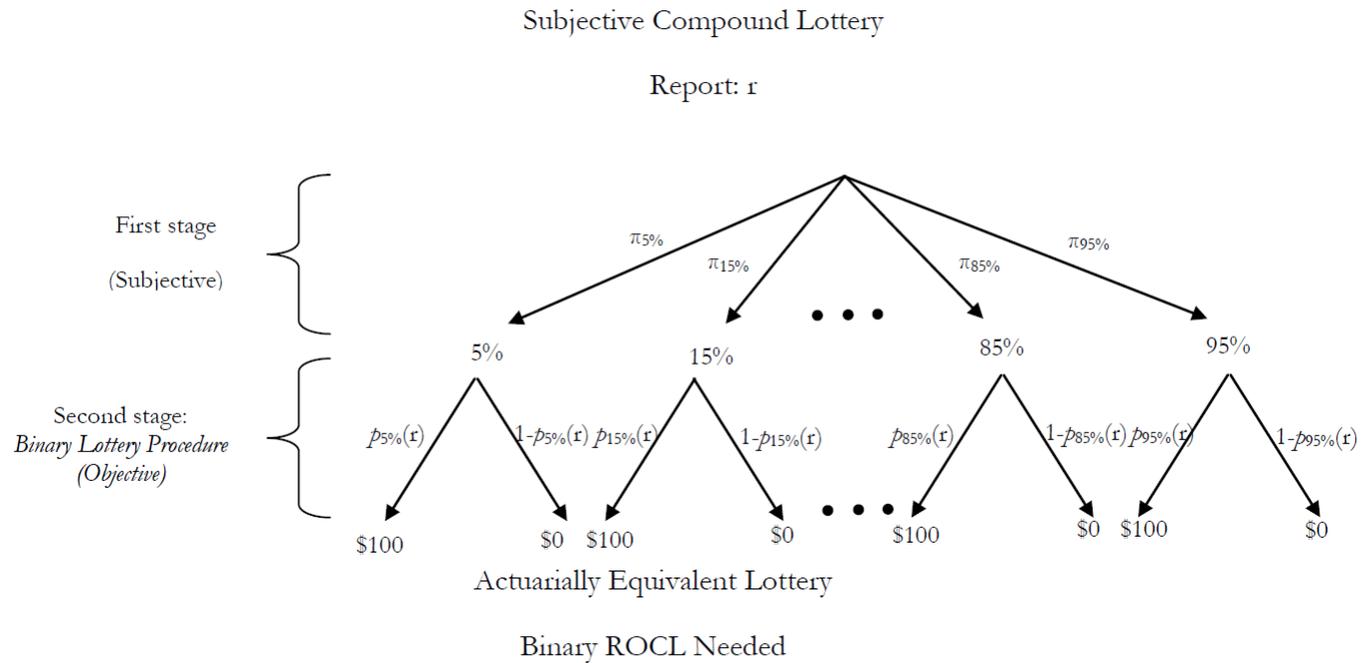
$$\begin{aligned}
[r_1] : & \pi_{5\%} \partial p_{5\%}(r) / \partial r_1 + \pi_{15\%} \partial p_{15\%}(r) / \partial r_1 + \dots + \pi_{85\%} \partial p_{85\%}(r) / \partial r_1 + \pi_{95\%} \partial p_{95\%}(r) / \partial r_1 = 0 \\
& \pi_{5\%} [50(2 \times (1) - 2r_1)] + \pi_{15\%} [50(-2r_1)] + \dots + \pi_{85\%} [50(-2r_1)] + \pi_{95\%} [50(-2r_1)] = 0 \\
& \pi_{5\%} - (\pi_{5\%} + \pi_{15\%} + \pi_{25\%} + \pi_{35\%} + \pi_{45\%} + \pi_{55\%} + \pi_{65\%} + \pi_{75\%} + \pi_{85\%} + \pi_{95\%}) r_1 = 0 \\
& \pi_{5\%} = r_1^*
\end{aligned}$$

$$\begin{aligned}
[r_2] : & \pi_{15\%} \partial p_{5\%}(r) / \partial r_2 + \pi_{15\%} \partial p_{15\%}(r) / \partial r_2 + \dots + \pi_{85\%} \partial p_{85\%}(r) / \partial r_2 + \pi_{95\%} \partial p_{95\%}(r) / \partial r_2 = 0 \\
& \pi_{15\%} [50(-2r_2)] + \pi_{15\%} [50(2(1) - 2r_2)] + \dots + \pi_{85\%} [50(-2r_2)] + \pi_{95\%} [50(-2r_2)] = 0 \\
& \pi_{15\%} - (\pi_{5\%} + \pi_{15\%} + \pi_{25\%} + \pi_{35\%} + \pi_{45\%} + \pi_{55\%} + \pi_{65\%} + \pi_{75\%} + \pi_{85\%} + \pi_{95\%}) r_2 = 0 \\
& \pi_{15\%} = r_2^*
\end{aligned}$$

·  
·  
·

$$\begin{aligned}
[r_{10}] : & \pi_{5\%} \partial p_{5\%}(r) / \partial r_{10} + \pi_{15\%} \partial p_{15\%}(r) / \partial r_{10} + \dots + \pi_{85\%} \partial p_{85\%}(r) / \partial r_{10} + \pi_{95\%} \partial p_{95\%}(r) / \partial r_{10} = 0 \\
& \pi_{5\%} [50(-2r_{10})] + \pi_{15\%} [50(-2r_{10})] + \dots + \pi_{85\%} [50(-2r_{10})] + \pi_{95\%} [50(2(1) - 2r_{10})] = 0 \\
& \pi_{95\%} - (\pi_{5\%} + \pi_{15\%} + \pi_{25\%} + \pi_{35\%} + \pi_{45\%} + \pi_{55\%} + \pi_{65\%} + \pi_{75\%} + \pi_{85\%} + \pi_{95\%}) r_{10} = 0 \\
& \pi_{95\%} = r_{10}^*
\end{aligned}$$

Figure 3.9: Generalized Scoring Rule Using the Binary Lottery Procedure



$$\begin{aligned}
 P &= \pi_{5\%} * p_{5\%}(r) + \pi_{15\%} * p_{15\%}(r) + \dots \\
 &+ \pi_{85\%} * p_{85\%}(r) + \pi_{95\%} * p_{95\%}(r) \\
 1-P &= (1-\pi_{5\%}) * (1-p_{5\%}(r)) + (1-\pi_{15\%}) * (1-p_{15\%}(r)) + \dots \\
 &+ (1-\pi_{85\%}) * (1-p_{85\%}(r)) + (1-\pi_{95\%}) * (1-p_{95\%}(r))
 \end{aligned}$$

The conclusion is that a subject that follows SEU and is given a generalized scoring rule with the BLP procedure to induce linear utility will optimally report his true subjective probability distribution, i.e.  $r^* = \Pi$ . This maximization problem has a unique optimum because the second order condition  $\partial^2 SEU(r)/(\partial r_j \partial r_j) < 0$  for all  $j = 1, \dots, 10$ . Thus a generalized scoring rule that relies on the BLP to induce linear utility is a proper scoring rule for a SEU maximizer. Harrison et al. (2012c) showed that in theory BLP can also induce linear utility for certain non-EU preference representations in belief elicitation tasks that use the QSR. These results hold trivially for the type of generalized scoring rule we explained above.

## 3.7.2 Experiments

### Experimental Design

Our design consist of two between-subjects treatments that are presented with the same stimuli in any given session, but with different scoring rules and belief elicitation questions. In **treatment 2Bin** subjects are presented with only one belief elicitation question that uses the QSR for binary events. In **treatment 10Bin** subjects are presented with only one belief elicitation question that uses the generalized scoring rule for non-binary events. The scores in each treatment are denominated in points that subsequently determine the objective probability of winning a binary lottery that pays either \$50 or \$0. In both treatments subjects are first presented with some information to generate subjective beliefs, and after that the choice is made. Subjects complete a demographic survey, but are explicitly told that their responses to that survey do not affect earnings. Then subjects receive monetary compensation according to a chance realization appropriate for each treatment and depending on their report in the belief elicitation task. There are no other salient tasks, before or after a subjects choices.

We used software we created in *Visual Basic .NET* to present the scoring rules we give to subjects and record their choices. Figures 3.10, 3.11, 3.12 and 3.13 illustrate the scoring rule interfaces we developed to implement these ideas, and generate our test of ROCL with subjective beliefs.

Subjects in treatment 10Bin are presented with the distributional scoring rule task shown in Figures 3.10 and 3.11. In this case we set  $I = 10$ , so that we can elicit subjective beliefs in deciles. The question we present to our subjects is “What is the fraction of red ping-pong balls in Bingo Cage 2?” In this case the response is bounded between 0% and 100%, but these general methods apply to beliefs about arbitrarily bounded variables as well. The interface starts by assuming a completely diffuse belief, as in Figure 3.10.<sup>33</sup> The subject has 100 tokens to allocate to each decile according to his beliefs. For instance, the subject might believe that the fraction of red balls in the Bingo Cage 2 is just below 60% but he is not completely certain about the true fraction of red balls. Therefore a subject might allocate the 100 tokens to several bars as shown in Figure 3.11. Payoffs to subjects depend

---

<sup>33</sup>One could have started with any random allocation of the 100 tokens, but this starting point seemed the most natural in many respects. The role of a starting point as an anchor is easily studied, and has been studied in Andersen et al. (2010) in the context of scoring rules defined over binary events.

on the number of points earned when the true composition of Bingo Cage 2 is revealed, and convert into greater probability of winning the highest outcome of the final binary lottery. Subjects in treatment 2Bin are presented with a binary scoring rule task shown in Figures 3.12 and 3.13. The question we present to our subjects in this interface is “What will be the color of the ping-pong ball drawn from Bingo cage 2?” The binary scoring rule illustrated in Figure 3.12 and 3.13 is a variant on the “slider interface” developed by Andersen et al. (2010), but adapted to look more like the interface for the distributional scoring rule in Figures 3.10 and 3.11. Subjects can move one or other of two sliders, and the other slider changes automatically so that 100 tokens are allocated.

It is important to emphasize the subtlety of the difference in the questions we ask subjects in each treatment. In treatment 10Bin we ask subjects to place a bet on the *composition* of a Bingo cage, while in treatment 2Bin we ask subjects to place bets *one realization* of that Bingo cage. This is an important feature of our design, which seeks to compare a degenerate and a non-degenerate probability distribution of the same underlying physical process. In treatment 2Bin we elicit a number that is the best estimate of the composition of the Bingo cage. In treatment 10Bin we elicit a full distribution that captures subjects beliefs about the composition of the Bingo cage. Our test of ROCL under subjective probabilities involves comparing the degenerate probability elicited in treatment 2Bin with the (weighted) average probability implied by the non-degenerate distribution elicited in treatment 10Bin.

One might conjecture that a concentrated, constrained version of the distributional scoring rule, in which all 100 tokens had to be allocated to one and only one decile, would be sufficient to elicit the weighted average subjective belief of the subject that obeyed ROCL. This would only be true if the latent subjective belief distribution could be assumed to be symmetric and unimodal. However, as explained earlier, ROCL implies that subjects are indifferent between *degenerate* subjective probability distributions and an infinite number of *non-degenerate* distributions, symmetric and unimodal or not, that have a weighted average equal to the degenerate distributions. Hence any number of latent, non-degenerate belief distributions could be consistent with the observed report from such a concentrated version of the 10Bin task.

An additional methodological issue is that we cannot compare responses to the distributional and binary scoring rules on an in-sample basis. This would necessitate the use the 1-in-K random lottery incentive procedure, which of course presumes the validity of ROCL over objective probabilities. Although we remain agnostic about whether ROCL over objective probabilities implies ROCL over subjective probabilities, and *vice versa*, we do not want to have to make any such assumption if we have to.<sup>34</sup> Therefore, each of our subjects is presented with only one choice, which allows us to avoid potential confounds that can arise from payment protocols that assume the same hypothesis we are trying to test.

---

<sup>34</sup>The evidence from Harrison et al. (2012b) suggests that ROCL is valid for objective lotteries only when  $K=1$ .

## Experimental Procedures

The general procedures during an experimental session are as follows. Upon arrival at the laboratory, each subject draw a number from a box which determines random seating position within the laboratory. After being seated and signing the informed consent document, subjects are given printed introductory instructions and allowed sufficient time to read these instructions. Then a Verifier is selected at random among subjects solely for the purpose of verifying that the procedures of the experiment are carried out according to the instructions. The Verifier is paid a fixed amount for this task. Each subject is assigned to one of two groups depending on whether the seating number was even or odd, where each group correspond to one of the treatments. One of the groups is then asked to leave and wait outside the room for a few minutes, always under the supervision of an experimenter. The other group remain in the laboratory and go over the main instructions with the experimenter. Simultaneously, subjects waiting outside are given instructions to read individually. Then the groups swap places and the experimenter read the main instructions designed for the other group. Once all instructions are finished, and both groups are together in the room again, we proceeded with the rest of the experiment. The reason for these procedures is to ensure that subjects in both treatments confront the same stimuli, defined below. We do not want to explain group-specific instructions in the presence of subjects of the other group and tell them not to pay attention, and *vice versa*. Subjects are told the basic reason for this step:

Part of this experiment is to test different computer screens. Therefore, we will divide you into two groups, and each group will be presented with a slightly different instructions and computer screens. If you are sitting in a computer station that has an odd number on it, you are part of the Odd group. If you are sitting in a computer station that has an even number on it, you are part of the Even group.

An important reason to assign subjects to treatments according to their station number in the laboratory is to avoid potential confounds in the results generated by subjects in each treatment having very different visual access to the stimuli. By mapping even or odd station numbers to treatment 2Bin or 10Bin, we ensure that if there exist any difference in subjects' vantage point, this difference was the same across treatments.

We use two bingo cages: Bingo Cage 1 and Bingo Cage 2. Bingo Cage 1 is loaded with balls numbered 1 to 99 in front of everyone.<sup>35</sup> A numbered ball is drawn from Bingo Cage 1, but the draw takes place behind a divider. The outcome of this draw is not verified in front of subjects until the very end of the experiment, after their decisions had been made. The number on the chosen ball from Bingo Cage 1 is used to construct Bingo Cage 2 behind the divider. The total number of balls in Bingo Cage 2 is always 100: the number of red balls matches the number drawn from Bingo Cage 1, and the number of white balls

---

<sup>35</sup>When shown in public, Bingo Cage 1 and 2, are always displayed in front of the laboratory where everyone could see them. We also use a high resolution video camera to display the bingo cages in three flat screen TVs distributed throughout the laboratory, and on the projection screen at the front of the room. Our intention is that everyone has equal chance of observing the bingo cages from a good perspective.

is 100 minus the number of red balls. Since the actual composition of Bingo Cage 2 is only revealed and verified in front of everybody at the end of the experiment, the Verifiers role is to confirm that the experimenter constructs Bingo Cage 2 according to the randomly chosen numbered ball. Once Bingo Cage 2 is constructed, the experimenter puts the chosen numbered ball in an envelope and affixes it to the front wall of the laboratory.

Bingo Cage 2 is then covered and placed on a platform in the front of the room. Bingo Cage 2 is then uncovered for subjects to see, spun for 10 turns, and covered again. Subjects then make their decisions about the number of red and white balls in Bingo Cage 2. After choices are made and subjects complete a non-salient demographic survey, the experimenter draws a ball from Bingo Cage 2. The sealed envelope is opened and the chosen numbered ball is shown to everyone, and the experimenter publicly counts the number of red and white balls in Bingo Cage 2. Then an experimenter approaches each subject and records earnings according to the betting choices made and the ball drawn from or the number of red balls in Bingo Cage 2. Then the number of points they earned in the belief elicitation task was recorded. If subjects are part of treatment 2Bin the number of points they earned is determined by the color of the ball drawn from Bingo cage 2. If subjects are part of treatment 10Bin the number of points they earned is determined by the actual number of red in Bingo cage 2. Then subjects in both treatments roll two 10-sided dice, and if the outcome is less or equal to the number of points earned they win \$50, otherwise they earn \$0 in the task. Finally, subjects leave the laboratory and are privately paid their earnings: a \$7.50 participation payment in addition to the monetary outcome of the belief elicitation task. The Verifiers are paid a flat \$25 fee plus the participation fee.

### **Evaluation of the Subjective ROCL Hypothesis**

We want to test if ROCL is satisfied in the domain of subjective probabilities. Since we rely on the BLP to induces linear utility in subjects, we conduct our tests directly on the elicited reports from subjects without the need to correct for risk attitudes. In our tests we assume that the structure of beliefs in the same in treatments 2Bin and 10Bin. Therefore any significant treatment effect can be attributed to violations of ROCL under subjective probabilities.

The test of our main hypothesis is, after all this preparation, surprisingly simple. We compare the elicited reports for red balls in treatment 2Bin with the weighted average proportion of red balls implied by the elicited distributions in treatment 10Bin. We use non-parametric statistical tests to study differences across treatments.

An ideal test of our hypothesis involves controlling for risk attitudes to estimate the latent subjective probabilities in both treatments and testing the equivalency implied by ROCL. This would require econometric techniques to recover the underlying subjective probabilities conditional on risk attitudes elicited in a different task. The tools to estimate latent probabilities in binary scoring rules are already available in the literature, but for the generalized scoring rule the econometric procedures are numerically challenging. We leave these issues for future research.

### 3.7.3 Conclusions

It is possible to design an operational test of ROCL in the domain of subjective beliefs. The test involves careful attention to the theoretical issues of belief elicitation for non-binary events, careful attention to defining elicitation tasks over the same underlying physical process, and careful attention to avoid payment protocols for experimental subjects that assume the validity of the axiom we are trying to test.

Figure 3.10: Distributional Scoring Rule Initialization



Figure 3.11: Illustrative Distributional Scoring Rule Response



Figure 3.12: Binary Scoring Rule Initialization

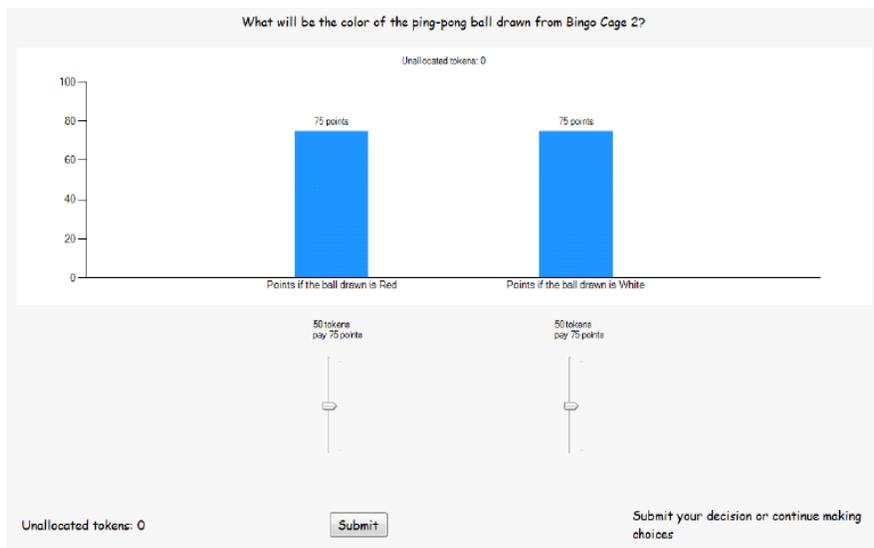
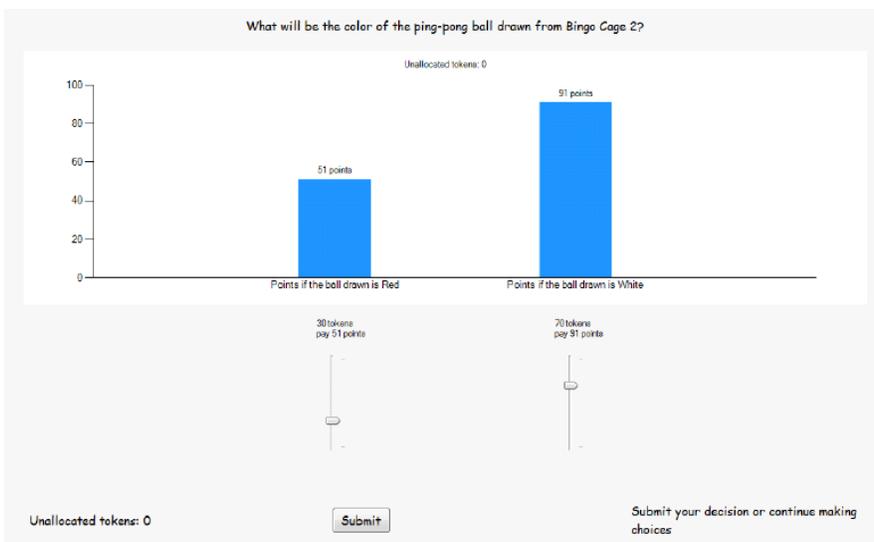


Figure 3.13: Illustrative Binary Scoring Rule Response



## 3.8 Appendix B. Instructions

### 3.8.1 Instructions for Treatment 1-in-1

1p

#### Choices Over Risky Prospects

This is a task where you will choose between prospects with varying prizes and chances of winning. You will be presented with one pair of prospects where you will choose one of them. You should choose the prospect you prefer to play. You will actually get the chance to play the prospect you choose, and you will be paid according to the outcome of that prospect, so you should think carefully about which prospect you prefer.

Here is an example of what the computer display of a pair of prospects might look like.



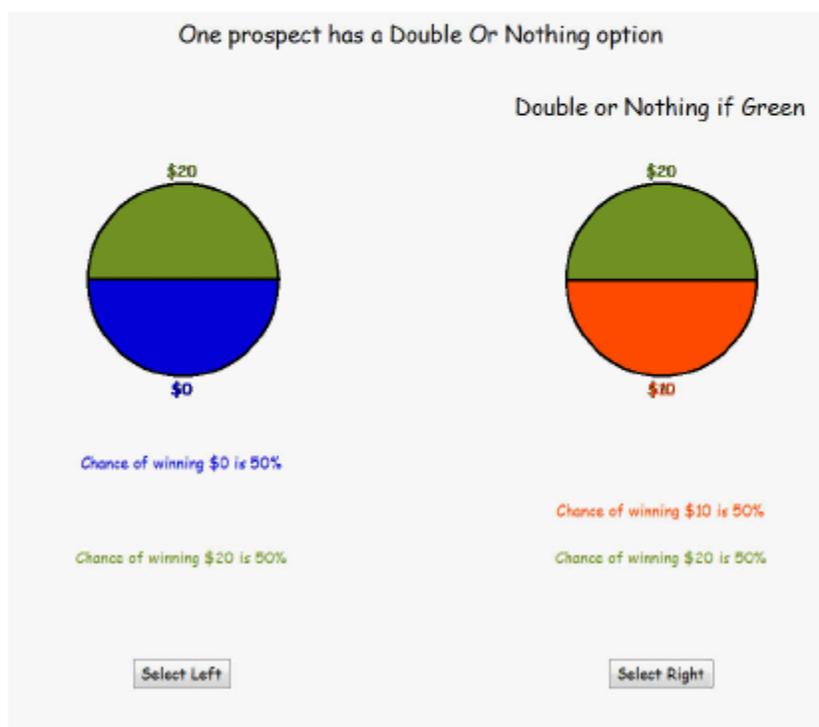
The outcome of the prospects will be determined by the draw of a random number between 1 and 100. Each number between, and including, 1 and 100 is equally likely to occur. In fact, you will be able to draw the number yourself using two 10-sided dice.

In the above example the left prospect pays five dollars (\$5) if the number drawn is between 1 and 40, and pays fifteen dollars (\$15) if the number is between 41 and 100. The blue color in the

pie chart corresponds to 40% of the area and illustrates the chances that the number drawn will be between 1 and 40 and your prize will be \$5. The orange area in the pie chart corresponds to 60% of the area and illustrates the chances that the number drawn will be between 41 and 100 and your prize will be \$15.

Now look at the pie in the chart on the right. It pays five dollars (\$5) if the number drawn is between 1 and 50, ten dollars (\$10) if the number is between 51 and 90, and fifteen dollars (\$15) if the number is between 91 and 100. As with the prospect on the left, the pie slices represent the fraction of the possible numbers which yield each payoff. For example, the size of the \$15 pie slice is 10% of the total pie.

You could also get a pair of prospects in which one of the prospects will give you the chance to play "Double or Nothing." For instance, the right prospect in the following screen image pays "Double or Nothing" if the Green area is selected, which happens if the number drawn is between 51 and 100. The right pie chart indicates that if the number is between 1 and 50 you get \$10. However, if the number is between 51 and 100 a coin will be tossed to determine if you get double the amount. If it comes up Heads you get \$40, otherwise you get nothing. The prizes listed underneath each pie refer to the amounts before any "Double or Nothing" coin toss.



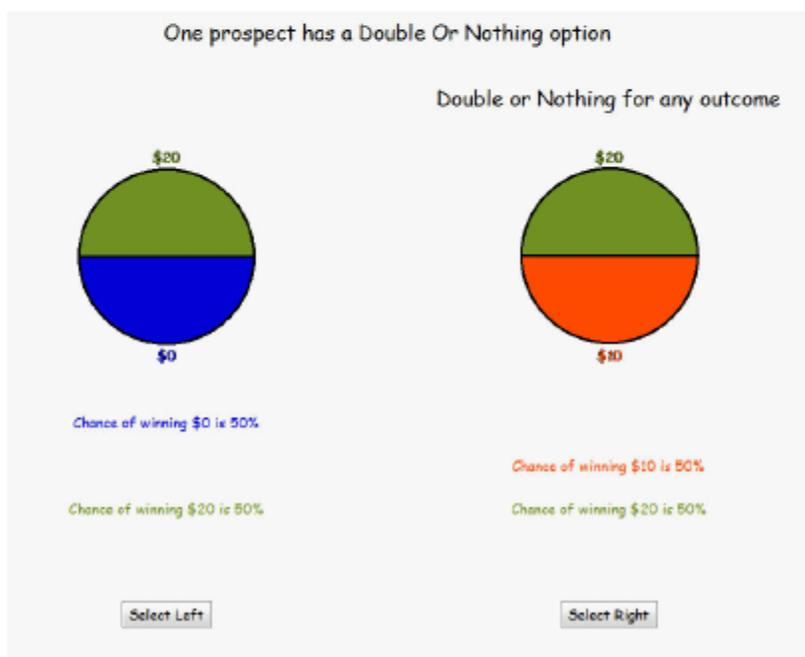
The pair of prospects you choose from is shown on a screen on the computer. On that screen, you should indicate which prospect you prefer to play by clicking on one of the buttons beneath the prospects.

After you have made your choice, raise your hand and an experimenter will come over. It is certain that your one choice will be played out for real. You will roll the two ten-sided dice to determine the outcome of the prospect you chose, and if necessary you will then toss a coin to determine if you get "Double or Nothing."

For instance, suppose you picked the prospect on the left in the last example. If the random number was 37, you would win \$0; if it was 93, you would get \$20.

If you picked the prospect on the right and drew the number 37, you would get \$10; if it was 93, you would have to toss a coin to determine if you get "Double or Nothing." If the coin comes up Heads then you get \$40. However, if it comes up Tails you get nothing from your chosen prospect.

It is also possible that you will be given a prospect in which there is a "Double or Nothing" option no matter what the outcome of the random number. The screen image below illustrates this possibility.



Therefore, your payoff is determined by three things:

- by which prospect you selected, the left or the right;
- by the outcome of that prospect when you roll the two 10-sided dice; and
- by the outcome of a coin toss if the chosen prospect outcome is of the “Double or Nothing” type.

Which prospects you prefer is a matter of personal taste. The people next to you may be presented with a different prospect, and may have different preferences, so their responses should not matter to you. Please work silently, and make your choices by thinking carefully about the prospect you are presented with.

All payoffs are in cash, and are in addition to the \$7.50 show-up fee that you receive just for being here. The only other task today is for you to answer some demographic questions. Your answers to those questions will not affect your payoffs.

### 3.8.2 Instructions for Treatment 1-in-40

40p

#### Choices Over Risky Prospects

This is a task where you will choose between prospects with varying prizes and chances of winning. You will be presented with a series of pairs of prospects where you will choose one of them. There are 40 pairs in the series. For each pair of prospects, you should choose the prospect you prefer to play. You will actually get the chance to play one of the prospects you choose, and you will be paid according to the outcome of that prospect, so you should think carefully about which prospect you prefer.



Here is an example of what the computer display of such a pair of prospects might look like.

The outcome of the prospects will be determined by the draw of a random number between 1 and 100. Each number between, and including, 1 and 100 is equally likely to occur. In fact, you will be able to draw the number yourself using two 10-sided dice.

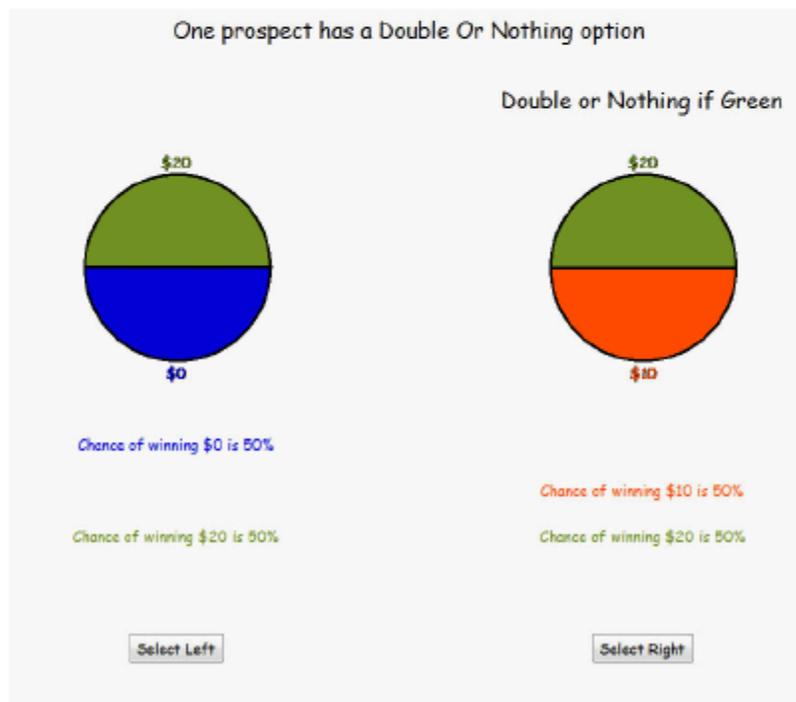
In the above example the left prospect pays five dollars (\$5) if the number drawn is between 1 and 40, and pays fifteen dollars (\$15) if the number is between 41 and 100. The blue color in the pie chart corresponds to 40% of the area and illustrates the chances that the number drawn will be between 1 and 40 and your prize will be \$5. The orange area in the pie chart corresponds to 60% of the area and illustrates the chances that the number drawn will be between 41 and 100 and your

prize will be \$15.

Now look at the pie in the chart on the right. It pays five dollars (\$5) if the number drawn is between 1 and 50, ten dollars (\$10) if the number is between 51 and 90, and fifteen dollars (\$15) if the number is between 91 and 100. As with the prospect on the left, the pie slices represent the fraction of the possible numbers which yield each payoff. For example, the size of the \$15 pie slice is 10% of the total pie.

Each pair of prospects is shown on a separate screen on the computer. On each screen, you should indicate which prospect you prefer to play by clicking on one of the buttons beneath the prospects.

You could also get a pair of prospects in which one of the prospects will give you the chance to play "Double or Nothing." For instance, the right prospect in the following screen image pays "Double or Nothing" if the Green area is selected, which happens if the number drawn is between 51 and 100. The right pie chart indicates that if the number is between 1 and 50 you get \$10. However, if the number is between 51 and 100 a coin will be tossed to determine if you get double the amount. If it comes up Heads you get \$40, otherwise you get nothing. The prizes listed underneath each pie refer to the amounts before any "Double or Nothing" coin toss.

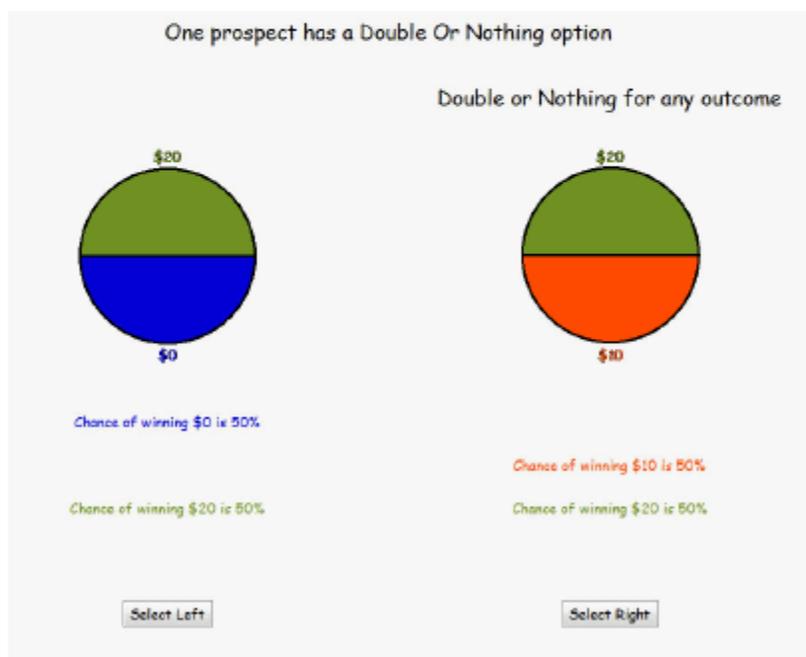


After you have worked through all of the 40 pairs of prospects, raise your hand and an experimenter will come over. You will then roll two 10-sided dice until a number between 1 and 40 comes up to determine which pair of prospects will be played out. Since there is a chance that any of your 40 choices could be played out for real, you should approach each pair of prospects as if it is the one that you will play out. Finally, you will roll the two ten-sided dice to determine the outcome of the prospect you chose, and if necessary you will then toss a coin to determine if you get "Double or Nothing."

For instance, suppose you picked the prospect on the left in the last example. If the random number was 37, you would win \$0; if it was 93, you would get \$20.

If you picked the prospect on the right and drew the number 37, you would get \$10; if it was 93, you would have to toss a coin to determine if you get "Double or Nothing." If the coin comes up Heads then you get \$40. However, if it comes up Tails you get nothing from your chosen prospect.

It is also possible that you will be given a prospect in which there is a "Double or Nothing" option no matter what the outcome of the random number. The screen image below illustrates this possibility.



Therefore, your payoff is determined by four things:

- by which prospect you selected, the left or the right, for each of these 40 pairs;
- by which prospect pair is chosen to be played out in the series of 40 such pairs using the two 10-sided dice;
- by the outcome of that prospect when you roll the two 10-sided dice; and
- by the outcome of a coin toss if the chosen prospect outcome is of the “Double or Nothing” type.

Which prospects you prefer is a matter of personal taste. The people next to you may be presented with different prospects, and may have different preferences, so their responses should not matter to you. Please work silently, and make your choices by thinking carefully about each prospect.

All payoffs are in cash, and are in addition to the \$7.50 show-up fee that you receive just for being here. The only other task today is for you to answer some demographic questions. Your answers to those questions will not affect your payoffs.

### 3.9 Appendix C. Parameters

To construct our battery of 40 lottery pairs, we used several criteria to choose the compound lotteries and their actuarially-equivalent lotteries used in our experiments:

1. The lottery compounding task should be as simple as possible. The instructions used by Halevy (2007) are a model in this respect, with careful picture illustrations of the manner in which the stages would be drawn. We wanted to avoid having physical displays, since we had many lotteries. We also wanted to be able to have the computer interface vary the order for us on a between-subject basis, so we opted for a simpler procedure that was as comparable as possible in terms of information as our simple lottery choice interface.
2. The lottery pairs should offer reasonable coverage of the Marschak-Machina (MM) triangle and prizes.
3. There should be choices/chords that assume parallel indifference curves, as expected under EUT, but the slope of the indifference curve should vary, so that the battery of lotteries can be used to test for a wide range of risk attitudes under the EUT null hypothesis (this criteria was employed for the construction of the basic 69 simple lotteries).
4. There should be a number of compound lotteries with their actuarially-equivalent counterparts in the interior of the triangle. Experimental evidence suggests that people tend to comply with the implications of EUT in the interior of the triangle and to violate it on the borders (Conlisk, 1989; Camerer, 1992; Harless, 1992; Gigliotti and Sopher, 1993; Starmer, 2000).
5. We were careful to choose lottery pairs with stakes and expected payoff per individual that are comparable to those in the original battery of 69 simple lotteries, since these had been used extensively in other samples from this population.

Our starting point was the battery of 69 lotteries in Table 3.10 used in Harrison and Swarthout (2012), which in turn were derived from Wilcox (2010). The lotteries were originally designed in part to satisfy the second and third criteria given above. Our strategy was then to “reverse engineer” the initial lotteries needed to obtain compound lotteries that would yield actuarially-equivalent prospects which already existed in the set of 69 pairs. For instance, the first pair in our battery of 40 lotteries was derived from pair 4 in the battery of 69 (contrast pair 1 in Table 3.11 with pair 4 in Table 3.10). We want the distribution of the “risky” lottery in the latter pair to be the actuarially-equivalent prospect of our compound lottery. To achieve this, we have an initial lottery that pays \$10 and \$0 with 50% probability each, and offering “Double or Nothing” if the outcome of the latter prospect is \$10. Hence it offers equal chances of \$20 or \$0 if the DON stage is reached. The \$5 stake was changed to \$0 because DON requires this prize to be among the

possible outcomes of the compound lotteries.<sup>36</sup> The actuarially-equivalent lottery of this compound prospect pays \$0 with 75% probability and \$20 with 25% probability, which is precisely the risky lottery in pair 4 of the default battery of 69 pairs. Except for the compound lottery in pair 10 in our set of lotteries, the actuarially-equivalent lotteries play the role of the “risky” lotteries.

Figure 3.14 shows the coverage of these lottery pairs in terms of the Marschak-Machina triangle. Each prize context defines a different triangle, but the patterns of choice overlap considerably. Figure 3.14 shows that there are many choices/chords that assume parallel indifference curves, as expected under EUT, but that the slope of the indifference curve can vary, so that the tests of EUT have reasonable power for a wide range of risk attitudes under the EUT null hypothesis (Loomes and Sugden, 1982; Harrison et al., 2007). These lotteries also contain a number of pairs in which the “EUT-safe” lottery has a higher EV than the “EUT-risky” lottery: this is designed deliberately to evaluate the extent of risk premia deriving from probability pessimism rather than diminishing marginal utility. The majority of our compound lotteries use a conditional version of the DON device because it allows to obtain good coverage of prizes and probabilities and keeps the compounding representation simple. As noted in the text, one can construct diverse compound lotteries with only two simple components: initial lotteries that either pay two outcomes with 50:50 odds or pay a given stake with certainty, and a conditional DON which pays double a predetermined amount with 50% probability or nothing with equal chance. In our design, if the subject has to play the DON option she will toss a coin to decide if she gets double the stated amount. One could use randomization devices that allow for probability distributions different from these 50:50 odds, but we want to keep the lottery compounding simple and familiar. Therefore, if one commits to 50:50 odds in the DON option, using exclusively unconditional DON will only allow one to generate compound lotteries with actuarially-equivalent prospects that assign 50% chance to getting nothing. For instance, suppose a compound prospect with an initial lottery that pays positive amounts \$X and \$Y with probability  $p$  and  $(1 - p)$ , respectively, and offers DON for any outcome. The corresponding actuarially-equivalent lottery pays \$2X, \$2Y and \$0 with probabilities  $p/2$ ,  $(1 - p)/2$  and 0.5, respectively.

The original 69 pairs use 10 contexts defined by three outcomes drawn from \$5, \$10, \$20, \$35 and \$70. For example, the first context consists of prospects defined over prizes \$5, \$10 and \$20, and the tenth context consists of lotteries defined over stakes \$20, \$35 and \$70.

As a result of using the DON device, we have to introduce \$0 to the set of stakes from which the contexts are drawn. However, some of the initial lotteries used prizes in contexts different from the ones used for final prizes, so that we could ensure that the stakes for the compounded lottery matched those of the simple lotteries. For example, pair 3 in Table B2 is defined over a context with stakes \$0, \$10 and \$35. The compound lottery of this pair offers an initial lottery that pays \$5 and \$17.50 with 50% chance each and a DON option for any outcome. This allows us to have as final prizes \$0, \$10 and \$35.

Our battery of 40 lotteries uses 6 of the original 10 contexts, but substitute the \$5 stake for \$0. We do not use the other 4 contexts: for them to be distinct from our 6 contexts they

---

<sup>36</sup>We contemplated using “double or \$5,” but this did not have the familiarity of DON.

would have to have 4 outcomes, the original 3 outcomes plus the \$0 stake required by the DON option. We chose to use only compound lotteries with no more than 3 final outcomes, which in turn requires initial lotteries with no more than 2 outcomes. Accordingly, the initial lotteries of compound prospects are defined over distributions that offer either 50:50 odds of getting any of 2 outcomes or certainty of getting a particular outcome which makes our design simple. It is worth noting that there are compound lotteries composed of initial prospects that offer an amount \$X with 100% probability and a DON option that pays \$2X and \$0 with 50% chance each (see pairs 5, 6 and 14 in Table 3.11 and pairs 34 and 40 in Table 3.13). By including this type of “trivial” compound lottery, we provide the basis for ROCL to be tested in its simplest form.

Finally, we included compound lotteries with actuarially-equivalent counterparts in the interior and on the border of the MM triangle, since previous experimental evidence suggests that this is relevant to test the implications of EUT. Pairs 3, 7, 10, 11, 32, 35 and 38 have compound lotteries with their actuarially-equivalent lotteries in the interior of the triangle.

Table 3.10: Default Simple Lotteries

Pair	Context	Prizes			"Safe" Lottery Probabilities			"Risky" Lottery Probabilities			EV Safe	EV Risky
		Low	Middle	High	Low	Middle	High	Low	Middle	High		
1	1	\$5	\$10	\$20	0	1	0	0.25	0	0.75	\$10.00	\$16.25
2	1	\$5	\$10	\$20	0.25	0.75	0	0.5	0	0.5	\$8.75	\$12.50
3	1	\$5	\$10	\$20	0	1	0	0.5	0	0.5	\$10.00	\$12.50
4	1	\$5	\$10	\$20	0.5	0.5	0	0.75	0	0.25	\$7.50	\$8.75
5	1	\$5	\$10	\$20	0	1	0	0.25	0.5	0.25	\$10.00	\$11.25
6	1	\$5	\$10	\$20	0.25	0.5	0.25	0.5	0	0.5	\$11.25	\$12.50
7	1	\$5	\$10	\$20	0	0.5	0.5	0.25	0	0.75	\$15.00	\$16.25
8	1	\$5	\$10	\$20	0	0.75	0.25	0.5	0	0.5	\$12.50	\$12.50
9	1	\$5	\$10	\$20	0.25	0.75	0	0.75	0	0.25	\$8.75	\$8.75
10	1	\$5	\$10	\$20	0	1	0	0.75	0	0.25	\$10.00	\$8.75
11	2	\$5	\$10	\$35	0	1	0	0.5	0	0.5	\$10.00	\$20.00
12	2	\$5	\$10	\$35	0	0.75	0.25	0.25	0	0.75	\$16.25	\$27.50
13	2	\$5	\$10	\$35	0.25	0.75	0	0.75	0	0.25	\$8.75	\$12.50
14	2	\$5	\$10	\$35	0	0.5	0.5	0.25	0	0.75	\$22.50	\$27.50
15	2	\$5	\$10	\$35	0	0.75	0.25	0.5	0	0.5	\$16.25	\$20.00
16	2	\$5	\$10	\$35	0	1	0	0.75	0	0.25	\$10.00	\$12.50
17	3	\$5	\$10	\$70	0.25	0.75	0	0.5	0	0.5	\$8.75	\$37.50
18	3	\$5	\$10	\$70	0	1	0	0.5	0	0.5	\$10.00	\$37.50
19	3	\$5	\$10	\$70	0.5	0.5	0	0.75	0	0.25	\$7.50	\$21.25
20	3	\$5	\$10	\$70	0	1	0	0.75	0	0.25	\$10.00	\$21.25
21	4	\$5	\$20	\$35	0	1	0	0.25	0	0.75	\$20.00	\$27.50
22	4	\$5	\$20	\$35	0	0.75	0.25	0.25	0	0.75	\$23.75	\$27.50

23	4	\$5	\$20	\$35	0	0.5	0.5	0.25	0	0.75	\$27.50	\$27.50
24	4	\$5	\$20	\$35	0	1	0	0.5	0	0.5	\$20.00	\$20.00
25	4	\$5	\$20	\$35	0.5	0.5	0	0.75	0	0.25	\$12.50	\$12.50
26	4	\$5	\$20	\$35	0	0.75	0.25	0.5	0	0.5	\$23.75	\$20.00
27	4	\$5	\$20	\$35	0.25	0.75	0	0.75	0	0.25	\$16.25	\$12.50
28	5	\$5	\$20	\$70	0.25	0.75	0	0.5	0	0.5	\$16.25	\$37.50
29	5	\$5	\$20	\$70	0	0.75	0.25	0.25	0	0.75	\$32.50	\$53.75
30	5	\$5	\$20	\$70	0.5	0.5	0	0.75	0	0.25	\$12.50	\$21.25
31	5	\$5	\$20	\$70	0.25	0.5	0.25	0.5	0	0.5	\$28.75	\$37.50
32	5	\$5	\$20	\$70	0.25	0.75	0	0.75	0	0.25	\$16.25	\$21.25
33	5	\$5	\$20	\$70	0	0.5	0.5	0.25	0	0.75	\$45.00	\$53.75
34	6	\$5	\$35	\$70	0	1	0	0.25	0	0.75	\$35.00	\$53.75
35	6	\$5	\$35	\$70	0.25	0.75	0	0.5	0	0.5	\$27.50	\$37.50
36	6	\$5	\$35	\$70	0	0.75	0.25	0.25	0	0.75	\$43.75	\$53.75
37	6	\$5	\$35	\$70	0.5	0.5	0	0.75	0	0.25	\$20.00	\$21.25
38	6	\$5	\$35	\$70	0	0.5	0.5	0.25	0	0.75	\$52.50	\$53.75
39	6	\$5	\$35	\$70	0	0.75	0.25	0.5	0	0.5	\$43.75	\$37.50
40	6	\$5	\$35	\$70	0.25	0.75	0	0.75	0	0.25	\$27.50	\$21.25
41	6	\$5	\$35	\$70	0	1	0	0.75	0	0.25	\$35.00	\$21.25
42	7	\$10	\$20	\$35	0	1	0	0.25	0	0.75	\$20.00	\$28.75
43	7	\$10	\$20	\$35	0.25	0.75	0	0.5	0	0.5	\$17.50	\$22.50
44	7	\$10	\$20	\$35	0	1	0	0.25	0.25	0.5	\$20.00	\$25.00
45	7	\$10	\$20	\$35	0	1	0	0.5	0	0.5	\$20.00	\$22.50
46	7	\$10	\$20	\$35	0	1	0	0.25	0.5	0.25	\$20.00	\$21.25
47	7	\$10	\$20	\$35	0	0.75	0.25	0.5	0	0.5	\$23.75	\$22.50
48	7	\$10	\$20	\$35	0	1	0	0.5	0.25	0.25	\$20.00	\$18.75
49	7	\$10	\$20	\$35	0.25	0.75	0	0.75	0	0.25	\$17.50	\$16.25
50	7	\$10	\$20	\$35	0	1	0	0.75	0	0.25	\$20.00	\$16.25

51	8	\$10	\$20	\$70	0.25	0.75	0	0.5	0	0.5	\$17.50	\$40.00
52	8	\$10	\$20	\$70	0.5	0.5	0	0.75	0	0.25	\$15.00	\$25.00
53	8	\$10	\$20	\$70	0.25	0.75	0	0.75	0	0.25	\$17.50	\$25.00
54	9	\$10	\$35	\$70	0	1	0	0.25	0	0.75	\$35.00	\$55.00
55	9	\$10	\$35	\$70	0.25	0.75	0	0.5	0	0.5	\$28.75	\$40.00
56	9	\$10	\$35	\$70	0	0.5	0.5	0.25	0	0.75	\$52.50	\$55.00
57	9	\$10	\$35	\$70	0	0.75	0.25	0.5	0	0.5	\$43.75	\$40.00
58	10	\$20	\$35	\$70	0	1	0	0.25	0	0.75	\$35.00	\$57.50
59	10	\$20	\$35	\$70	0.25	0.75	0	0.5	0	0.5	\$31.25	\$45.00
60	10	\$20	\$35	\$70	0	0.75	0.25	0.25	0	0.75	\$43.75	\$57.50
61	10	\$20	\$35	\$70	0	1	0	0.5	0	0.5	\$35.00	\$45.00
62	10	\$20	\$35	\$70	0.5	0.5	0	0.75	0	0.25	\$27.50	\$32.50
63	10	\$20	\$35	\$70	0	1	0	0.25	0.5	0.25	\$35.00	\$40.00
64	10	\$20	\$35	\$70	0.25	0.5	0.25	0.5	0	0.5	\$40.00	\$45.00
65	10	\$20	\$35	\$70	0	0.5	0.5	0.25	0	0.75	\$52.50	\$57.50
66	10	\$20	\$35	\$70	0	1	0	0.5	0.25	0.25	\$35.00	\$36.25
67	10	\$20	\$35	\$70	0.25	0.75	0	0.75	0	0.25	\$31.25	\$32.50
68	10	\$20	\$35	\$70	0	0.75	0.25	0.5	0	0.5	\$43.75	\$45.00
69	10	\$20	\$35	\$70	0	1	0	0.75	0	0.25	\$35.00	\$32.50

Figure 3.14: Default Simple Lotteries

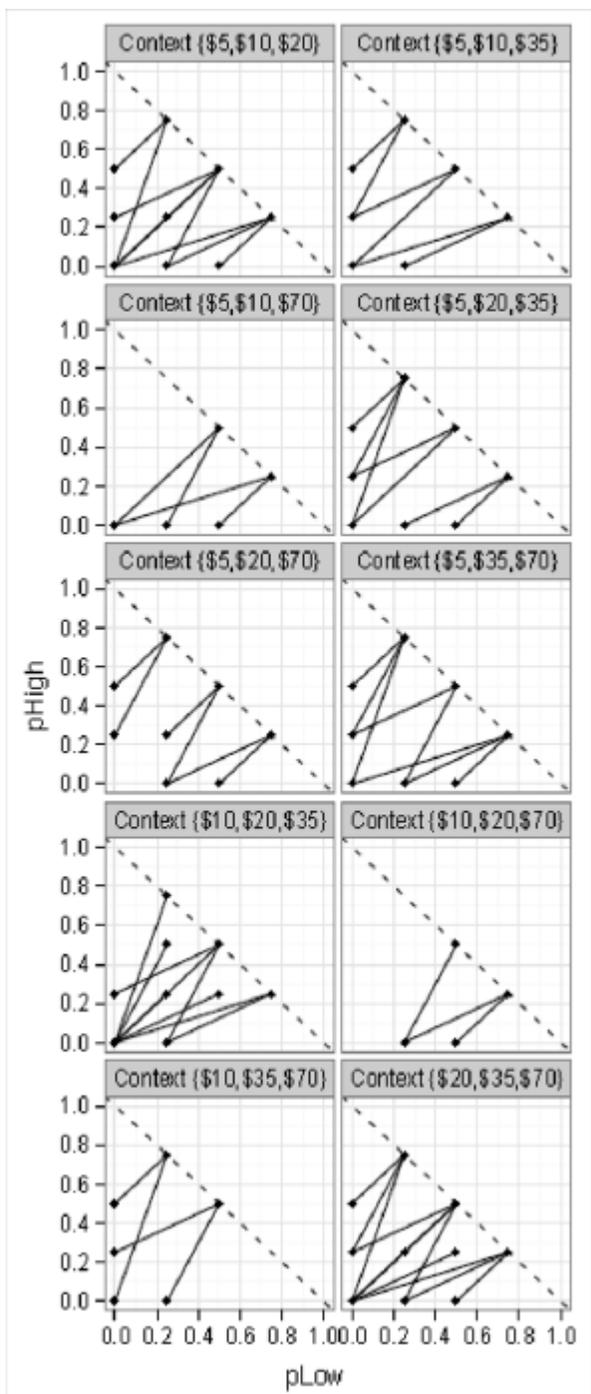


Table 3.11: Simple Lotteries vs. Compound Lotteries (Pairs 1-15)

Pair	Final Prizes				Simple Lottery			Compound Lottery							EV Simple	EV Compound
	Context	Low	Middle	High	Probabilities			Initial Lottery			Initial Lottery			"Double or Nothing" option		
					Low	Middle	High	Low	Middle	High	Low	Middle	High			
1	1	\$0	\$10	\$20	0.5	0.5	0	\$0	\$10	\$20	0.5	0.5	0	DON if middle	\$5.00	\$5.00
2	1	\$0	\$10	\$20	0	1	0	\$0	\$10	\$20	0.5	0.5	0	DON if middle	\$10.00	\$5.00
3	2	\$0	\$10	\$35	0	1	0	\$0	\$5	\$17.50	0	0.5	0.5	DON for any outcome	\$10.00	\$11.25
4	2	\$0	\$10	\$35	0.25	0.75	0	\$0	\$17.50	\$35	0.5	0.5	0	DON if middle	\$7.50	\$8.75
5	3	\$0	\$10	\$70	0.25	0.75	0	\$0	\$35	\$70	0	1	0	DON for any outcome	\$7.50	\$35.00
6	3	\$0	\$10	\$70	0	1	0	\$0	\$35	\$70	0	1	0	DON for any outcome	\$10.00	\$35.00
7	4	\$0	\$20	\$35	0	1	0	\$0	\$10	\$35	0	0.5	0.5	DON if middle	\$20.00	\$22.50
8	4	\$0	\$20	\$35	0	0.75	0.25	\$0	\$17.50	\$35	0	0.5	0.5	DON if middle	\$23.75	\$8.75
9	5	\$0	\$20	\$70	0	0.5	0.5	\$0	\$35	\$70	0	0.5	0.5	DON if middle	\$45.00	\$52.50
10	5	\$0	\$20	\$70	0.5	0	0.5	\$0	\$20	\$35	0	0.5	0.5	DON if high	\$35.00	\$27.50
11	5	\$0	\$20	\$70	0	1	0	\$0	\$20	\$35	0	0.5	0.5	DON if high	\$20.00	\$27.50
12	5	\$0	\$20	\$70	0	0.75	0.25	\$0	\$35	\$70	0	0.5	0.5	DON if middle	\$32.50	\$52.50
13	6	\$0	\$35	\$70	0	1	0	\$0	\$35	\$70	0	0.5	0.5	DON if middle	\$35.00	\$52.50
14	6	\$0	\$35	\$70	0	0.75	0.25	\$0	\$35	\$70	0	1	0	DON for any outcome	\$43.75	\$35.00
15	6	\$0	\$35	\$70	0	0.75	0.25	\$0	\$35	\$70	0	0.5	0.5	DON if middle	\$43.75	\$52.50

Table 3.12: Simple Lotteries vs. Actuarially-Equivalent Lotteries (Pairs 16-30)

Pair	Context	Final Prizes			Simple Lottery Probabilities			Actuarially-Equivalent Lottery Probabilities			EV Simple	EV Actuarially-Equivalent
		Low	Middle	High	Low	Middle	High	Low	Middle	High		
16	1	\$0	\$10	\$20	0.5	0.5	0	0.75	0	0.25	\$5.00	\$5.00
17	1	\$0	\$10	\$20	0	1	0	0.75	0	0.25	\$10.00	\$5.00
18	2	\$0	\$10	\$35	0	1	0	0.5	0.25	0.25	\$10.00	\$11.25
19	2	\$0	\$10	\$35	0.25	0.75	0	0.75	0	0.25	\$7.50	\$8.75
20	3	\$0	\$10	\$70	0.25	0.75	0	0.5	0	0.5	\$7.50	\$35.00
21	3	\$0	\$10	\$70	0	1	0	0.5	0	0.5	\$10.00	\$35.00
22	4	\$0	\$20	\$35	0	1	0	0.25	0.25	0.5	\$20.00	\$22.50
23	4	\$0	\$20	\$35	0	0.75	0.25	0.25	0	0.75	\$23.75	\$8.75
24	5	\$0	\$20	\$70	0	0.5	0.5	0.25	0	0.75	\$45.00	\$52.50
25	5	\$0	\$20	\$70	0.5	0	0.5	0.25	0.5	0.25	\$35.00	\$27.50
26	5	\$0	\$20	\$70	0	1	0	0.25	0.5	0.25	\$20.00	\$27.50
27	5	\$0	\$20	\$70	0	0.75	0.25	0.25	0	0.75	\$32.50	\$52.50
28	6	\$0	\$35	\$70	0	1	0	0.25	0	0.75	\$35.00	\$52.50
29	6	\$0	\$35	\$70	0	0.75	0.25	0.5	0	0.5	\$43.75	\$35.00
30	6	\$0	\$35	\$70	0	0.75	0.25	0.25	0	0.75	\$43.75	\$52.50

Table 3.13: Actuarially-Equivalent Lotteries vs. Compound Lotteries (Pairs 31-40)

Pair	Final Prizes				Actuarially-Equivalent Lottery Probabilities			Compound Lottery							EV Actuarially-Equivalent	EV Compound
	Context	Low	Middle	High	Low	Middle	High	Initial Lottery Prizes			Initial Lottery Probabilities			"Double or Nothing" option		
								Low	Middle	High	Low	Middle	High			
31	1	\$0	\$10	\$20	0.75	0	0.25	\$0	\$10	\$20	0.5	0.5	0	DON if middle	\$5.00	\$5.00
32	2	\$0	\$10	\$35	0.5	0.25	0.25	\$0	\$5	\$17.50	0	0.5	0.5	DON for any outcome	\$11.25	\$11.25
33	2	\$0	\$10	\$35	0.75	0	0.25	\$0	\$17.50	\$35	0.5	0.5	0	DON if middle	\$8.75	\$8.75
34	3	\$0	\$10	\$70	0.5	0	0.5	\$0	\$35	\$70	0	1	0	DON for any outcome	\$35.00	\$35.00
35	4	\$0	\$20	\$35	0.25	0.25	0.5	\$0	\$10	\$35	0	0.5	0.5	DON if middle	\$22.50	\$22.50
36	4	\$0	\$20	\$35	0.25	0	0.75	\$0	\$17.50	\$35	0	0.5	0.5	DON if middle	\$8.75	\$8.75
37	5	\$0	\$20	\$70	0.25	0	0.75	\$0	\$35	\$70	0	0.5	0.5	DON if middle	\$52.50	\$52.50
48	5	\$0	\$20	\$70	0.25	0.5	0.25	\$0	\$20	\$35	0	0.5	0.5	DON if high	\$27.50	\$27.50
30	6	\$0	\$35	\$70	0.25	0	0.75	\$0	\$35	\$70	0	0.5	0.5	DON if middle	\$52.50	\$52.50
40	6	\$0	\$35	\$70	0.5	0	0.5	\$0	\$35	\$70	0	1	0	DON for any outcome	\$35.00	\$35.00

### 3.10 Appendix D. Related Literature

Cubitt et al. (1998b) studied the Reduction of Compound Lotteries Axiom [ROCL] in a 1-in-1 design that gave each subject one and only one problem for real stakes and was conceived to test principles of dynamic choice. Also, Starmer and Sugden (1991), Beattie and Loomes (1997) and Cubitt et al. (1998a) have studied the Random Lottery Incentive Method (RLIM), and as a by-product have tested the ROCL axiom. We focus on the results related to the ROCL axiom: Harrison and Swarthout (2012) review the results related to RLIM.

Cubitt et al. (1998b) gave to one group of subjects one problem that involved compound lotteries and gave to another group the reduced compound version of the same problem. If ROCL is satisfied, one should see the same pattern of choice in both groups. They cannot find statistically significant violations of ROCL in their design.

Starmer and Sugden (1991) gave their subjects two pairs of lotteries that were designed to test common consequence violations of EUT. In each pair  $i$  there was a risky ( $R_i$ ) option and a safe ( $S_i$ ) option. They recruited 160 subjects that were divided into four groups of equal number. Two groups faced one of the two pairs in 1-in-1 treatments, while the other two groups were given both pairs to make a choice over using the RLIM to choose the pair for final payoff. We focus on the latter two groups since RLIM induces four possible compound lotteries: i) (0.5,  $R_1$ ; 0.5,  $R_2$ ), ii) (0.5,  $R_1$ ; 0.5,  $S_2$ ), iii) (0.5,  $R_2$ ; 0.5,  $S_1$ ) and iv) (0.5,  $S_1$ ; 0.5,  $S_2$ ). The lottery parameters were chosen to make compound lotteries ii) and iii) have equal actuarially-equivalent prospects.

They hypothesize that if a *reduction principle* holds, and if any of the induced compound lotteries ii) and iii) above is preferred by a subject, then the other one must be preferred.<sup>37</sup> The rejection of this hypothesis is a violation of ROCL, since this axiom implies that two compound lotteries with the same actuarially-equivalent prospects should be equally preferred. Therefore, the null hypothesis in Starmer and Sugden (1991, p. 976) is that “the choice between these two responses to be made at random; as a result, these responses should have the same expected frequency.” From the 80 subjects that faced the 1-in-2 treatments, 32.5% of the individuals chose (0.5,  $R_1$ ; 0.5,  $S_2$ ) and 15% chose (0.5,  $R_2$ ; 0.5,  $S_1$ ), thus Starmer and Sugden reject the null hypothesis of equal frequency in choices based on a one-tail test with a binomial distribution and  $p$ -value=0.017. This pattern is very similar in each of the 1-in-2 treatments; in one of them the proportions are 30% and 15%, whereas in the other they are 35% and 15%. A two-sided Fisher Exact test yields a  $p$ -value of 0.934, which suggest that these patterns of choices are very similar in both 1-in-2 treatments. Therefore, there is no statistical evidence to support ROCL in their experiment.

Beattie and Loomes (1997) examined 4 lottery choice tasks. The first 3 tasks involved a binary choice between two lotteries, and the fourth task involved the subject selecting one

---

<sup>37</sup>Following Holt (1986), Starmer and Sugden (1991, p. 972) define the reduction principle to be when “compound lotteries are reduced to simple ones by the calculus of probabilities and that choices are determined by the subject’s preferences over such reduced lotteries.” This is what we call ROCL, in addition to some axioms that are maintained for present purpose.

of four possible lotteries, two of which were compound lotteries.<sup>38</sup> They recruited 289 subjects that were randomly assigned to six groups. The first group faced a hypothetical treatment and was paid a flat fee for completing all four tasks. The second group was given a 1-in-4 treatment, and each of the other four groups faced one of the four tasks in 1-in-1 treatments. Sample sizes were 49 for the hypothetical treatment and the 1-in-4 treatment, and a total of 191 in the four 1-in-1 treatments.

Beattie and Loomes (1997, p. 164) find that “there is no support for the idea that two problems involving the same ‘reduced form’ alternatives and therefore involving the same difference between expected values will be treated equivalently.” On this basis, their Question 3 in the 1-in-4 treatment would be actuarially-equivalent to their Question 1 in the 1-in-1 treatment. They found that the pattern of choices in both treatments are so different that a chi-square test rejects with a very great confidence ( $p < .001$ ) the hypothesis that they are treated equivalently (p. 164). The  $p$ -value  $< 0.001$  of the Fisher Exact test provides further support for this violation of ROCL.

Their Question 4 is a task that is similar to the method developed by Binswanger (1980): subjects are offered an ordered set of choices that increase the average payoff while increasing variance. The difference with the Binswanger procedure is that two of the four choices involved compound lotteries: one paid a given amount of money if two Heads in a row were flipped, and the other paid a higher amount if three Heads in a row were flipped. For responses in Question 4, Beattie and Loomes (1997, p. 162)

...conjecture that the REAL [1-in-1] treatment might stimulate the greatest effort to picture the full sequential process [of coin flipping in the compound prospects] and, as a part of that, to anticipate feelings at each stage in the sequence; whereas the HYPO [hypothetical] treatment would be most conducive to thinking of the alternatives in their reduced form as a set of simple lotteries... The RPSP [1-in-4 treatment] might then, both formally and psychologically, represent an intermediate position, making the process less readily imaginable by adding a further stage (the random selection of the problem) to the beginning of the sequence, and reducing but not eliminating the incentive to expend the necessary imaginative effort.

On this basis, they predict that, when answering Question 4, subjects in the hypothetical treatment are more likely to think in *reduced form probability distributions*. Beattie and Loomes consider that this might enhance the salience of the high-payoff option, and thus the compound lotteries are expected to be chosen more frequently in the hypothetical treatment than in the 1-in-1 and 1-in-4 treatments.

Beattie and Loomes (1997, p. 165) found support for these conjectures: Their subjects tend to choose the compound lotteries more often in the hypothetical treatment than in the ones

---

<sup>38</sup>Beattie and Loomes (1997) use nine prospects: A= (0.2, £15; 0.8, £0), B= (0.25, £10; 0.75, £0), C= (0.8, £0; 0.2, £30), D= (0.8, £5; 0.2, £0), E=(0.8, £15; 0.2, £0), F = (1, £10), G=(1, £4), H=(0.5,£10; 0.5,£0), I=(£25 if two Heads in a row are flipped; otherwise nothing) and J=(£62.50 if three Heads in a row are flipped; otherwise nothing). Questions 1 through 3 are binary choices that offer, respectively, A or B, C or D and E or F. In Question 4, the subject must choose the prospect that she prefers the most among G, H, I or J. Options I and J are compound lotteries with 2 and 3 stages, respectively.

with economic incentives (i.e., 1-in-1 and 1-in-4 treatments). They found that under the hypothetical treatment more than 1 in 3 of the sample opted for the compound lotteries; this proportion was reduced in the 1-in-4 treatment to just over 1 in 5; and in the 1-in-1 treatment the proportion fell to 1 in 12. A chi-square test rejects ( $p$ -value  $< 0.01$ ) the hypothesis that there is no difference in patterns across treatments. The Fisher Exact test is consistent with this result.<sup>39</sup>

Cubitt et al. (1998a) use common consequence and common ratio pairs of pairs in three experiments. We focus in the first two since the third experiment has no treatments relevant to test ROCL. In the first experiment they compare 1-in-1 choices with 1-in-2 choices. Their comparison rests on subjects not having extreme risk-loving preferences over the pairs of lotteries in the 1-in-2 treatment designed to capture this behavior. Given that this *a priori* assumption is true, and it is generally supported by their data, the lottery pairs in each of the 1-in-2 treatments were chosen to generate compound prospects with actuarially-equivalent lotteries equal to the prospects in each of the 1-in-1 treatments. If ROCL is satisfied, the distribution of responses between risky and safe lotteries should be the same in both treatments. The  $p$ -value from the Fisher Exact test in one of the 1-in-1 and 1-in-2 treatment comparisons<sup>40</sup> is 0.14, which suggests that ROCL is most likely violated.<sup>41</sup>

Similarly, in the second experiment the 1-in-2 treatment induced compound lotteries with actuarially-equivalent prospects equal to the lottery choices in one of their 1-in-1 treatment. In the latter, 52% of the 46 subjects chose the risky lottery, whereas 38% of the 53 subjects in the 1-in-2 treatment chose the risky prospect. These choice patterns suggest that ROCL does not hold in the second experiment.<sup>42</sup>

---

<sup>39</sup>We test the similarity between treatments of the proportions of subjects that chose each of the four prospects in Question 4. The two-sided Fisher Exact test applied to the hypothetical and the 1-in-1 treatments rejects the hypothesis of no difference in choice patterns ( $p$ -value = 0.001). The  $p$ -value for the comparison of the same four choices between the hypothetical and the 1-in-4 treatments is 0.473.

<sup>40</sup>Groups 1.1 and 1.3 in their notation.

<sup>41</sup>The proportions of subjects that chose the risky prospect in the other 1-in-1 and 1-in-2 treatments (groups 1.2 and 1.4 in their notation) are close: 50% and 55%, respectively. However, we cannot perform the Fisher Exact test for this 1-in-1 and 1-in-2 comparison, since the compound lotteries induced by the 1-in-2 treatment have actuarially-equivalent prospects equal to the ones in the 1-in-1 treatment only if the subjects do not exhibit extreme risk-loving preferences. Since 8% of the subjects in this 1-in-2 treatment exhibited risk-loving preferences, one cannot perform the Fisher test because this contaminates the comparison between the compound lotteries and their actuarially-equivalent counterparts.

<sup>42</sup>Since one of the subjects in the 1-in-2 treatment (group 2.3) exhibited risk-loving preferences, we cannot perform the Fisher Exact test for the reasons explained earlier.

## 3.11 Appendix E. Nonparametric Tests

### *A. Choice Patterns Where ROCL Predicts Indifference*

*Research Hypothesis:* Subjects are indifferent between a compound (C) lottery and its paired actuarially-equivalent (AE) lottery, and therefore the choice between both lotteries is made at random in any of our 10 AE-C pairs in Table 3.13. As a result we should observe equiprobable response proportions between C and AE lotteries. ROCL is rejected if we can provide statistical evidence that the proportion of observations in which subjects chose a C lottery over its AE lottery is different from the proportion of choices in which subjects chose the AE lottery over the C lottery.

*Structure of data sets:* We analyze the observed responses from subjects who were presented with any of the 10 pairs described in Table 3.13 and which contain both a C lottery and its AE lottery. First, we study the responses from 32 subjects who were presented with one and only one of the AE-C pairs in 1-in-1 treatment. We also study the 620 responses from the 62 subjects that were presented with all of the 10 AE-C pairs in the 1-in-40 treatment. In terms of the statistical literature, the responses to each of the AE-C pairs in the 1-in-1 constitute an “independent” sample. This means that subjects are presented with one and only one choice, thus one observation does not affect any other observation in the sample. Conversely, the responses to the AE-C pairs in the 1-in-40 constitute 10 “dependent” samples since each of the 62 subjects responded to each of the 10 AE-C pairs.

We analyze the data separately because, in contrast to the 1-in-40 treatment which uses the random lottery incentive mechanism (RLIM) as payment protocol, any conclusion drawn from the 1-in-1 treatment does not depend in the independence axiom assumed by the RLIM. We want to control for the possibility that the observed choice patterns in the 1-in-40 treatment are affected by this payment protocol. This means that any failure to see indifference between a C lottery and its AE lottery in our data could be explained by confounds created by the payment protocol. By analyzing data from the 1-in-1 treatment only, we avoid possible confounds created by the RLIM.

#### 1-in-1 Treatment

We apply the Binomial probability test to each of the AE-C pair for which there is sufficient data to conduct the test. The latter allows us to test individually for each AE-C pair if subjects choose the C lottery and the AE lottery in equal proportions. We also use a generalized version of the Fisher Exact test that allows us to jointly test the statistical null hypothesis that the proportions of subjects that chose the C lottery over the AE lottery in each of the AE-C lottery pairs are the same. Both test can provide statistical evidence of the overall performance of the ROCL indifference prediction.

*Statistical Null Hypothesis of the Binomial Probability Test.* For a given AE-C lottery pair, the proportion of subjects that choose the C lottery is 50 %. This is equivalent to test the claim that subjects choose the AE lottery and the C lottery in equal proportions when they are presented with a given AE-C lotteries.

If this statistical null hypothesis is not rejected then we conclude that there is evidence to support the claim that, for a given AE-C pair, subjects choose the AE and the C lotteries in equal

proportions. If the null hypothesis of the test is rejected, then we concluded that subjects choose the AE and the C lotteries in different proportions. The rejection (acceptance) of the statistical null hypothesis implies that the research hypothesis is rejected (accepted) and we conclude that there is evidence to support the claim that the basic ROCL indifference prediction is violated (satisfied) in a given AE-C pair.

This is an appropriate test to use for this treatment because it allows us to test the equality of proportions implied by the ROCL prediction. More importantly, the basic assumptions of the test are satisfied by the data. As described by Sheskin (2004, p. 245), the assumptions are: (i) each of the  $n$  observations are independent (i.e., the outcome of one observation is not affected by the outcome of another observation), (ii) each observation is randomly selected from a population, and (iii) each observation can be classified only two into mutually exclusive categories. Assumption (i) is satisfied because each subject makes one and only one choice. Assumption (ii) is also satisfied because each subject is randomly recruited from the extensive subject pool of the EXCEN laboratory at Georgia State University. Finally (iii) is also satisfied because subjects can only choose either the C lottery or the AE lottery in each of the 10 AE-C pairs.

*Statistical Null Hypothesis of the Generalized Fisher Exact Test.* The proportion of individuals choosing the C lotteries is the same for all the AE-C lottery pairs.

The contingency table used in the test has the following structure:

Pair	Choice		Total
	AE Lottery	C Lottery	
AE-C pair 1	$n_{11}$	$n_{12}$	$n_{11} + n_{12}$
AE-C pair 2	$n_{21}$	$n_{22}$	$n_{21} + n_{22}$
.	.	.	.
AE-C pair $r$	$n_{r1}$	$n_{r2}$	$n_{r1} + n_{r2}$
Total	$\sum_{i=1,2,\dots,r} (n_{i1})$	$\sum_{i=1,2,\dots,r} (n_{i2})$	$\sum_{i=1,2,\dots,r} (n_{i1} + n_{i2})$

The number in each cell is defined as follows. The symbol  $n_{11}$  represents the number of individuals that chose the AE lottery when they were presented with the AE-C lottery pair 1. The symbol  $n_{12}$  represents the number of individuals that chose the C lottery when they were presented with the same AE-C pair 1. The sum  $n_{11} + n_{12}$  represents the total number of subjects that were presented with the AE-C lottery pair 1. The interpretation of  $n_{i1}$  and  $n_{i2}$ , for  $i=2, 3, \dots, r$ , can be similarly derived. The generalized Fisher Exact test tests the null hypothesis that the proportion of subjects that choose the C lottery is statistically the same for all of the  $r$  AE-C lottery pairs used in the table. Formally, the statistical null hypothesis is  $H_0: p_1 = p_2 = \dots = p_r$ , where  $p_i = n_{i2} / (n_{i1} + n_{i2})$  for  $i=1, 2, 3, \dots, r$ .

We use this test in conjunction with the Binomial probability test applied individually to each of the AE-C lottery pairs to make stronger claims of the overall performance of ROCL. If the

Binomial probability test does not reject its statistical null hypothesis for each of the AE-C lottery pairs and if the statistical null hypothesis of the generalized Fisher Exact test is not rejected, we can conclude that the proportion of subjects that chose the C lottery is statistically the same for all AE-C and therefore the ROCL indifference prediction is supported. If we can reject the statistical null hypothesis of the generalized Fisher Exact test, we can conclude that at least in two of the AE-C pairs the proportions of subjects that chose the C lottery are different. For example, suppose there were only 2 AE-C lottery pairs. If the Fisher Exact test rejects the null hypothesis, we conclude that the proportions of subjects choosing the C lottery are not the same in the two lottery pairs. Therefore, even if one of the proportions was equal to 50%, as ROCL predicts for any given AE-C lottery pair, the rejection of the statistical null would imply that the other proportion is not statistically equal to 50%. Consequently, we would reject the research hypothesis that subjects satisfy ROCL and choose at random between the AE and the C lottery in all of the AE-C lottery pairs.

The generalized Fisher Exact test is appropriate to test the joint hypothesis that the proportion of subjects that chose the C lottery is the same in all of the AE-C lottery pairs. The basic assumptions of the test are satisfied by the data. As described by Sheskin (2004, p. 424 and 506), the assumptions are: (i) each of the  $n$  observations are independent (i.e., the outcome of one observation is not affected by the outcome of another observation), (ii) each observation is randomly selected from a population, (iii) each observation can be classified only into mutually exclusive categories, (iv) the Fisher Exact test is recommended when the size of the sample is small, and (v) many sources note that an additional assumption is that the sum of the rows and columns of the contingency table used in the Fisher Exact test are predetermined by the researcher. Assumptions (i)-(iii) are satisfied for the same reasons we explained in the Binomial probability test. The Fisher Exact test is commonly used for small samples like ours instead of the Chi-square test of homogeneity, which relies on large samples to work appropriately. Finally, the last assumption is not met by our data, however, Sheskin (2004, p. 506) claim that this is rarely met in practice and, consequently, the test is used in contingency tables when “one or neither of the marginal sums is determined by the researcher.”

#### 1-in-40 Treatment

The strategy to test the basic prediction of indifference in the “Pay-1-in-40 compound is different from the one used in the “Pay-1-in-1-compound.” The reason is that the structure of the data in each of the treatments is different. In the case of the 1-in-1 treatment, each of the 10 AE-C lottery pairs generated, using the terminology of the statistical literature, an “independent” sample in the sense that there was no subject that made choices over more than one AE-C lottery pair. On the contrary, in the 1-in-40 treatment we have multiple “dependent” samples. This means that several subjects made choices over each of our 10 AE-C lottery pairs, and therefore, we obtain 10 “dependent” samples. This subtle difference has relevant implications for the type of nonparametric test that one should use to test any hypothesis with the structure of the data we described. We use the Cochran Q test to test the basic ROCL prediction of indifference in the 1-in-40 treatment.

*Statistical Null Hypothesis of the Cochran Q Test.* The proportion of subjects that choose the C is the same in each of the 10 AE-C lottery pairs.

The information needed to perform this test is captured in a table of the following type:

	AE-C pair 1	AE-C pair 2	.	AE-C pair 9	AE-C pair 10
Subject 1	$c_{1,1}$	$c_{1,2}$	.	$c_{1,9}$	$c_{1,10}$
Subject 2	$c_{2,1}$	$c_{2,2}$	.	$c_{2,9}$	$c_{2,10}$
.	.	.	.	.	.
Subject n	$c_{n,1}$	$c_{n,2}$	.	$c_{n,9}$	$c_{n,10}$
Total	$\sum_{i=1,2,\dots,n} (c_{i,1})$	$\sum_{i=1,2,\dots,n} (c_{i,2})$	.	$\sum_{i=1,2,\dots,n} (c_{i,9})$	$\sum_{i=1,2,\dots,n} (c_{i,10})$

The number in each cell is defined as follows. The symbol  $c_{i,j}$  is a dichotomous variable that can be either 0 or 1 and records the choice that was made when of subject  $i$  was presented with the AE-C lottery pairs. If subject  $i$  chooses the AE lottery,  $c_{i,j}$  is equal to 0; but if the subject chooses the C lottery instead,  $c_{i,j}$  is equal to 1. The symbol  $c_{i,j}$  represents the number of individuals that chose the AE lottery when they were presented with the AE-C lottery pair  $j$ . The symbols  $c_{1,2}, c_{1,3}, \dots$  and  $c_{1,10}$  are similarly defined and record subject 1's choices in AE-C pairs 2 through 10. Similarly, the symbols  $c_{i,1}, c_{i,2}, \dots$  and  $c_{i,10}$  record the choices that subject  $i$  made in each of the 10 AE-C lottery pairs. The sum  $\sum_{i=1,2,\dots,n} (c_{i,k})$  represents the total number of subjects, out of the  $n$  subjects, that chose the C lottery when they were presented with the AE-C lottery pair  $k$ . The Cochran Q test tests if the proportion of subjects that chose the C lottery in each of the 10 AE-C lottery pairs are the same. Thus the statistical null hypothesis is  $H_0: p_1 = p_2 = \dots = p_{10}$ , where  $p_i = \sum_{i=1,2,\dots,n} (c_{i,k}) / n$  for  $i=1, 2, 3, \dots, 10$ . The actual statistic of the Cochran Q test involves information of these proportions, as well as information per subject.

This joint hypothesis is enough to reject the indifference prediction. For example, suppose there were only 2 AE-C lottery pairs. If the Cochran Q test rejects the null hypothesis, we conclude that the proportions of subjects choosing the C lottery are not the same in the two lottery pairs. Therefore, even if one of the proportions is equal to 50% as ROCL predicted, the test provides evidence that the other proportion is not equal to 50%. Consequently, we would reject the research hypothesis that subjects satisfy ROCL and choose at random between the AE and the C lottery in any of the AE-C lottery pairs. In the text we provide confidence intervals on the number of subjects (out of the 62 in our sample) that chose the S lottery in each of the AE-C lottery. If for a given AE-C lottery pair the number 31 is not contained in the confidence interval, it implies that with 95% probability the proportion of the 62 subjects that chose the C lottery will not be 50%. We could have apply the Binomial probability test to each of the 10 AE-C pairs in the 1-in-40 treatment. However, this would not be appropriate since the Binomial test assumes "independence" in the sample in the statistical sense which is not satisfied in the present treatment.

The Cochran Q test is an appropriate test in this treatment because it allows us to jointly reject the null hypothesis that subjects choose the C lottery and the AE lottery in equal proportions when the data set is composed by multiple dependent samples. The basic assumptions of the test are satisfied by the data. As described by Sheskin (2004, p. 245), the assumptions are: (i) each of the subjects respond to each of the 10 AE-C lottery pairs, (ii) one has to control for order effects and (iii) each observation can be classified only into mutually exclusive categories. Assumption (i) is satisfied since in the 1-in-40 treatment each subjects respond to all 40 lottery pairs, which include the 10 AE-C pairs. Assumption (ii) is also satisfied because in our experiments each subject is presented with the 40 lotteries in random order. Assumption (iii) is trivially satisfied since in each of the AE-C

### *B. Choice Patterns Where ROCL Predicts Consistent Choices*

*Research Null Hypothesis:* Subjects choose the S lottery when presented with the S-C lottery pair if and only if they also choose the S lottery when presented with the corresponding S-AE lottery pair. This is equivalent to state the null hypothesis using the C and AE lotteries but we chose to work with the S lottery for simplicity. Therefore, ROCL is satisfied if we can provide statistical evidence that the proportion of subjects that choose the S lottery when presented with a S-C pair is equal to the proportion of subjects that choose also the S lottery when presented with the paired S-AE pair.

*Structure of data sets:* We use data from the 62 subjects in the 1-in-40 treatment who were presented with each of the 30 lottery pairs in Tables 3.11 and 3.12. Each of the 15 S-C lottery pairs in Table 3.11 has a corresponding S-AE pair in Table 3.12. Therefore, we can construct 15 comparisons of pairs that constitute 15 consistency tests of ROCL. In the 1-in-40 we again have to assume that the independence axiom holds. Therefore, we also use data from the 1-in-1 treatment to control for possible confounds created by the RLIM. However, we have to assume homogeneity in risk preferences for the analysis of this particular treatment. The reason is that the response of any subject to a particular S-C lottery pair, is going to be compared with the response of another subject to the paired S-AE lottery pair. In terms of the statistical literature, the responses to each of the S-C or S-AE pairs in the 1-in-1 constitute an “independent” sample. Conversely, the responses to each of the S-C or S-AE pairs in the 1-in-40 constitute 30 “dependent” samples since each of the 62 subjects responded to each of the 15 S-C and the 15 S-AE. Also, each of the 15 comparisons is constructed by matching a S-C pair with its corresponding S-AE pair.

#### Analysis of data from the 1-in-1 treatment

We use the Fisher Exact test to evaluate the consistency predicted by ROCL in each of the paired comparisons of S-C pairs and S-AE pairs for which we have enough data to conduct the test. We also use the Cochran-Mantel-Haenszel (CMH) as a joint test of the 15 paired comparisons to evaluate the overall performance of the ROCL consistency prediction.

*Statistical Null Hypothesis of the Fisher Exact Test:* For any given paired comparison, subjects choose the S lottery in the same proportion when presented with a S-C pair and with its corresponding S-AE lottery pair.

The table shows statistical tests on contingency tables of the following form:

	Choice		Total
	S	AE/C	
S-AE pair	$a$	$b$	$a+b$
S-C pair	$c$	$d$	$c+d$
Total	$a+c$	$b+d$	$a+b+c+d$

The positions in each cell are defined as follows. The letter  $a$  represents the number of individuals that chose the simple lottery when they were presented with a S-AE pair. The letter  $c$  represents the number of individuals that chose the simple lottery when they were presented with the corresponding S-C pair. The letter  $b$  represents the number of subjects that chose the AE lottery when they were presented with the S-AE lottery pair. Similarly, the letter  $d$  represents the number of subjects that chose the C lottery when they were presented with the corresponding S-C lottery pair. In the notation of the previous note, the proportions used in the Fisher Exact test are defined as  $\pi_1 = a/(a+b)$  and  $\pi_2 = c/(c+d)$ .

The Fisher Exact test for  $2 \times 2$  contingency tables is appropriate to test individually in each of the 15 matched pairs the hypothesis that the proportion of subjects that chose the S lottery is the same when they are presented with the S-C or its corresponding S-AE pair. The basic assumptions of the test are satisfied by the data. As described by Sheskin (2004, p. 424 and 506), the assumptions are: (i) each of the  $n$  observations are independent (i.e., the outcome of one observation is not affected by the outcome of another observation), (ii) each observation is randomly selected from a population, (iii) each observation can be classified only into mutually exclusive categories, (iv) the Fisher Exact test is recommended when the size of the sample is small, and (v) many sources note that an additional assumption is that the sum of the rows and columns of the contingency table used in the Fisher Exact test are predetermined by the researcher. Assumptions (i)-(iv) are satisfied for the same reasons we explained in the in the case of the generalized Fisher exact test. Finally, as we explained before assumption (v) is not satisfied in our data.

*Statistical Null Hypothesis of the Cochran-Mantel-Haenszel test:* In all of the 15 paired comparisons subjects choose in the same proportion the S lottery when presented with the S-C lottery pair and its paired S-AE lottery pairs. More formally, the odds ratio of each of the 15 contingency tables constructed from the 15 paired comparisons are jointly equal to 1.

If the CMH test rejects the null hypothesis, then we interpret this as evidence of ROCL-inconsistent observed behavior. However, if we cannot reject the null, we conclude that subjects make choices according to the ROCL consistency predictions in the 15 pair comparisons even if we find that the Fisher Exact tests rejects its null hypothesis for some of the paired comparisons.

The CMH is the appropriate joint test to apply since it allows us to pool the data of multiple contingency tables that satisfy the assumptions needed for the Fisher Exact test and test jointly the homogeneity of the tables.

#### Analysis of data from the 1-in-40 treatment

We use the Cochran Q test coupled with the Bonferroni-Dunn (B-D) method to test the statistical hypothesis that subjects choose the S lottery in the same proportion when presented with a S-C lottery pair and with the corresponding S-AE lottery pair. The B-D method allows us to test if, in each of the 15 paired comparisons of S-C and S-AE lottery pairs, the observed difference in the proportion of subjects that chose the S lottery is statistically significant.

The B-D method is a *post-hoc* procedure that is conducted after calculating the Cochran Q test. The first step is to conduct the latter test to reject or not reject the null hypothesis that the proportions of individuals that choose the S lottery are the same in all 15 S-C and 15 S-AE lottery pairs. If this null is rejected the B-D method involves calculating a critical value  $d$  (see Sheskin, 2004, p. 871 for the definition) that allows to evaluate the statistical significance of the difference in proportions and that takes into account all the information of the 30 lottery pairs and a confidence level of  $\alpha$ .

*Statistical Null Hypothesis in each of the Pair Comparisons using the B-D method:* Define  $p_1$  as the proportion of subjects that choose the S lottery when presented with a given S-AE lottery pair. Similarly, define  $p_2$  as the proportion of subjects that chose the S lottery in the paired S-C lottery pair. The statistical null hypothesis is that, for a given paired comparison,  $p_1 = p_2$ .

The B-D method rejects the statistical null hypothesis if  $|p_1 - p_2| > d$ . In this case we would conclude that the observed difference in proportions in a given paired comparison is statistically significant. This is a more powerful test than conducting individual tests for each paired comparison because the critical value  $d$  takes into account the information of all of the 15 comparisons. See Sheskin (2004, p. 871) for further details of the B-D method.

The Cochran Q test coupled with the B-D method are appropriate to test in this treatment the null hypothesis that subjects choose the S lottery in the same proportion when presented with a given S-C pair and with its corresponding S-AE lottery, and the data set is composed by multiple dependent samples in the sense we explained above. The basic assumptions of the Cochran Q test are satisfied by the data. As described by Sheskin (2004, p. 245), the assumptions are: (i) each of the subjects respond to each of the 15 S-C lottery pairs and the 15 S-AE lottery pairs, (ii) one has to control for order effects and (iii) each observation can be classified only into mutually exclusive categories. Assumption (i) is satisfied since in the 1-in-40 treatment each subjects respond to all 40 lottery pairs, which include the 15 S-C pairs and the 15 S-AE pairs. Assumption (ii) is also satisfied because in our experiments each subject is presented with the 40 lotteries in random order. Assumption (iii) is trivially satisfied since in each of the AE-C pairs the subjects have to make a dichotomous choice. The B-D method applied to the Cochran Q test does not require any extra assumptions. However, the calculation of the critical value to make the comparisons requires to define a family wise Type I error rate ( $\alpha_{FW}$ ). Sheskin (2004, p. 871) claims that “[w]hen a limited number of comparisons are planned prior to collecting the data, most sources take the position that a researcher is not obliged to control the value of  $\alpha_{FW}$ . In such a case, the per comparison Type I error rate ( $\alpha_{PC}$ ) will be equal to the prespecified value of alpha [the confidence level].”

We are also interested in studying the patterns in the violation of ROCL. We want to test the statistical validity of differences in switching behavior. A pattern inconsistent with ROCL would be

subjects choosing the S lottery when presented with a given S-C lottery pair, but switching to prefer the AE lottery when presented with the matched S-AE pair. We construct  $2 \times 2$  contingency tables that show the number of subjects in any given matched pair who exhibit each of the four possible choice patterns: (i) always choosing the S lottery; (ii) choosing the S lottery when presented with a S-C pair and switching to prefer the AE lottery when presented with the matched S-AE pair; (iii) choosing the C lottery when presented with a S-C pair and switching to prefer the S lottery when presented with the matched S-AE; and (iv) choosing the C lottery when presented with the S-C lottery and preferring the AE lottery when presented with the matched S-AE. We use the McNemar test to evaluate the statistical significance of patterns in the violations of ROCL.

*Statistical Null Hypothesis of the McNemar Test.* Subjects exhibit the discordant choice patterns (ii) and (iii) in equal proportions within each set of matched pairs.

If the statistical null hypothesis is rejected then we can claim that there is an statistical difference in the two possible patterns of switching behavior that violate ROCL.

The test requires to construct a contingency tables of the following form:

		Simple Lottery vs. Actuarially-Equivalent		
		Left Lottery	Right Lottery	Total
Simple Lottery vs. Compound Lottery	Left lottery	$a$	$b$	$a+b$
	Right lottery	$c$	$d$	$c+d$
	Total	$a+c$	$b+d$	$a+b+c+d$

The positions in each cell are defined as follows. The letter  $a$  represents the number of individuals that chose the left lottery both when they were presented with a pair of a simple lottery and a compound lottery (S-C) and a corresponding pair that has the same simple lottery and the actuarially-equivalent lottery (S-AE) of the compound lottery. The simple lotteries, and therefore the compound and their actuarially equivalent lotteries, are always in the same position across. For the purpose of the statistical tests, the simple lotteries are always the left lotteries; the compound and their actuarially-equivalent lotteries are always the right lotteries. Therefore,  $a$  is the number of individuals that chose the simple lottery when they were presented with a given pair of S-C and its corresponding pair of S-AE. The letter  $c$  represents the number of individuals that chose the simple lottery when they were presented with a given pair of S-AE but that chose the compound lottery when they were presented with corresponding pair of S-C.

The McNemar test is an appropriate test to apply in this context. The assumptions of the test are (See Sheskin, 2004, p. 634): (i) the sample of  $n$  subjects has been randomly selected from the population, (ii) each of the  $n$  observations in the contingency table is independent of other

observations, (iii) each observation can be classified only into mutually exclusive categories, and (iv) the test should not be used with extremely small samples. Assumptions (i) and (iii) are satisfied for reasons we explained before. Even though there is no agreement of what an small sample is for the McNemar test, we follow the recommendation of the literature and provide in our results the exact probability of the test. Assumption (ii) is not satisfied since each subject respond makes choice over more than one pair. However, the test still allows us to draw conclusions about the discordant switching patterns but does not allow to make causal inferences, which is enough for our purposes. In fact it will allows us to conclude if there is an statistical difference between the two possible choice patterns that contradict ROCL Nevertheless, it will not allow us to conclude anything about the source of this difference (see Sheskin, 2004, p. 639)].

## 3.12 Appendix F. Additional Econometric Analysis

### 3.12.1 Analysis of Data from the 1-in-1 Treatment

Assuming homogeneity in preferences, we find that the model that best describes the data is the one that allows for the source-dependent version of RDU, and conditional on this model there is no evidence of violations of ROCL. Both the Vuong test and the Clarke test provide statistical evidence that the model that allows for the source-dependent version of the RDU is the best model to explain the data in the 1-in-1 treatment.<sup>43</sup> Table 3.14 shows the parameter estimates of the two models we consider. In particular, panel A shows the estimates for the model that allows for the source-dependent version of RDU. We find that the estimates for parameters  $r$ ,  $rc$ ,  $\gamma$  and  $\gamma c$  are 0.62, 0.14, 0.77 and -0.19, respectively. A test of the joint null hypothesis that  $rc = \gamma c = 0$  results in a  $p$ -value of .29. This implies that there is no statistical evidence for source-dependency both in the utility and the probability weighting functions, and therefore no evidence of violations of ROCL when homogeneity is assumed. If we had assumed that subjects behave according to the sourcedependent EUT, we would have incorrectly concluded that there is evidence of violations of ROCL as suggested by the estimate of  $rc$  equal to 0.27 which has a  $p$ -value of 0.008 (see panel B of Table 3.14). A joint test of  $r$  and  $rc$  results in a  $p$ -value less than 0.0001. This highlights the importance of choosing the preference representation that best characterizes the way in which individuals make choices.

Although the model that best characterizes behavior in the 1-in-1 treatment is the sourcedependent version of the RDU model, there is evidence of marginal diminishing returns but no evidence of probability weighting.<sup>44</sup> The parameter estimates for  $r$  and  $\gamma$  are equal to 0.62 and 0.77. Figure 3.15 shows the functions implied by these estimates and are plotted in the relevant domains. A test for the hypothesis that  $\gamma = 1$  results in a  $p$ -value of 0.23, which provides no evidence of probability weighting.

### 3.12.2 Analysis of Data from the 1-in-40 Treatment

Assuming homogeneity, we find that the model that best describes the data is the one that allows for the source-dependence version of RDU, and conditional on this model we find evidence of violations of ROCL. Both the Vuong test and the Clarke test provide evidence to support the source-dependent RDU as the best model to explain the data in the 1-in-40 treatment.<sup>45</sup> An statistical test for the joint null hypothesis that  $rc = \gamma c = 0$  results in a  $p$ -value less than 0.0001.

---

<sup>43</sup>When we assume homogeneity in risk attitudes, the Vuong test statistic is -1.45 in favor of the source-dependent RDU, with a  $p$ -value of .073. Further, the Clarke test also gives evidence in favor of the source-dependent RDU with a test statistic equal to 44.

<sup>44</sup>The Vuong and the Clarke tests provide evidence to choose the model that bests characterize data between two models but are agnostic about the statistical significance of the winning model.

<sup>45</sup>When we assume homogeneity in risk attitudes, the Vuong test statistic is -5.16 in favor of the source-dependent RDU, with a  $p$ -value less than 0.001. Further, the Clarke test also gives evidence in favor of the source-dependent RDU with a test statistic equal to 935.

Table 3.14: Estimates of Source-Dependent RDU and EUT Model Allowing for Heterogeneity

Data from the 1-in-1 treatment (N=133). Estimates from the Fechner error parameter omitted

A. Source-Dependent RDU(LL=-81.82)

	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
r	.6182909	.0669022	9.24	0.000	.4871649	.7494169
r_rocl	.1410226	.0922965	1.53	0.127	-.0398752	.3219204
$\gamma$	.7699848	.193429	3.98	0.000	.3908708	1.149099
$\gamma c$	-.1955013	.1819329	-1.07	0.283	-.5520832	.1610806

(Ho:  $rc = \gamma c = 0$ ;  $p$ -value = 0.2848)  
(Ho:  $r = \gamma = 0$ ;  $p$ -value < 0.0001)

B. Source-Dependent Version of EUT (LL=-84.24)

	Coef.	Robust Std. Err.	z	P> z	[95% Conf. Interval]	
r	.5799532	.0731694	7.93	0.000	.4365438	.7233626
rc	.2717936	.1028005	2.64	0.008	.0703083	.4732789

(Ho:  $r=rc=0$ ;  $p$ -value < 0.0001)

Table 3.15 shows the estimates for  $r$ ,  $rc$ ,  $\gamma$  and  $\gamma c$  are 0.57, 0.11, 1.09 and -0.40, respectively. This implies that the nature of the violation has two components. First, the estimates suggests that when a typical individual is presented with a compound lottery he behaves as if the utility function was more concave. The linear combination of parameters  $r + rc$  results in an estimated coefficient of 0.68 with a  $p$ -value less than 0.0001 and a test of the null hypothesis of  $r + rc = 0.57$  results in a  $p$ -value equal to 0.0001. Thus a typical subject would increase his utility risk aversion parameter from 0.57 to 0.68 when presented with a compound lottery. Second, there is no evidence of probability weighting when subjects are presented with simple lotteries but there is evidence of probability optimism when subjects evaluate compound lotteries. A test on the probability weighting parameter for  $\gamma = 1$  results in a  $p$ -value equal to 0.28 and the linear combination  $\gamma + \gamma c$  results in an estimated parameter equal to 0.69 with a  $p$ -value less than 0.001. Hence, when presented with a simple lottery a typical subject displays no probability weighting but does exhibits diminishing marginal returns; however, when facing a compound lottery, a typical subject behaves as if the utility function was more concave and the probability weighting function

displays probability optimism. Figure 3.16 shows how the concavity of the utility function and the probability weighting function differ when individuals are presented with a simple or a compound lottery.

Table 3.15: Estimates of Source-Dependent RDU and EUT Model Allowing for Heterogeneity

Data from the 1-in-40 treatment (N=2480= 62 Subjects  $\times$  40 choices). Estimates from the Fechner error parameter omitted

A. Source-Dependent RDU (LL=-1460.57)

	Coef.	Robust Std. Err.	z	P> z	[95% Conf. Interval]	
r	.5670508	.024286	23.35	0.000	.519451	.6146505
rc	.1090157	.0231279	4.71	0.000	.0636858	.1543456
$\gamma$	1.088315	.081601	13.34	0.000	.9283802	1.24825
$\gamma c$	-.3957716	.0587485	-6.74	0.000	-.5109165	-.2806266

(Ho: rc =  $\gamma c$  = 0; p-value<0.0001)

(Ho: r =  $\gamma$  = 0; p-value=0.0001)

B. Version of EUT (LL=-1512.45)

	Coef.	Robust Std. Err.	z	P> z	[95% Conf. Interval]	
r	.5906296	.0352857	16.74	0.000	.5214708	.6597883
rc	-.0071859	.0194071	-0.37	0.711	-.0452233	.0308514

Figure 3.15: Estimated Functions from the RDU Specification in the 1-in-1 Treatment Assuming Homogeneity in Preferences

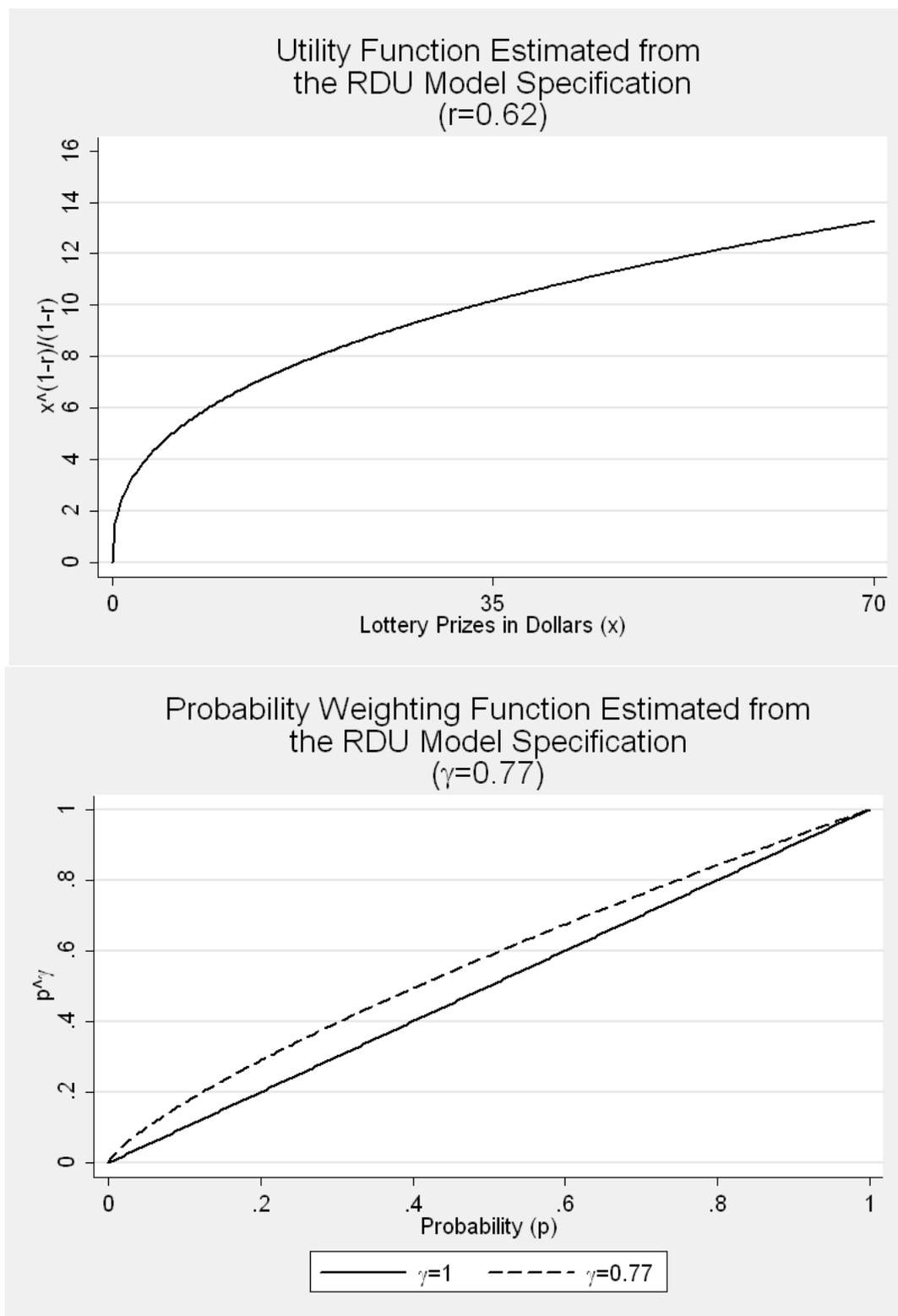
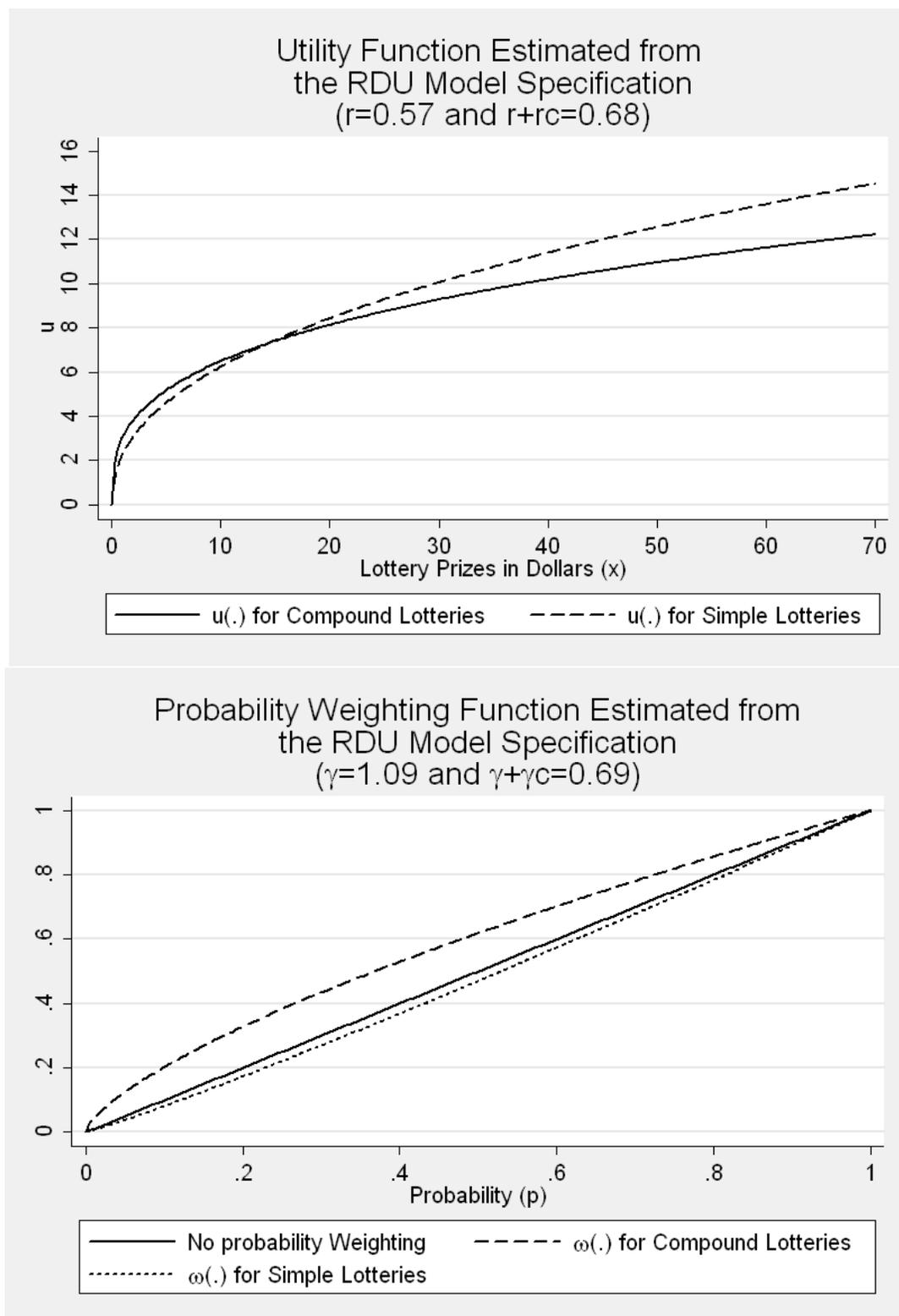


Figure 3.16: Estimated Functions from the RDU Specification in the 1-in-1 Treatment Assuming Homogeneity in Preferences



### 3.13 Appendix G. Detailed Binomial Test Results

The following tests are conducted over the actuarially-equivalent and compound lottery pairs in Table 3.13 (Pairs 31-40).

#### 3.13.1 Binomial tests for treatment 1-in-1

We do not report the test for pairs 31, 35 and 35 because for the first one there was only one observation and for the last two there were no observations at all.

Note: In the following Binomial tests “N” is the total size of the sample for each lottery pair and “Observed k” is the number of subjects (out of the N subjects) that chose the compound lottery.

##### Pair 32

Variable	N	Observed k	Expected k	Assumed p	Observed p
choice_aed-2	3	3	1.5	0.50000	1.00000
Pr(k >= 3)	= 0.125000	(one-sided test)			
Pr(k <= 3)	= 1.000000	(one-sided test)			
Pr(k <= 0 or k >= 3)	= 0.250000	(two-sided test)			

##### Pair 33

Variable	N	Observed k	Expected k	Assumed p	Observed p
choice_aed-3	7	5	3.5	0.50000	0.71429
Pr(k >= 5)	= 0.226563	(one-sided test)			
Pr(k <= 5)	= 0.937500	(one-sided test)			
Pr(k <= 2 or k >= 5)	= 0.453125	(two-sided test)			

##### Pair 36

Variable	N	Observed k	Expected k	Assumed p	Observed p
choice_aed-6	5	1	2.5	0.50000	0.20000
Pr(k >= 1)	= 0.968750	(one-sided test)			
Pr(k <= 1)	= 0.187500	(one-sided test)			
Pr(k <= 1 or k >= 4)	= 0.375000	(two-sided test)			

##### Pair 37

Variable	N	Observed k	Expected k	Assumed p	Observed p
choice_aed-7	2	1	1	0.50000	0.50000
Pr(k >= 1)	= 0.750000	(one-sided test)			
Pr(k <= 1)	= 0.750000	(one-sided test)			
Pr(k <= 1 or k >= 1)	= 1.000000	(two-sided test)			

Pair 38

Variable	N	Observed k	Expected k	Assumed p	Observed p
choice_aed~8	3	1	1.5	0.50000	0.33333
Pr(k >= 1)	=	0.875000	(one-sided test)		
Pr(k <= 1)	=	0.500000	(one-sided test)		
Pr(k <= 1 or k >= 2)	=	1.000000	(two-sided test)		

Pair 39

Variable	N	Observed k	Expected k	Assumed p	Observed p
choice_aed~9	5	4	2.5	0.50000	0.80000
Pr(k >= 4)	=	0.187500	(one-sided test)		
Pr(k <= 4)	=	0.968750	(one-sided test)		
Pr(k <= 1 or k >= 4)	=	0.375000	(two-sided test)		

Pair 40

Variable	N	Observed k	Expected k	Assumed p	Observed p
choice_ae~10	6	3	3	0.50000	0.50000
Pr(k >= 3)	=	0.656250	(one-sided test)		
Pr(k <= 3)	=	0.656250	(one-sided test)		
Pr(k <= 3 or k >= 3)	=	1.000000	(two-sided test)		

### 3.14 Appendix H. Detailed Fisher Exact Test Results

The following tests are conducted by matching each of the 15 simple and compound lottery pairs in Table 3.13 (Pairs 1-15) with a corresponding pair in the set of the 15 simple and actuarially-equivalent lottery pairs in Table 3.13 (Pairs 16-30).

#### Pair 1 vs. Pair 16

Pairs	Choice		Total
	S	C/AE	
S-AE Pair 16	3	0	3
S-C Pair 1	1	1	2
Total	4	1	5

Fisher's exact = 0.400  
1-sided Fisher's exact = 0.400

#### Pair 3 vs. Pair 18

Pairs	Choice		Total
	S	C/AE	
S-AE Pair 18	3	3	6
S-C Pair 3	0	2	2
Total	3	5	8

Fisher's exact = 0.464  
1-sided Fisher's exact = 0.357

#### Pair 5 vs. Pair 20

Pairs	Choice		Total
	S	C/AE	
S-AE Pair 18	0	1	1
S-C Pair 5	2	0	2
Total	2	1	3

Fisher's exact = 0.333  
1-sided Fisher's exact = 0.333

Pair 6 vs. Pair 21

Pairs	Choice		Total
	S	C/AE	
S-AE Pair 21	2	1	3
S-C Pair 6	2	2	4
Total	4	3	7

Fisher's exact = 1.000  
 1-sided Fisher's exact = 0.629

Pair 7 vs. Pair 22

Pairs	Choice		Total
	S	C/AE	
S-AE Pair 22	4	0	4
S-C Pair 7	5	4	9
Total	9	4	13

Fisher's exact = 0.228  
 1-sided Fisher's exact = 0.176

Pair 8 vs. Pair 23

Pairs	Choice		Total
	S	C/AE	
S-AE Pair 23	1	2	3
S-C Pair 8	2	2	4
Total	3	4	7

Fisher's exact = 1.000  
 1-sided Fisher's exact = 0.629

Pair 9 vs. Pair 24

Pairs	Choice		Total
	S	C/AE	
S-AE Pair 24	0	3	3
S-C Pair 9	5	1	6
Total	5	4	9

Fisher's exact = 0.048  
 1-sided Fisher's exact = 0.048

Pair 11 vs. Pair 26

Pairs	Choice		Total
	S	C/AE	
S-AE Pair 26	3	2	5
S-C Pair 11	5	4	9
Total	8	6	14

Fisher's exact = 1.000  
 1-sided Fisher's exact = 0.657

Pair 12 vs. Pair 27

Pairs	Choice		Total
	S	C/AE	
S-AE Pair 27	4	1	5
S-C Pair 11	2	0	2
Total	6	1	7

Fisher's exact = 1.000  
 1-sided Fisher's exact = 0.714

Pair 13 vs. Pair 28

Pairs	Choice		Total
	S	C/AE	
S-AE Pair 28	2	2	4
S-C Pair 13	1	0	1
Total	3	2	5

Fisher's exact = 1.000  
 1-sided Fisher's exact = 0.600

Pair 15 vs. Pair 30

Pairs	Choice		Total
	S	C/AE	
S-AE Pair 30	3	0	3
S-C Pair 15	0	1	1
Total	3	1	4

Fisher's exact = 0.250  
 1-sided Fisher's exact = 0.250

### 3.15 Appendix I. Detailed McNemar Test Results

#### All Simple-Compound pairs vs. all Simple-Actuarially-Equivalent pairs

Cases	Controls Exposed	Unexposed	Total
Exposed	447	97	544
Unexposed	182	204	386
Total	629	301	930

McNemar's chi2(1) = 25.90 Prob > chi2 = 0.0000  
 Exact McNemar significance probability = 0.0000

#### Proportion with factor

Cases	.5849462			
Controls	.6763441	[95% Conf. Interval]		
difference	-.0913978	-.1271815	-.0556142	
ratio	.8648649	.8177913	.9146481	
rel. diff.	-.282392	-.405559	-.1592251	
odds ratio	.532967	.4121939	.6855842	(exact)

Note: the contingency table above was constructed by summing each of the positions in the 15 contingency tables below. For example, the number in the first position (column 1, row 1) in the table above was calculated by summing the numbers in the first position in each of the 15 contingency tables below.

#### Pair 1 vs. Pair 16

Cases	Controls Exposed	Unexposed	Total
Exposed	22	2	24
Unexposed	32	6	38
Total	54	8	62

McNemar's chi2(1) = 26.47 Prob > chi2 = 0.0000  
 Exact McNemar significance probability = 0.0000

#### Proportion with factor

Cases	.3870968			
Controls	.8709677	[95% Conf. Interval]		
difference	-.483871	-.6395385	-.3282034	
ratio	.4444444	.3235546	.6105024	
rel. diff.	-3.75	-6.863468	-.6365323	
odds ratio	.0625	.0072572	.2449784	(exact)

Pair 2 vs. Pair 17

Cases	Controls		Total
	Exposed	Unexposed	
Exposed	58	1	59
Unexposed	3	0	3
Total	61	1	62

McNemar's chi2(1) = 1.00 Prob > chi2 = 0.3173  
 Exact McNemar significance probability = 0.6250

## Proportion with factor

Cases	.9516129		
Controls	.983871	[95% Conf. Interval]	
difference	-.0322581	-.1110998	.0465837
ratio	.9672131	.9060347	1.032522
rel. diff.	-2	-8.789514	4.789514
odds ratio	.3333333	.0063495	4.151441 (exact)

Pair 3 vs. Pair 18

Cases	Controls		Total
	Exposed	Unexposed	
Exposed	38	1	39
Unexposed	17	6	23
Total	55	7	62

McNemar's chi2(1) = 14.22 Prob > chi2 = 0.0002  
 Exact McNemar significance probability = 0.0001

## Proportion with factor

Cases	.6290323		
Controls	.8870968	[95% Conf. Interval]	
difference	-.2580645	-.3919297	-.1241993
ratio	.7090909	.5925527	.8485489
rel. diff.	-2.285714	-4.438998	-.1324308
odds ratio	.0588235	.0014075	.3754091 (exact)

Pair 4 vs. Pair 19

Cases	Controls		Total
	Exposed	Unexposed	
Exposed	7	6	13
Unexposed	7	42	49
Total	14	48	62

McNemar's chi2(1) = 0.08 Prob > chi2 = 0.7815  
 Exact McNemar significance probability = 1.0000

## Proportion with factor

Cases	.2096774		
Controls	.2258065	[95% Conf. Interval]	
difference	-.016129	-.1461672	.1139091
ratio	.9285714	.5499486	1.567864
rel. diff.	-.0208333	-.169583	.1279163
odds ratio	.8571429	.2379799	2.978588 (exact)

Pair 5 vs. Pair 20

Cases	Controls		Total
	Exposed	Unexposed	
Exposed	12	6	18
Unexposed	13	31	44
Total	25	37	62

McNemar's chi2(1) = 2.58 Prob > chi2 = 0.1083  
 Exact McNemar significance probability = 0.1671

## Proportion with factor

Cases	.2903226		
Controls	.4032258	[95% Conf. Interval]	
difference	-.1129032	-.2639309	.0381244
ratio	.72	.4813126	1.077055
rel. diff.	-.1891892	-.4409851	.0626067
odds ratio	.4615385	.1438515	1.301504 (exact)

Pair 6 vs. Pair 21

Cases	Controls		Total
	Exposed	Unexposed	
Exposed	35	6	41
Unexposed	11	10	21
Total	46	16	62

McNemar's chi2(1) = 1.47 Prob > chi2 = 0.2253  
 Exact McNemar significance probability = 0.3323

## Proportion with factor

Cases	.6612903			
Controls	.7419355	[95% Conf. Interval]		
difference	-.0806452	-.2255601	.0642697	
ratio	.8913043	.7399666	1.073594	
rel. diff.	-.3125	-.8911317	.2661317	
odds ratio	.5454545	.1656336	1.609034	(exact)

Pair 7 vs. Pair 22

Cases	Controls		Total
	Exposed	Unexposed	
Exposed	26	8	34
Unexposed	16	12	28
Total	42	20	62

McNemar's chi2(1) = 2.67 Prob > chi2 = 0.1025  
 Exact McNemar significance probability = 0.1516

## Proportion with factor

Cases	.5483871			
Controls	.6774194	[95% Conf. Interval]		
difference	-.1290323	-.2966623	.0385978	
ratio	.8095238	.6278837	1.04371	
rel. diff.	-.4	-.9680515	.1680515	
odds ratio	.5	.1852586	1.238236	(exact)

Pair 8 vs. Pair 23

Cases	Controls		Total
	Exposed	Unexposed	
Exposed	23	11	34
Unexposed	11	17	28
Total	34	28	62

McNemar's chi2(1) = 0.00 Prob > chi2 = 1.0000  
 Exact McNemar significance probability = 1.0000

Proportion with factor

Cases	.5483871			
Controls	.5483871	[95% Conf. Interval]		
difference	0	-.164404	.164404	
ratio	1	.7630866	1.310467	
rel. diff.	0	-.3283231	.3283231	
odds ratio	1	.3931661	2.543454	(exact)

Pair 9 vs. Pair 24

Cases	Controls		Total
	Exposed	Unexposed	
Exposed	37	6	43
Unexposed	9	10	19
Total	46	16	62

McNemar's chi2(1) = 0.60 Prob > chi2 = 0.4386  
 Exact McNemar significance probability = 0.6072

Proportion with factor

Cases	.6935484			
Controls	.7419355	[95% Conf. Interval]		
difference	-.0483871	-.1863563	.0895821	
ratio	.9347826	.7881077	1.108755	
rel. diff.	-.1875	-.7045	.3295	
odds ratio	.6666667	.1952634	2.097224	(exact)

Pair 8 vs. Pair 23

Cases	Controls		Total
	Exposed	Unexposed	
Exposed	23	11	34
Unexposed	11	17	28
Total	34	28	62

McNemar's chi2(1) = 0.00 Prob > chi2 = 1.0000  
 Exact McNemar significance probability = 1.0000

Proportion with factor

Cases	.5483871		
Controls	.5483871	[95% Conf. Interval]	
difference	0	-.164404	.164404
ratio	1	.7630866	1.310467
rel. diff.	0	-.3283231	.3283231
odds ratio	1	.3931661	2.543454 (exact)

Pair 9 vs. Pair 24

Cases	Controls		Total
	Exposed	Unexposed	
Exposed	37	6	43
Unexposed	9	10	19
Total	46	16	62

McNemar's chi2(1) = 0.60 Prob > chi2 = 0.4386  
 Exact McNemar significance probability = 0.6072

Proportion with factor

Cases	.6935484		
Controls	.7419355	[95% Conf. Interval]	
difference	-.0483871	-.1863563	.0895821
ratio	.9347826	.7881077	1.108755
rel. diff.	-.1875	-.7045	.3295
odds ratio	.6666667	.1952634	2.097224 (exact)

Pair 10 vs. Pair 25

Cases	Controls		Total
	Exposed	Unexposed	
Exposed	45	3	48
Unexposed	12	2	14
Total	57	5	62

McNemar's chi2(1) = 5.40 Prob > chi2 = 0.0201  
 Exact McNemar significance probability = 0.0352

## Proportion with factor

Cases	.7741935		
Controls	.9193548	[95% Conf. Interval]	
difference	-.1451613	-.2782711	-.0120515
ratio	.8421053	.7283504	.9736266
rel. diff.	-1.8	-4.340404	.7404037
odds ratio	.25	.0452729	.9263782 (exact)

Pair 11 vs. Pair 26

Cases	Controls		Total
	Exposed	Unexposed	
Exposed	27	11	38
Unexposed	9	15	24
Total	36	26	62

McNemar's chi2(1) = 0.20 Prob > chi2 = 0.6547  
 Exact McNemar significance probability = 0.8238

## Proportion with factor

Cases	.6129032		
Controls	.5806452	[95% Conf. Interval]	
difference	.0322581	-.1250174	.1895335
ratio	1.055556	.832837	1.337834
rel. diff.	.0769231	-.2469752	.4008214
odds ratio	1.222222	.460447	3.336929 (exact)

Pair 12 vs. Pair 27

Cases	Controls		Total
	Exposed	Unexposed	
Exposed	26	14	40
Unexposed	9	13	22
Total	35	27	62

McNemar's chi2(1) = 1.09 Prob > chi2 = 0.2971  
 Exact McNemar significance probability = 0.4049

## Proportion with factor

Cases	.6451613		
Controls	.5645161	[95% Conf. Interval]	
difference	.0806452	-.0857564	.2470467
ratio	1.142857	.888976	1.469244
rel. diff.	.1851852	-.1290666	.4994369
odds ratio	1.555556	.6271247	4.074174 (exact)

Pair 13 vs. Pair 28

Cases	Controls		Total
	Exposed	Unexposed	
Exposed	44	1	45
Unexposed	10	7	17
Total	54	8	62

McNemar's chi2(1) = 7.36 Prob > chi2 = 0.0067  
 Exact McNemar significance probability = 0.0117

## Proportion with factor

Cases	.7258065		
Controls	.8709677	[95% Conf. Interval]	
difference	-.1451613	-.2597136	-.030609
ratio	.8333333	.7303802	.9507986
rel. diff.	-1.125	-2.309497	.0594969
odds ratio	.1	.0023043	.702939 (exact)

Pair 14 vs. Pair 29

Cases	Controls		Total
	Exposed	Unexposed	
Exposed	34	8	42
Unexposed	12	8	20
Total	46	16	62

McNemar's chi2(1) = 0.80 Prob > chi2 = 0.3711  
 Exact McNemar significance probability = 0.5034

## Proportion with factor

Cases	.6774194			
Controls	.7419355	[95% Conf. Interval]		
difference	-.0645161	-.2211047	.0920725	
ratio	.9130435	.7479736	1.114542	
rel. diff.	-.25	-.8624887	.3624887	
odds ratio	.6666667	.2363844	1.773597	(exact)

Pair 15 vs. Pair 30

Cases	Controls		Total
	Exposed	Unexposed	
Exposed	13	13	26
Unexposed	11	25	36
Total	24	38	62

McNemar's chi2(1) = 0.17 Prob > chi2 = 0.6831  
 Exact McNemar significance probability = 0.8388

## Proportion with factor

Cases	.4193548			
Controls	.3870968	[95% Conf. Interval]		
difference	.0322581	-.1385308	.2030469	
ratio	1.083333	.7376116	1.591096	
rel. diff.	.0526316	-.1933086	.2985718	
odds ratio	1.181818	.4885561	2.913432	(exact)

### 3.16 Appendix J. Detailed Wald Test Results for Predictions

1-in-1 Treatment: Wald tests for the null hypothesis of  $r = 0$

p-value	Predicted r																# of Subjects	
	0.45	0.46	0.49	0.53	0.59	0.62	0.64	0.67	0.75	0.78	0.80	0.83	0.89	0.93	0.94	0.97		
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	28	0	28
1.14E-34	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	16	16
1.33E-26	0	0	0	0	0	0	0	0	0	0	0	0	22	0	0	0	0	22
5.60E-25	0	0	0	0	0	0	0	9	0	0	0	0	0	0	0	0	0	9
6.78E-25	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	2
2.90E-22	0	0	0	0	0	0	17	0	0	0	0	0	0	0	0	0	0	17
2.02E-19	0	0	0	0	0	0	0	0	0	0	0	0	0	9	0	0	0	9
7.87E-19	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	2
1.12E-15	0	0	0	0	8	0	0	0	0	0	0	0	0	0	0	0	0	8
2.89E-13	0	0	0	0	0	5	0	0	0	0	0	0	0	0	0	0	0	5
3.05E-11	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	2
3.12E-09	0	0	0	0	0	0	0	0	0	3	0	0	0	0	0	0	0	3
4.42E-07	0	0	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0	3
6.11E-07	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2
0.0002934	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4
0.0003798	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
# of Subjects	4	1	2	3	8	5	17	9	2	3	2	2	22	9	28	16	133	

1-in-1 Treatment: Wald tests for the null hypothesis of gamma = 1

p-value	Predicted gamma															# of Subjects	
	0.23	0.24	0.25	0.26	0.29	0.30	0.31	0.32	0.46	0.47	0.48	0.49	0.52	0.53	0.54		0.55
7.02E-06	22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	22
0.000047	0	0	0	0	0	0	0	0	0	17	0	0	0	0	0	0	17
0.0000525	0	28	0	0	0	0	0	0	0	0	0	0	0	0	0	0	28
0.0001667	0	0	0	0	0	0	0	0	8	0	0	0	0	0	0	0	8
0.0002153	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	2
0.0004045	0	0	0	0	0	0	0	0	0	0	0	9	0	0	0	0	9
0.000434	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	2
0.0004769	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	2
0.0005309	0	0	0	0	0	0	3	0	0	0	0	0	0	0	0	0	3
0.0007535	0	0	9	0	0	0	0	0	0	0	0	0	0	0	0	0	9
0.001047	0	0	0	16	0	0	0	0	0	0	0	0	0	0	0	0	16
0.0022308	0	0	0	0	0	0	0	0	0	5	0	0	0	0	0	0	5
0.0147585	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	3
0.0306056	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1
0.0315525	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	2
0.0396612	0	0	0	0	0	0	0	0	0	0	0	0	4	0	0	0	4
# of Subjects	22	28	9	16	2	2	3	2	8	17	5	9	4	2	1	3	133

1-in-40 Treatment: Wald tests for the null hypothesis of r = 0

p-value	Predicted r											# of Subjects	
	0.40	0.44	0.47	0.49	0.52	0.62	0.64	0.67	0.67	0.71	0.82		0.86
1.96E-27	0	0	0	0	0	0	7	0	0	0	0	0	7
1.6E-24	0	0	0	0	0	0	0	0	0	0	8	0	8
1.56E-20	0	0	0	0	0	0	0	0	3	0	0	0	3
2.35E-18	0	0	0	0	0	0	0	0	0	11	0	0	11
1.25E-15	0	0	0	0	8	0	0	0	0	0	0	0	8
5.45E-14	0	0	0	0	0	0	0	4	0	0	0	0	4
3.92E-13	0	0	0	0	0	0	0	0	0	0	0	3	3
2.07E-10	0	0	0	11	0	0	0	0	0	0	0	0	11
8.76E-10	0	0	0	0	0	1	0	0	0	0	0	0	1
1.22E-08	0	0	3	0	0	0	0	0	0	0	0	0	3
0.00000917	0	1	0	0	0	0	0	0	0	0	0	0	1
0.0000284	2	0	0	0	0	0	0	0	0	0	0	0	2
# of Subject	2	1	3	11	8	1	7	4	3	11	8	3	62

1-in-40 Treatment: Wald tests for the null hypothesis of  $r + rc = 0$

p-value	Predicted $r + rc$												# of Subjects
	0.54	0.57	0.59	0.63	0.66	0.67	0.71	0.73	0.74	0.76	0.81	0.84	
0.00E+00	0	0	0	0	0	0	0	3	0	0	0	0	3
9.27E-32	0	0	0	0	0	0	7	0	0	0	0	0	7
2.86E-25	0	0	0	0	0	0	0	0	4	0	0	0	4
3.55E-22	0	0	0	0	0	0	0	0	0	11	0	0	11
5.59E-19	0	0	0	0	0	0	0	0	0	0	0	3	3
7.64E-18	0	0	0	0	0	0	0	0	0	0	8	0	8
2.03E-17	0	0	0	11	0	0	0	0	0	0	0	0	11
7.91E-14	0	1	0	0	0	0	0	0	0	0	0	0	1
2.32E-12	0	0	0	0	0	1	0	0	0	0	0	0	1
1.5E-10	0	0	0	0	8	0	0	0	0	0	0	0	8
0.000000094	0	0	3	0	0	0	0	0	0	0	0	0	3
0.000000886	2	0	0	0	0	0	0	0	0	0	0	0	2
# of Subject	2	1	3	11	8	1	7	3	4	11	8	3	62

1-in-40 Treatment: Wald tests for the null hypothesis of  $\gamma = 1$

p-value	Predicted gamma											# of Subjects	
	0.37	0.41	0.68	0.75	0.79	0.95	0.99	1.06	1.26	1.30	1.33		1.37
7.41E-03	0	8	0	0	0	0	0	0	0	0	0	0	8
0.0110372	3	0	0	0	0	0	0	0	0	0	0	0	3
0.0924481	0	0	0	0	0	0	0	0	0	0	0	11	11
0.1921943	0	0	0	0	0	0	0	0	0	0	8	0	8
0.1973165	0	0	0	11	0	0	0	0	0	0	0	0	11
0.2324818	0	0	1	0	0	0	0	0	0	0	0	0	1
0.2682751	0	0	0	0	0	0	0	0	0	2	0	0	2
0.3140454	0	0	0	0	0	0	0	0	1	0	0	0	1
0.3357557	0	0	0	0	3	0	0	0	0	0	0	0	3
0.8049443	0	0	0	0	0	0	0	3	0	0	0	0	3
0.8290983	0	0	0	0	0	4	0	0	0	0	0	0	4
0.9432108	0	0	0	0	0	0	7	0	0	0	0	0	7
# of Subjects	3	8	1	11	3	4	7	3	1	2	8	11	62

1-in-40 Treatment: Wald tests for the null hypothesis of  $\gamma + \gamma c = 1$

p-value	Predicted gamma + gammac											# of Subjects	
	0.29	0.34	0.42	0.55	0.56	0.61	0.61	0.63	0.69	0.69	0.82		0.87
4.02E-05	0	0	0	7	0	0	0	0	0	0	0	0	7
0.0001762	0	0	1	0	0	0	0	0	0	0	0	0	1
0.0002076	8	0	0	0	0	0	0	0	0	0	0	0	8
0.008463	0	3	0	0	0	0	0	0	0	0	0	0	3
0.0122949	0	0	0	0	3	0	0	0	0	0	0	0	3
0.0125595	0	0	0	0	0	0	11	0	0	0	0	0	11
0.0382169	0	0	0	0	0	4	0	0	0	0	0	0	4
0.062012	0	0	0	0	0	0	0	0	0	3	0	0	3
0.0686239	0	0	0	0	0	0	0	0	1	0	0	0	1
0.0960365	0	0	0	0	0	0	0	2	0	0	0	0	2
0.2874661	0	0	0	0	0	0	0	0	0	0	11	0	11
0.3762546	0	0	0	0	0	0	0	0	0	0	0	8	8
# of Subject	8	3	1	7	3	4	11	2	1	3	11	8	62

## Chapter 4

# Inducing Risk Neutral Preferences with Binary Lotteries: A Reconsideration

*by Glenn W. Harrison, Jimmy Martínez-Correa and Todd J. Swarthout*

### **Abstract**

We evaluate the binary lottery procedure for inducing risk neutral behavior. We strip the experimental implementation down to bare bones, taking care to avoid any potentially confounding assumption about behavior having to be made. In particular, our evaluation does not rely on the assumed validity of any strategic equilibrium behavior, or even the customary independence axiom. We show that subjects sampled from our population are generally risk averse when lotteries are defined over monetary outcomes, and that the binary lottery procedure does indeed induce a statistically significant shift towards risk neutrality. This striking result generalizes to the case in which subjects make several lottery choices and one is selected for payment.

## 4.1 Introduction

Experimental economists would love to have a procedure to induce linear utility functions. Many inferences in economics depend on risk premia and the extent of diminishing marginal utility.<sup>1</sup> In fact, the settings in which these do not play a confounding role are the special case. Procedures to induce linear utility functions have a long history, with the major contributions being Smith (1961), Roth and Malouf (1979) and Berg et al. (1986). Unfortunately, these “lottery procedures” have come under attack on behavioral grounds: the consensus appears to be that they may be fine in theory, but simply do not work as advertised.

We review that evidence. The first point to note is that the consensus is simply an oft-repeated falsehood. There are several instances where the lottery procedures have indeed shifted choices in the predicted direction, and simple explanations provided to explain why others have generated negative findings (e.g., Rietz, 1993). The second point is the most important for us: none of the prior tests have been pure tests of the lottery procedure. Every previous test requires one or more of three auxiliary assumptions:

- That the utility functions defined over money, or other consequences, are in fact non-linear, so that there is a behavioral problem to be solved with the lottery procedure;
- That behavior is characterized according to some strategic equilibrium concept, such

---

<sup>1</sup>Risk attitudes are only synonymous with diminishing marginal utility under expected utility theory. But diminishing marginal utility also plays a confounding role under many of the prominent alternatives to expected utility theory, such as rank-dependent utility theory and prospect theory.

as Nash Equilibrium; and/or

- That the independence axiom holds when subjects in experiments are paid for 1 in  $K$  choices, where  $K > 1$ .

Selten et al. (1999, Table 1) pointed out the existence of the first two confounds in the previous literature. Their own test employed the third assumption as an auxiliary assumption, and found no evidence to support the use of the lottery procedure. We propose tests that avoid all three assumptions.

Our procedures are very simple. First, we ask subjects in one treatment to make a single choice over a pair of lotteries defined over money and objective probabilities. They make no other choices, hence we do not need to rely on the Random Lottery Incentive Method (RLIM) and the Independence Axiom (IA); these terms are defined more formally later. This treatment constitutes a theoretical and behavioral baseline, to allow us to establish that the typical decision maker in our population exhibits a concave utility function over these prizes.<sup>2</sup>

Second, we ask subjects in another treatment to make the same choices but where they earn points instead of money, and these points convert into increased probability of winning some later, binary lottery. The choices are the same in the sense that they have the same numerical relationship between consequences (e.g., if one lottery had prizes of \$70 or \$35, the variant would have prizes of 70 or 35 points), and the same objective probabilities. The subjects are also drawn at random from the same population as the

---

<sup>2</sup>This is also true if we estimate a structural model with rank-dependent probability weighting. All of our choices involve the gain domain, so the traditional form of loss aversion in prospect theory does not apply.

control treatment. Between-subjects tests are necessary, of course, if one is to avoid the RLIM procedure and having to assume the IA.

At this point there are two ways to evaluate the lottery procedure. One is to see if behavior in the points tasks matches the theoretical prediction of choosing whichever lottery had the highest Expected Value (EV). The other is to see if it induces significantly less concave utility functions than the baseline task, and indeed generates statistical estimates consistent with a linear utility function. We apply both approaches, which have relative strengths and weaknesses, and find that the lottery procedure works virtually exactly as advertized.

In section 4.2.1 we review the literature on the lottery procedure, in section 4.3 we review the theory underlying the procedure, in section 4.4 we present our experimental design, and in section 4.5 we evaluate the results. Section 4.2.1 makes the point that the procedure has an impressive lineage in statistics, and that all of the previous tests in economics require auxiliary assumptions. Section 4.3 clarifies the axiomatic basis of behavior and the distinct roles in the experiment for the Independence Axiom and a special binary version of the Reduction of Compound Lotteries (ROCL) axiom. It also explains the relationship between the lottery procedure and non-standard models of decision making under risk: does the lottery procedure help induce *risk neutrality* under rank-dependent models and prospect theory? The answer is “yes,” under some weak conditions.

## 4.2 Literature

### 4.2.1 Literature in Statistics

Smith (1961) appears to have been the first to explicitly pose the lottery procedure as a way of inducing risk neutral behavior. He considers the issue of two individuals placing bets over some binary event. The person whose subjective probability we seek to elicit is Bob, and the experimenter is Charles. There is a third person, an Umpire, who is the funding agency providing the subject fees. Choices over Savage-type lotteries are elicited from Bob, and inferences then made about his subjective probabilities. But the reward can, of course, affect the utilities of Bob, so how does one control for that confound when inferring Bob's subjective probabilities?

Stated differently, is there some way to make sure that the choices over bets do not depend on the reward, but only on the subjective probability of the event? Smith (1961, p. 13)

proposes a solution:

To avoid these difficulties it is helpful to use the following device, adapted from Savage [1954]. Instead of presenting cash to Bob and Charles, the Umpire takes 1 kilogram of beeswax (of negligible value) and hides within it at random a very small but valuable diamond. He divides the wax into two parts, presenting one to each player, and instructs them to use it for stakes. After all bets have been settled, the wax is melted down and whoever has the diamond keeps it. Effectively this means that if, say, Bob gives Charles  $y$  grams of wax, he increases Charles's chance of winning the diamond by  $y/1000$ . [...] Hence using beeswax or "probability currency" the acceptability of a bet depends on the odds [...], and not on the stake ...

It should be noted that this solution does two things, each of which play a role in the later economics literature. Not only can we infer Bob's subjective probabilities from his choices

over the bets without knowing (much about) his utility function, but the same is true with respect to Charles. Thus, in the hands of Roth and Malouf (1979), we can evaluate the expected utility of two bargaining agents with this device.<sup>3</sup> We are interested here just in the first part of this, the knowledge it provides of Bob's utility scale over these two prizes. Of course, the reference to Savage (1954) is tantalizing, but that is a large and dense book! There are three places in which the concept appears to be implied. The first, of course, is the core axiom (P5), introduced in §3.2, and its use in many proofs. This axiom requires that there be at least two consequences such that the decision-maker strictly prefers one consequence to the other.<sup>4</sup> The formal mathematical use of this axiom in settings in which there are three or more consequences makes it clear that probabilities defined over any such pair can be used to define utilities that can be scaled by some utility function when other axioms deal with the other consequences. Of course, this formal use is far from the operational lottery procedure, but is suggestive.

The second is the related discussion in §5.5 of the application of axiom (P5) in a "small world" setting in which there are in fact many consequences. Specifically, he offers the metaphor of tickets in distinct lotteries for nothing, a sedan, a convertible, or a thousand dollars. The decision maker selects one of these four lotteries, and wins one of these four consequences with some (subjective) probability.<sup>5</sup> So the lottery is between the status quo

---

<sup>3</sup>Strictly speaking there is one change from the betting metaphor developed by Smith (1961, p. 4). He has the Umpire pose subjective probabilities, not Charles. In bargaining games, Bob and Charles directly negotiate on these probabilities under some protocol.

<sup>4</sup>The need for (P5), however minimal, is why we referred in the previous paragraph to the lottery procedure not requiring that we know much about the utility function of the decision maker. It does require that (P5) apply for the two prizes, so that we can assign distinct real values to them.

<sup>5</sup>The consequence "nothing" is used in context to locate this small world experiment in the grand world that the decision maker inhabits. Thus "nothing" means nothing from the experimental task, or the maintenance of the grand world status quo. Little would be lost in our context by replacing "nothing" with "one

and the status quo plus the single consequence associated with the lottery ticket type chosen.

The third is more explicit, and pertains to the discussion of controlling for the utility of the decision maker in applications of the minimax rule in §13.4. He proposed (p. 202) three solutions to this issue, the first of which defines what he is after (a linear utility scale) and the third of which presents the lottery procedure:

Three special circumstances are known to me under which escape from this dilemma is possible. First, there are problems in which some straightforward commodity, such as money, lives, man hours, hospital bed days, or submarines sighted, is obviously so nearly proportional to utility as to be substitutable for it. [...] Third, there are many important problems, not necessarily lacking in richness of structure, in which there are exactly two consequences, typified by overall success or failure in a venture. In such a problem, as I have heard J. von Neumann stress, the utility can, without loss of generality, be set equal to 0 on the less desired and equal to 1 on the more desired of the two consequences.

Yet another tantalizing bibliographic thread!

## 4.2.2 Literature in Economics

Roth and Malouf (1979) (RM) independently introduced the Smith (1961) procedure into the economics literature. The procedure is simple, and has been employed by several experimenters. Their experiment involved two subjects bargaining over some pie. Since most of the cooperative game theoretic solution concepts require that subjects bargain directly over utilities or expected utilities, RM devised a procedure for ensuring that this was the case if subjects obeyed the axioms of expected utility theory. Their idea was to provide each subject  $i$  with a high prize  $M_i$  and a low prize  $m_i$ , where  $M_i > m_i$  for each

---

penny.”

subject  $i$ . Although not essential, let these prizes be money. Each subject was then to engage in a bargaining process to divide 100 lottery tickets between the two subjects. Each lottery ticket that the subject received from the bargaining process resulted in them having a 1 percentage point chance of receiving the high prize instead of the low prize. Thus, if subject  $i$  received 83 of the lottery tickets, he would receive the high prize with probability 0.83 and the low prize with residual probability 0.17. Since utility functions are arbitrary up to a linear transformation, one could set the utility value of the high prize to 1 for each player and the utility value of the low prize to 0 for each player. Thus bargaining over the division of 100 lottery tickets means that the subjects are bargaining over the expected utility to themselves and the other player.

There are several remarkable and related features of this elegant design. First, no player has to know the value of the prizes available to the other player in order to bargain over expected utility uniquely. Whether the other person's high prize is the same or double my high prize, I can set his utility of receiving that prize to 1. All that is required are the assumptions of non-satiation in the prize and the invariance of equivalent utility representations. Second, and related to this first point but separable, the prizes can differ. Third, the subject does not even have to know the value of the monetary prizes to himself, just that there will be "more of it" if he wins the lottery and that "it" is something in which he is not satiated.

The experiments of RM also revealed some important behavioral features of applying this procedure in experiments. When subjects bargained, in a relatively unstructured manner, in a setting in which they did not know the value of the prizes to the other player, they

generally tended to bargain to equal-split outcomes of the lottery tickets, which translate into equal splits of expected utility to each player. But when subjects received more information than received (cooperative) theory typically required, specifically the value of the monetary prizes to the other subjects, outcomes converged even more clearly to the equal-split outcome when the prizes were identical. But when the prizes were not identical, there were two outcomes, reflected in a bi-modality of the data. One mode involved subjects bargaining to an equal-split of tickets, and the other mode involved subjects bargaining to an unequal-split of tickets that tended to equalize the expected *monetary* gain to each player. In other words, the subjects behaved as if using the information on the value of the prizes, and the interpersonal comparability of the utility<sup>6</sup> of those prizes, to arrive at an outcome that was fair in terms of expected monetary gain. Of course, this fair outcome in terms of expected monetary gain coincided with the fair outcome in terms of expected utility when subjects were told the value of monetary prizes and that they were the same for both players.

There are two important insights from their results for our purposes. First, it is feasible to modify an experimental game to ensure that the payoffs of subjects are defined in terms of utility and expected utility. We review procedures employed by several experimenters interested in noncooperative games below. Second, the provision of information that allows subjects to make interpersonal comparisons of utility can add a possible confound. That is, the provision of “more information than theory assumes is needed for subjects to know utility payoffs” can lead to subjects employing fairness rules or norms that rely on

---

<sup>6</sup>A dollar note given to me is the same dollar note that could have been given to you, and the transform from money to utility is unique.

interpersonal comparability of utility.<sup>7</sup>

The RM procedure was generalized by Berg et al. (1986), albeit in the context of games against Nature. Their idea was that subjects would make choices over lotteries defined in terms of points instead of pennies, and that their accumulated points earnings would be then converted to money using an exchange rate function. If this function was linear, then risk neutrality would be induced. If this function was convex (concave), risk-loving (risk averse) preferences would be induced. By varying the function one could, in principle, induce any specific risk attitude.

Unfortunately, the Berg et al. (1986) procedure came under fire immediately from Cox et al. (1985). They applied the procedure in two treatments in which they also had identical, paired treatments that did not use the procedure. The context of their test was an experiment in which four subjects bid for a single object using first-price sealed-bid rules, and values were induced randomly in an independent and private manner. In one treatment they generated random values over 20 periods, and paid subjects their monetary profits; in the paired treatment they used the same 80 random valuations, applied in the same order but to a different pool of subjects, but used the lottery procedure to generate risk-neutral bidding. They found no support for the hypothesis that the lottery procedure generated risk neutral bidding. Related tests of the lottery procedure, conditional on assumptions about bidding behavior in auctions, were provided by Walker et al. (1990) and Cox and

---

<sup>7</sup>RM point this out quite clearly, and proceed to develop an alternative to the standard cooperative bargaining solution concept that allows subjects to make such interpersonal comparisons. These differences are of some significance for policy. For example, Harrison and Rutström (1991) apply the two concepts developed by RM to predict outcomes of international trade negotiations, showing how comparable information on the U.S. dollar -equivalent of the “equivalent variation” of alternative trade policies can be used to influence negotiated outcomes.

Oaxaca (1995). One important feature of the experimental tests of Walker et al. (1990)] is that 5 of their 15 experiments used subjects that had demonstrated, in past experiments, “tight consistency” with Nash Equilibrium bidding predictions. Thus the use of those subjects could be viewed *a priori* as recognizing, and mitigating, the confounding effects of those auxiliary assumptions on tests of the lottery procedure. Rietz (1993) provides a careful statement of the detailed procedural features of these earlier, discouraging tests of the lottery procedure, and their role in its efficacy; we review his main findings below. The controversy over the use of the risk-inducement technique led many experimental economists at the time to abandon it. Although not often stated, the folklore was clear: since it had not been advocated as necessary to use, why bother? Moreover, the procedures for inducing risk aversion or risk loving behavior did add a cognitive layer of complexity to procedures that one might want to avoid unless necessary.

Several experimenters did use the lottery procedure in tandem with experiments that did not attempt to control for risk aversion: in effect, staying directly out of the debate over the validity of the procedure but checking if it made any difference.<sup>8</sup> For example, Harrison (1989) ran his first-price sealed bid auction with and without the lottery procedure to induce risk neutrality, and managed to generate enough debate on other grounds that nobody cared if the procedure had any effect! Similarly, Harrison and McCabe (1992) ran their alternating-offer, non-cooperative bargaining experiments “both ways,” and found no difference in behavior.<sup>9</sup>

---

<sup>8</sup>Braunstein and Schotter (1982) employed an early “with and without” design, in the context of individual choice experiments examining job search.

<sup>9</sup>On the other hand, the weight of experimental procedure was against the use of such procedures, leading Harrison and McCabe (1996, p. 315) to cave in and offer an invalid rationalization of their choice not to use the lottery procedure: “We elected not to use the lottery procedure of Roth and Malouf (1979) to

Ochs and Roth (1989) is an important study because it was the first foray of Alvin Roth, the “R” in RM, into non-cooperative experimental games, and did not employ the binary lottery procedure developed by RM. They explicitly make “... the assumption that the bargainer’s utility is measured by their monetary payoffs” (p. 359), but have nothing else to say on the matter. This methodological discontinuity between RM and Ochs and Roth (1989) is an interesting puzzle, and may have been prompted by the acrimonious debate generated by Cox et al. (1985) and the fact that none of the prior non-cooperative bargaining experiments that Ochs and Roth (1989) were generalizing had worried about the possible difference.

There have been several experiments in which the lottery procedure has been employed exclusively, most notably Cooper et al. (1989, 1990, 1992, 1993).<sup>10</sup> They had a very clear sense of why some such procedure was needed, and implemented it in a simple manner:

Each games was defined to be one of complete information, because each player’s payoff matrix was common knowledge, and the numerical payoffs represented a player’s utility if the corresponding strategies were chosen. To accomplish this, we induced payoffs in terms of utility using the procedure of [RM...]. With this procedure, each player’s payoff is given in points; these points determine the probability of the player winning a monetary prize. At the end of each period of each game, we conducted a lottery in which ‘winning’ players received \$1.00 or \$2.00, depending on the session, and “losing” players received \$0.00. The probability of winning was given by dividing the points the player had earned by 1,000. Since expected utility is invariant with respect to linear transformations, this procedure ensures that, when players maximize

---

induce risk-neutral behaviour. None of the previous studies of Ultimatum bargaining have used it, and risk attitudes should not matter for the standard game-theoretic prediction that we are testing.” The final phrase is technically correct, but only because the subgame perfect Nash equilibrium prediction calls for one player to offer essentially nothing to the other player, and to take essentially all of the pie for himself. Thus one does not need to know what utility function each player has, since the prediction calls for the players to get utility outcomes that can always be normalized to “essentially zero” and “essentially one.”

<sup>10</sup>Harrison (1994) employed it in tests of a non-strategic setting, where the predictions of expected utility theory depended on risk attitudes. He recognized that the experiment therefore became a test of the joint hypothesis that the risk inducement procedure worked and that expected utility theory applied to the lottery choices under study.

their expected utility, they maximize the expected number of points in each game, regardless of their attitude to risk. [1993; p.1307, footnotes omitted]

Their experiments involved simple normal form games in which the points payoffs ranged from 0 up to 1000, with many around the 300 to 600 range, and subject participated sequentially in 30 games against different opponents. One important feature of their implementation is that the players could engage in interpersonal comparisons of utility, since they knew that the prizes each subject faced were the same.

Rietz (1993) examines the lottery procedure in the context of auxiliary assumptions about equilibrium bidding behavior in first-price sealed bid auctions, as in Harrison (1989), but uncovers some interesting and neglected behavioral properties of the procedure.<sup>11</sup> First, if subjects are exposed to the task with monetary prizes, it is difficult to change their behavior with the lottery procedure. Thus there is a behavioral hysteresis or order effect. Second, if subjects have not been previously exposed to the task with monetary prizes, then the lottery procedure works as advertized. Finally, if one “trains subjects up” in the lottery procedure in a dominant-strategy context (e.g., a second-price sealed bid auction), then it’s performance “travels” to a different setting and it works as advertized in a strategic context in which there is no dominant strategy (e.g., a first-price sealed bid auction).

On the other hand, Cox and Oaxaca (1995) criticize the estimator used by Rietz (1993). He used a Least-Absolute-Deviations (LAD) estimator that was applied to data that had already been normalized by dividing observed bids by item values for the bidder, in contrast to the earlier use of Ordinary Least Squares (OLS) on untransformed data by

---

<sup>11</sup>These properties were identified in an attempt to explain the different conclusions drawn from the same general environment by Cox et al. (1985) and Walker et al. (1990).

Walker et al. (1990). Cox and Oaxaca (1995) argue that OLS is not obviously inferior to LAD in this context, and that there are tradeoffs to one over the other (e.g., if heteroskedasticity is not eliminated, which of OLS or LAD is easier to evaluate for heteroskedasticity, and which has better out-of-sample predictive accuracy?). It is apparent that both OLS and LAD are decidedly second-best if one could estimate a structural model that directly respects the underlying theory, as in Harrison and Rutström (2008, §3.6). Cox and Oaxaca (1995) also point out that the lottery procedure implies both a “zero intercept” and a “unit slope” in behavior compared to the risk-neutral Nash equilibrium bid predictions, and that Rietz (1993) only tested for the latter. Hence his tests are incomplete as a conceptual matter, even if one puts aside questions about the “best” estimator for these tests.

Berg et al. (2008) collect and review all of the studies testing the lottery procedure, and argue that the evidence against its efficacy is not so clear as many have claimed.

Selten et al. (1999) is the first study to stress that all previous tests of the lottery procedure have involved confounding assumptions, even if there had been attempts in some, such as Walker et al. (1990), to mitigate them. They presented subjects with 36 paired lottery choices, and 14 lottery valuation tasks. The latter valuation tasks employed the Becker- DeGroot-Marschak elicitation procedure, which has poor behavioral incentives even if it is theoretically incentive compatible (Harrison, 1992).<sup>12</sup> They calculate a statistic for each subject over all 50 tasks: the difference between the maximum EV over all 50 choices minus the actual EV for the observed choices. If the lottery procedure is generating

---

<sup>12</sup>Given these concerns, and the detailed listing of data by Selten et al. (1999, Appendix B), it would be useful to re-evaluate their conclusions by just looking at the 36 binary choices.

risk neutral behavior then it should lead to a reduction in this statistic, compared to treatments using monetary prizes directly. Focusing on their treatments in which statistical measures about the lotteries were not made available, they had 48 subjects in each treatment. They find that the subjects in the lottery procedure actually had *larger* losses relative to the maximum if they had been following a strategy of choosing in a risk neutral manner. These differences were statistically evaluated using non-parametric two-sample Wilcoxon- Mann-Whitney tests of the null hypothesis that they were drawn from the same distribution; the onesided  $p$ -value was lower than 0.05. Not only is the lottery procedure failing to induce risk neutrality, it appears to be moving subjects in the wrong direction!

### 4.3 Theory

The Reduction of Compound Lotteries (ROCL) axiom states that a decision-maker is indifferent between a compound lottery and the actuarially-equivalent simple lottery in which the probabilities of the two stages of the compound lottery have been multiplied out. To use the language of Samuelson (1952, p. 671), the former generates a *compound income-probability-situation*, and the latter defines an *associated income-probability-situation*, and that “...only algebra, not human behavior, is involved in this definition.”

To state this more explicitly, let  $X$ ,  $Y$  and  $Z$  denote simple lotteries,  $A$  and  $B$  denote compound lotteries,  $\succ$  express strict preference, and  $\sim$  express indifference. Then the ROCL axiom says that  $A \sim X$  if the probabilities in  $X$  are the actuarially-equivalent

probabilities from A. Thus let the initial lottery pay \$10 if a coin flip is a head and \$0 if the coin flip is a tail. Then if A is the compound lottery that pays double the outcome of the coin-flip lottery if a die roll is a 1 or a 2; triple the outcome of the coin-flip lottery if a die roll is a 3 or 4; and quadruple the outcome of the coin-flip lottery if a die roll is a 5 or 6. In this case X would be the lottery that pays \$20 with probability  $(1/2) \times (1/3) = 1/6$ , \$30 with probability  $1/6$ , \$40 with probability  $1/6$ , and nothing with probability  $1/2$ .

The Binary ROCL axiom restricts the application of ROCL to compound binary lotteries and the actuarially-equivalent, simple, binary lottery. In the words of Selten et al. (1999, p. 221ff)

It is sufficient to assume that the following two conditions are satisfied. [...] *Monotonicity.* The decision makers utility for simple binary lotteries involving the same high prize with a probability of p and the same low prize with the complementary probability 1-p is monotonically increasing in p. [...] *Reduction of compound binary lotteries.* The decision maker is indifferent between a compound binary lottery and a simple binary lottery involving the same prizes and the same probability of winning the high prize. Both postulates refer to binary lotteries only. Reduction of compound binary lotteries is a much weaker requirement than an analogous axiom for compound lotteries in general.

To use the earlier example, with the Binary ROCL axiom we would have to restrict the compound lottery A to consist of only two final prizes, rather than four prizes (\$20, \$30, \$40 or \$0). Thus the initial stage of compound lottery A might pay 70 points if a 6-sided die roll comes up 1 or 2 or 3, 30 points if the die roll comes up 4, and 15 points if the die roll comes up 5 or 6, and the second stage might then pay \$16 or \$5 depending on the points earned in the initial lottery. For example, if a subject earns 15 points and a 100-sided die with faces 1 through 100 comes up 15 or lower then she would earn \$16, and \$5 otherwise. There are only two final prizes to this binary compound lottery, \$16 or \$5,

and the actuarially equivalent lottery X pays \$16 with probability 0.45 ( $=1/2 \times 0.7 + 1/6 \times 0.3 + 1/3 \times 0.15$ ) and \$5 with probability 0.55 ( $=1/2 \times 0.3 + 1/6 \times 0.7 + 1/3 \times 0.85$ ). The binary lottery procedure generates risk neutral behavior even if the decision maker violates EUT in the “probabilistically sophisticated manner” as defined by Machina and Schmeidler (1992, 1995). For example, assume that the decision maker uses a Rank-Dependent Utility model with a simple, monotonically increasing probability weighting function, such as  $w(p) = p^\gamma$  for  $\gamma \neq 1$ . Then the higher prize receives decision weight  $w(p)$ , where  $p$  is the probability of the higher prize, and the lower prize receives decision weight  $1 - w(p)$ . EUT is violated in this case, but neither of the axioms needed for the binary lottery procedure to induce risk neutrality are violated.<sup>13</sup>

## 4.4 Experiment

Table 4.1 summarizes our experimental design, and the sample size of subjects and choices in each treatment. All sessions were conducted in 2011 at the ExCEN experimental lab of Georgia State University (<http://excen.gsu.edu/Laboratory.html>). Subjects were recruited from a database of volunteers from classes in all undergraduate colleges at Georgia State University initiated at the beginning of the 2010-2011 academic year.

In **treatment A** we have subjects undertake one binary choice, where the one pair they face is drawn at random from a set of 24 lottery pairs shown in Table 4.3 of section 4.8.

---

<sup>13</sup>Berg et al. (2008) argue that the lottery procedure requires a model of decision making under risk that assumes linearity in probabilities. This is incorrect as a theoretical matter, if the objective is solely to induce risk neutrality. Their remarks are valid if the objective is to induce a specific risk attitude other than risk neutrality, following Berg et al. (1986).

Table 4.1: Experimental Design

All choices drawn from the same battery of 24 lottery pairs at random.  
 All subjects receive a \$7.50 show-up fee.  
 Subjects were told that there would be no other salient task in the experiment.

Treatment	Subjects (Choices)
A. Monetary prizes with only one binary choice (Figure 1)	55 (55)
B. Binary lottery points with only one binary choice (Figure 2)	69 (69)
C. Monetary prizes with one binary choice out of $K \gg 1$ selected for payment (Figure 1)	208 (2104)
D. Binary lottery points with one binary choice out of $K \gg 1$ selected for payment (Figure 2)	39 (936)
E. Binary lottery points with only one binary choice and with EV information provided for each lottery (Figure 3)	34 (34)
F. Binary lottery points with only one binary choice and with EV information provided for each lottery (Figure 3), as well as “cheap talk” instructions	38 (38)

Figure 4.1 shows the interface used, showing the probabilities of each monetary prize.

The lottery pairs span five monetary prize amounts, \$5, \$10, \$20, \$35 and \$70, and five probabilities, 0, 0.25, 0.5, 0.75 and 1. They are based on a subset of a battery of lottery pairs developed by Wilcox (2010) for the purpose of robust estimation of RDU models.<sup>14</sup>

These lotteries contain some pairs in which the “EUT-safe” lottery has a *higher* EV than the “EUT-risky” lottery: this is designed deliberately to evaluate the extent of risk premia deriving from probability pessimism rather than diminishing marginal utility. None of the lottery pairs have prospects with equal EV, and the range of EV differences is wide. Each lottery in treatment A is a simple lottery, with no compounding.

<sup>14</sup>The original battery includes repetition of some choices, to help identify the “error rate” and hence the behavioral error parameter, defined later. In addition, the original battery was designed to be administered in its entirety to every subject.

Figure 4.1: Default Binary Choice Interface



In Treatment A we do not have to assume that the Independence Axiom applies for the payment protocol in order for observed choices to reflect risk preferences under EUT or RDU.<sup>15</sup> In effect, it represents the behavioral Gold Standard benchmark, against which the

<sup>15</sup>Following Segal (1988b, 1990, 1992), the Mixture Independence Axiom (MIA) says that the preference ordering of two simple lotteries must be the same as the actuarially-equivalent simple lottery formed by adding a common outcome in a compound lottery of each of the simple lotteries, where the common outcome has the same value and the same (compound lottery) probability. Let  $X$ ,  $Y$  and  $Z$  denote simple lotteries and  $\succ$  express strict preference. The MIA says that  $X \succ Y$  iff the actuarially-equivalent simple lottery of  $\alpha X + (1-\alpha)Z$  is strictly preferred to the actuarially-equivalent simple lottery of  $\alpha Y + (1-\alpha)Z$ ,  $\forall \alpha \in (0,1)$ . The verbose language used to state the axiom makes it clear that MIA embeds ROCL into the usual independence axiom construction with a common prize  $Z$  and a common probability  $(1-\alpha)$  for that prize. When choices only involve simple lotteries, as in Treatment A, a weaker version of the independence axiom, called the Compound Independence Axiom, can be applied to justify the use of the RLIM. In general, we will be considering choices over compound lotteries when we apply the BLP, so the MIA is needed to justify the use of the RLIM when we extend Treatment A to allow for several lottery choices in Treatment C. Treatment A, to repeat, does not need the RLIM. Although we say “Independence Axiom” in the text, the context should make clear which version of the axiom is involved.

other payment protocols are to be evaluated, following Starmer and Sugden (1991), Beattie and Loomes (1997), Cubitt et al. (1998a), Cox et al. (2011) and Harrison and Swarthout (2012). For our purposes the critical feature of our design is that we do not test the binary lottery procedure conditional on some needlessly restrictive axiom being valid.

The standard language in the instructions for treatment A that describe the lotteries sets the stage for the variants in other treatments:

The outcome of the prospects will be determined by the draw of a random number between 1 and 100. Each number between, and including, 1 and 100 is equally likely to occur. In fact, you will be able to draw the number yourself using two 10-sided dice.

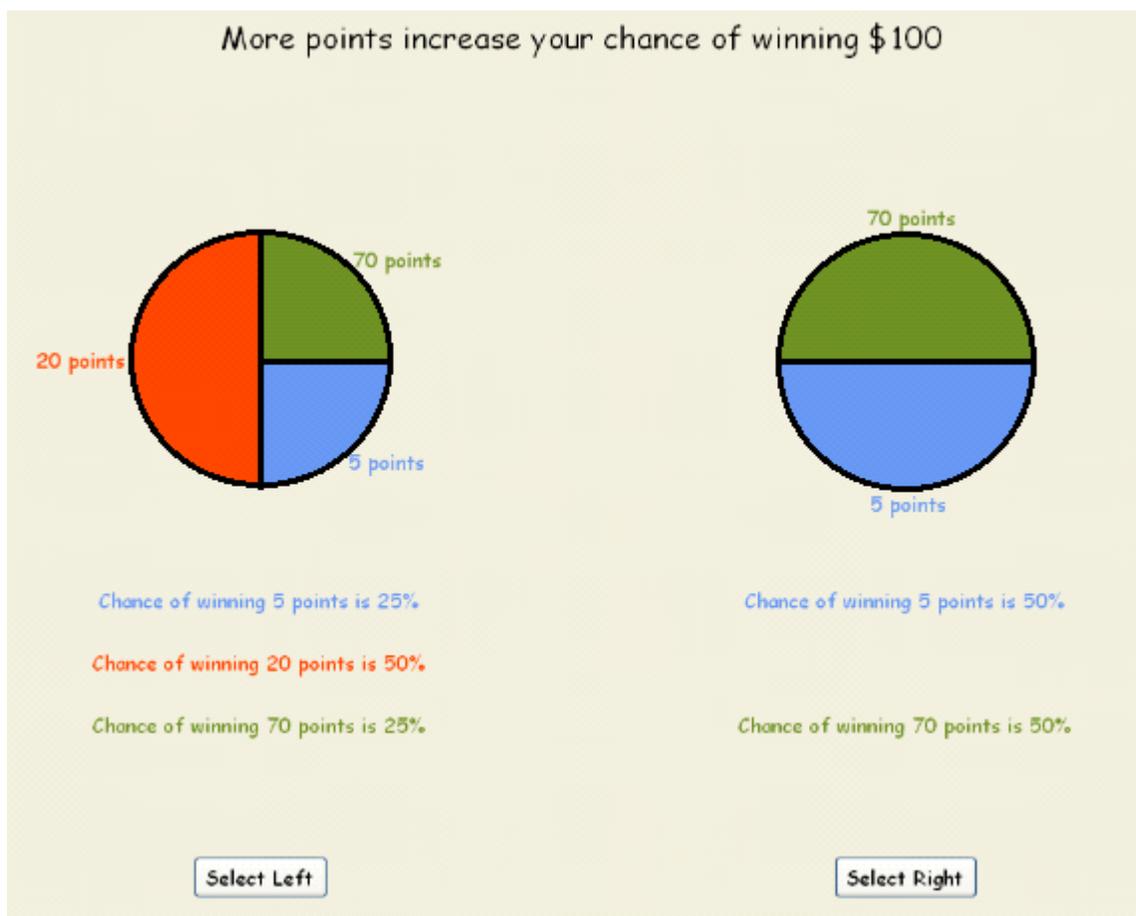
In the above example the left prospect pays five dollars (\$5) if the number drawn is between 1 and 40, and pays fifteen dollars (\$15) if the number is between 41 and 100. The blue color in the pie chart corresponds to 40% of the area and illustrates the chances that the number drawn will be between 1 and 40 and your prize will be \$5. The orange area in the pie chart corresponds to 60% of the area and illustrates the chances that the number drawn will be between 41 and 100 and your prize will be \$15.

Now look at the pie in the chart on the right. It pays five dollars (\$5) if the number drawn is between 1 and 50, ten dollars (\$10) if the number is between 51 and 90, and fifteen dollars (\$15) if the number is between 91 and 100. As with the prospect on the left, the pie slices represent the fraction of the possible numbers which yield each payoff. For example, the size of the \$15 pie slice is 10% of the total pie.

This language is changed in as simple a manner as possible to introduce the lotteries defined over points in the following treatments.

**Treatment B** introduces the binary lottery procedure in which the initial lottery choice is over prizes defined in points, matching the monetary prizes used in treatment A. We use same lotteries in treatment A to construct the *initial* lotteries in treatment B, but with the interim prizes defined in terms of points as shown in Figure 4.2.

Figure 4.2: Choice Interface for Points



We construct the lotteries in our Treatment B battery by interpreting the dollar amounts as points that define the probability of getting the highest prize of \$100. For example, consider the lottery pair from Treatment A where the left lottery is (\$20, 0%; \$35, 75%; \$70, 25%) and the right lottery is (\$20, 25%; \$35, 0%; \$70, 75%). We then construct a lottery pair that the subject sees in Treatment B by defining the monetary prizes as points: so the left lottery becomes (20 points, 0%; 35 points, 75%; 70 points, 25%) and the right lottery becomes (20 points, 25%; 35 points, 0%; 70 points, 75%). The outcomes of these lotteries are points that determine the probability of winning the highest prize.

Therefore, these initial lotteries defined in terms of points are in fact binary compound lotteries in Treatment B, mapping into the two final monetary prizes of \$100 and \$0.<sup>16</sup> The left lottery is a compound lottery that gives the subject 75% chance of playing the lottery (\$100, 35%; \$0, 65%) and 25% probability of playing (\$100, 70%; \$0, 30%). Similarly, the right lottery is a compound lottery that offers the subject 25% chance of playing (\$100, 20%; \$0, 80%) and 75% chance of playing (\$100, 70%; \$0, 30%). The actuarially-equivalent simple lotteries of these compound lotteries are (\$100, 43.75%; \$0, 56.25%) and (\$100, 57.50%; \$0, 42.50%), respectively, but these actuariallyequivalent simple lotteries are obviously not presented to subjects as such.

The relevant part of the instructions mimics the information given for treatment A, but with respect to points, and then explains how points are converted to monetary prizes:

You earn points in this task. We explain below how points are converted to cash payoffs.

The outcome of the prospects will be determined by the draw of two random numbers between 1 and 100. The first random number drawn determines the number of points you earn in the chosen prospect, and the second random number determines whether you win the high or the low amount according to the points earned. The high amount is \$100 and the low amount is \$0. Each random number between, and including, 1 and 100 is equally likely to occur. In fact, you will be able to draw the two random numbers yourself by rolling two 10-sided dice twice.

The payoffs in each prospect are points that give you the chance of winning the \$100 high amount. The more points you earn, the greater your chance of winning \$100. In the left prospect of the above example you earn five points (5) if the outcome of the first dice roll is between 1 and 25, twenty points (20) if the outcome of the dice roll is between 26 and 75, and seventy points (70) if the outcome of the roll is between 76 and 100. The blue color in the pie chart corresponds to 25% of the area and illustrates the chances that the number drawn will be between 1 and 25 and your prize will be 5 points. The orange area in the pie chart corresponds to 50% of the area and illustrates the chances

---

<sup>16</sup>To be verbose, to anticipate the extension to Treatment D, each of the lotteries in *points* are simple lotteries, and each of the lotteries in *money* are now compound lotteries. Thus the MIA would be needed to justify the RLIM in Treatment D; the RLIM is not needed in Treatment B.

that the number drawn will be between 26 and 75 and your prize will be 20 points. Finally, the green area in the pie chart corresponds to the remaining 25% of the area and illustrates that the number drawn will be between 76 and 100 and your prize is 70 points.

Now look at the pie in the chart on the right. You earn five points (5) if the first number drawn is between 1 and 50 and seventy points (70) if the number is between 51 and 100. As with the prospect on the left, the pie slices represent the fraction of the possible numbers which yield each payoff. For example, the size of the 5 points pie slice is 50% of the total pie.

Every point that you earn gives you greater chance of being paid for this task. If you earn 70 points then you have a 70% chance of being paid \$100. If you earn 20 points then you have a 20% chance of being paid \$100. After you determine the number of points that you earn by rolling the two 10-sided dice once, you will then roll the same dice for a second time to determine if you get \$100 or \$0. If your second roll is a number that is less than or equal to the number of points that you earned, you win \$100. If the second roll is a number that is greater than the number of points that you earned, you get \$0. If you do not win \$100 you receive nothing from this task, but of course you get to keep your show-up fee. Again, the more points you earn the greater your chance of winning \$100 in this task.

**Treatment C** extends Treatment A by asking subjects to make  $K > 1$  binary lottery choices over prizes defined by monetary prizes and then selecting one of the  $K$  at random for resolution and payment.<sup>17</sup> This is the case that is most widely used in the experimental literature, and relies on the RLIM procedure for the choice patterns to be comparable to those in Treatment A. In turn, the RLIM procedure rests on the validity of the IA, as noted earlier.

**Treatment D** extends Treatment B and applies the lottery procedure to the situation in which the subject makes  $K > 1$  binary lottery choices over prizes defined initially by points.<sup>18</sup> Hence it also relies on the validity of the RLIM procedure for choices in

---

<sup>17</sup> $K=30$  or  $40$  in all tasks in Treatment C. Lottery pairs were selected from a wider range than those used in Treatments B and D, but only lottery pairs that match those found in Treatments B and D are reported to ensure comparability.

<sup>18</sup> $K=24$  in all tasks in Treatment D.

Treatment D and Treatment B to be the same. The test of the binary lottery procedure that is generated by comparing Treatments C and D is therefore a joint test of the Binary ROCL axiom and the IA.

Treatment E extends Treatment B by adding information on the expected value of each lottery in the choice display. The only change in the interface is to add the text atop each lottery shown in Figure 4.3. We deliberately introduce the notion of expectation using a natural frequency representation, as in the statement, “If this prospect were played 1000 times, on average the payoff would be 37.5 points.” The instructions augmented those for Treatment B with just this extra paragraph:

Above each prospect you will be told what the average payoff would be if this prospect was played 1000 times. You will only play the prospect once if you choose it.

No other changes in procedures were employed compared to Treatment B.

Finally, **Treatment F** added some “cheap talk” explanation as to why it might be in the best interest of the decision maker to choose lotteries so as to maximize expected points:

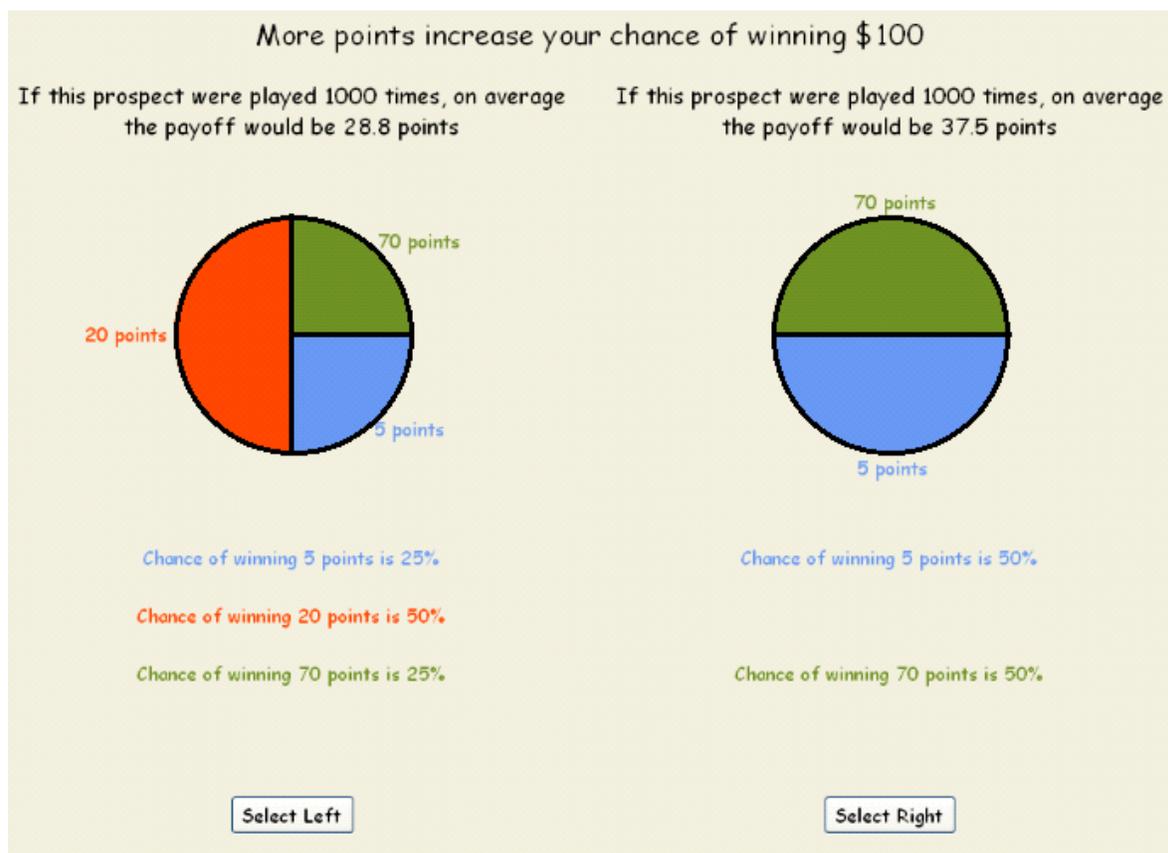
You maximize your chance of winning \$100 by choosing the prospect that gives you on average the highest number of points. However, this may not be perfectly clear, so we will now explain why this is true.

Continue with the example above, and suppose you choose the prospect on the left. You can expect to win 28.8 points on average if you played it enough times. This means that your probability of winning \$100 would be 28.8% on average. However, if you choose the prospect on the right you can expect to win more points on average: the expected number of points is 37.5. Therefore, you can expect to win \$100 with 37.5% probability on average.

You can see in the example above that by choosing the prospect on the left you would win on average less points than in the prospect on the right. Therefore, your chances of winning \$100 in the prospect on left are lower on average than your chances of winning \$100 in the prospect on the right.

Therefore, you maximize your chances of winning \$100 by choosing the prospect that offers the highest expected number of points.

Figure 4.3: Choice Interface for Points with Expected Value Information



These instructions necessarily build on the notion of the expected value, so it would not be natural to try to generate a treatment with cheap talk without providing the EV information.

We acknowledge openly that these normative variants might end up working in the desired direction but for the wrong reason. Providing the EV might simply “anchor” behavior directly, and both might generate linear utility because of “demand effects.” In one sense, we do not care what the explanation is, as long as the procedures reliably generate behavior consistent with linear utility functions. In another sense, we do care, because the observed behavior might not be reliable for normative evaluation of behavior.<sup>19</sup>

<sup>19</sup>The issue is subtle, but should not be glossed. It is akin to evaluating preferences revealed by choices

## 4.5 Results

The basic results can be presented in terms of choice patterns that are consistent or not with the prediction that subjects will pick the lottery with the greatest EV. We then extend the analysis to allow for a cardinal measure of the extent of deviation from EV maximization, as well as structural models of behavior, to better evaluate the effect of the treatments. Evaluating choice patterns has the advantage that one can remain agnostic about the particular model of decision making under risk being used, but it has the disadvantage that one does not use all of the information in the stimuli. The information that is not used is the difference in EV between the two lotteries: intuitively, a deviation from EV maximization should be more serious if the EV difference is large than when it is minuscule. Of course, to use that information one has to make some assumptions about what determines the probability of any predicted choice.

A structural model of behavior, using Expected Utility Theory for example, allows one to use information on the size of errors from the perspective of the null hypothesis. For example, choices that are inconsistent with the null hypothesis but that involve statistically insignificant errors from the perspective of that hypothesis are not treated with the same weight as statistically significant errors. One setting in which this could arise is if we had some subjects that were approximately risk neutral over monetary prizes, and some who were decidedly risk averse. In a statistical sense, we should care more about the validity of the choices of the latter subjects: a structural model allows that, conditional of course on

---

after individuals have been exposed to advertizing. We add this rhetorical warning, since modern behaviorists are fond of casually referring to “constructed preferences” as if the concept had some operational meaning.

“the assumed structure,” but the evaluation of choice patterns treats these choices equally. In addition, it is relatively easy to extend the structural model to allow for varying degrees of heterogeneity of preferences, which is essential for between-subject tests of the lottery procedure.

In the end, we draw essentially the same conclusions from evaluating choice patterns and structural estimates of preferences.

#### **4.5.1 Do Subjects Pick the Lottery With the Higher Expected Value?**

The primary hypothesis is crisp: that the binary lottery procedure generates more choices that are consistent with risk neutral behavior. We calculate the EV for each lottery, and then simply tabulate how many choices were consistent with that prediction. Table 4.2 contains these results.

The fraction of choices consistent with risk neutrality in Table 4.2 starts out in the control Treatment A at 60.0%, and increases to 73.9% in Treatment B. This difference is statistically significant, and in the predicted direction. A Fisher Exact test rejects the hypothesis that treatments A and B generate the same choice patterns with a (one-sided)  $p$ -value of 0.073. Because the binary lottery procedure predicts the direction of differences in choices, a one-sided test is the appropriate one to use.

Turning to the comparison of choice patterns in Treatments C and D, one observes the same trend. The fraction of choices consistent with risk neutrality increases from 63.5% in Treatment C increases to 68.7% in Treatment D. Even though this is a smaller increment

Table 4.2: Observed Choice Patterns

Treatment	Risk neutral choices	Other choices	All choices
A. Monetary prizes with one choice (Figure 1)	33 (60%)	22 (40%)	55 (100%)
B. Binary lottery points with one choice (Figure 2)	51 (74%)	18 (26%)	69 (100%)
C. Monetary prizes with $K \gg 1$ choices (Figure 1)	1,336 (63%)	768 (37%)	2,104 (100%)
D. Binary lottery points with $K \gg 1$ choices (Figure 2)	643 (69%)	293 (31%)	936 (100%)
E. Binary lottery points with one choice and EV information (Figure 3)	24 (71%)	10 (29%)	34 (100%)
F. Binary lottery points with one choice and EV information (Figure 3), plus “cheap talk” instructions	30 (79%)	8 (21%)	38 (100%)

in percentage points than for Treatments A and B, the sample sizes are significantly larger: by design,  $K$  times larger per subject. If we momentarily ignore the fact that each subject contributed several choices to these data, we can again apply a one-sided Fisher Exact test and reject the hypothesis that Treatments C and D generate the same choice patterns with a  $p$ -value of 0.003. However, we do need to correct for this clustering at the level of the individual, and an appropriate test statistic in this case is the Pearson  $\chi^2$  statistic adjusted for clustering with the second-order correction of Rao and Scott (1984). This test leads one to reject the null hypothesis with a one-sided  $p$ -value of 0.031.

Treatments E and F add normative tweaks to the binary lottery procedure, to see if one can nudge the fraction of risk neutral choices even higher than in Treatment B. The effects are mixed, although parallel to the effect of Treatment B compared to Treatment A.

Adding information on the EV does not make much of a difference to the “vanilla” binary

lottery procedure, nor does adding “cheap talk.” Of course, this is completely consistent with the hypothesis that the subjects that moved towards risk neutral choices already understood how to guesstimate or calculate the EV, and that this would be how they maximize their chance of winning the \$100. Pooling Treatments B, E and F together, and comparing to Treatment A, we can reject the null hypothesis of no change compared to Treatment A using a Fisher Exact test and a  $p$ -value of 0.036.

#### 4.5.2 Effect on Expected Value Maximization

As noted earlier, Selten et al. (1999) developed a statistic to test the strength of the deviation from risk neutrality and EV maximization. For all choices by a subject, it simply takes the difference between the maximum EV that could have been earned and the EV that was chosen. A risk neutral subject would have a statistic value of zero, and a risk averse subject would generally have a positive statistic value. So the null hypothesis is that the lottery procedure moves the value of this statistic to zero, or at least in that direction, compared to the treatment with direct monetary prizes. This statistic aggregates all choices by a given subject, so can be calculated in a similar manner for all of our treatments. Statistical significance is then tested by conducting nonparametric tests of the hypothesis that the distribution of these statistics is the same across treatments.

The average values for this statistic for Treatments A, B, C, D, E and F are \$2.57, 1.87 points, \$2.79, 2.31 points, 1.29 points and 1.28 points, respectively. So there is movement in the predicted direction for the binary lottery treatments B, E and F when compared to Treatment A, and for Treatment D compared to Treatment C. For the treatments with

only one choice task, we find overall that the statistic moves in the right direction, and significantly. Pooling over Treatments B, E and F, the average statistic is 1.57 points, compared to \$2.57 for Treatment A. This difference in means is statistically significant in a t-test with a one-sided alternative hypothesis test and a  $p$ -value of 0.066, assuming unequal variances. Using the Wilcoxon-Mann-Whitney two-sample test of rank sums, we also conclude that the distributions are different, with a one-sided  $p$ -value of only 0.025. For Treatments C and D we find that the statistic again moves in the right direction and that the differences are statistically significant, using either the rank sum test of the distributions or the t-test and  $p$ -values less than 0.01.

### 4.5.3 Effect on Expected Value Maximization

Section 4.9 outlines a simple specification of a structural model to estimate risk preferences, assuming Expected Utility Theory (EUT). The specification is by now quite standard, and is explained in detail by Harrison and Rutström (2008). We generally assume a Constant Relative Risk Aversion (CRRA) utility function with coefficient  $r$ , such that  $r = 0$  denotes risk neutrality and  $r > 0$  denotes risk aversion under EUT.

The estimates are striking. Initially assume that differences in risk preferences were randomized across treatments, so that the average effect of the treatment can be reliably estimated without controlling for heterogeneity of preferences. Under Treatment A we estimate  $r$  to be 0.981, with a 95% confidence interval between 0.54 and 1.42, and the effect of Treatment B is to lower that by 0.912 such that the estimated  $r$  for Treatment B is only 0.069 with a 95% confidence interval between -0.45 and 0.59. The  $p$ -value on the test that

the Treatment B risk aversion coefficient is different from zero is 0.793, so we cannot reject the hypothesis that the lottery procedure worked as advertized.

The effect of the lottery procedure is not so sharp when we consider the designs of Treatments C and D that employ the RLIM payment protocol. In this case the risk aversion coefficient  $r$  for Treatment C is estimated to be 0.725 with a 95% confidence interval between 0.66 and 0.79, and the effect of the lottery procedure is to lower that by 0.45. Hence the estimated risk aversion coefficient under Treatment D is 0.161, with a 95% confidence interval between 0.15 and 0.17, and a  $p$ -value on the one-sided hypothesis of risk neutrality of only 0.032. So we observe clear movement in the direction of risk neutrality, but not the attainment of risk neutrality. The estimated effect of the lottery procedure, -0.45, has a 95% confidence interval between -0.75 and -0.15.

We can extend these structural models to provide some allowance for subject heterogeneity. Because the data for each subject in Treatments A and B consist of just one observation, one loses degrees of freedom rapidly with too many demographic characteristics. For example, in samples of 55, how many Asian females are Seniors? Larger samples would obviously mitigate this issue, but for present purposes a simpler solution is to merge in data from comparable tasks and samples drawn at random from the same population. In this case we were able to use data for Treatment A using lotteries that use the same prizes and probabilities, but in different combinations than the 24 we focus on in the comparisons of choice patterns.<sup>20</sup> This increases the sample size for estimation from 55 to 149 under Treatment A.<sup>21</sup> This is not appropriate when one is comparing choice patterns, since the

---

<sup>20</sup>The additional lotteries are documented in Harrison and Swarthout (2012).

<sup>21</sup>The fraction of choices consistent with risk neutrality drops slightly, from 60.0% to 58.4%., with the

stimuli are different in nature, but is appropriate when one is estimating risk preferences. Allowing for subject heterogeneity confirms our qualitative conclusions from assuming that randomization to treatment led to the same distribution of preferences across treatments. Detailed estimation results are provided in section 4.9, and control for a number of binary characteristics: **blp** is 1 for choices in Treatment B or Treatment D; **female** is 1 for women, and 0 otherwise; **sophomore** and **senior** are 1 for whether that was the current stage of undergraduate education at GSU, and 0 otherwise; and **asian** and **white** are 1 based on self-reported ethnic status, and 0 otherwise.

Controlling for observable characteristics in this manner, for Treatments A and B we estimate the coefficient on the lottery procedure dummy to be -0.70, with a  $p$ -value of 0.034, and the constant term for Treatment A to be 0.74 with a  $p$ -value of less than 0.001. The net effect, the estimated coefficient for Treatment B after controlling for the demographic covariates, is -0.042 with a  $p$ -value of 0.90, so we again cannot reject the null hypothesis that the lottery procedure induces risk neutral behavior. Predicting risk attitudes using these estimates, the average  $r$  for Treatment A is 0.63, and for Treatment B is -0.077. Figure 4.4 displays kernel densities of the predicted risk attitudes over all subjects, demonstrating the dramatic effect of the binary lottery procedure. These conclusions stay the same if we pool in the choices from Treatments E and F; again, the normative variants in the binary lottery procedure displays and instructions do not, by themselves, make much of a difference.

For Treatments C and D we estimate the effect of the lottery procedure on the risk

---

enhanced sample.

aversion coefficient to be  $-0.48$  with a  $p$ -value of  $0.003$ , compared to the constant term of  $0.67$  with a  $p$ -value also less than  $0.001$ . So the net effect, demographics aside, is for the lottery procedure to lower the estimated risk aversion to  $0.20$  with a 95% confidence interval between  $-0.14$  and  $0.53$  and a  $p$ -value of  $0.25$ . Figure 4.5 shows the distribution of estimated risk attitudes from predicted values that account for heterogeneity of preferences. The average predicted risk aversion in Treatment C is  $0.73$  and in Treatment D is  $0.26$ . The effect is not as complete as estimated for Treatments A and B, but clearly in the predicted direction.

Figure 4.4: Estimated Risk Attitudes in Treatments A and B

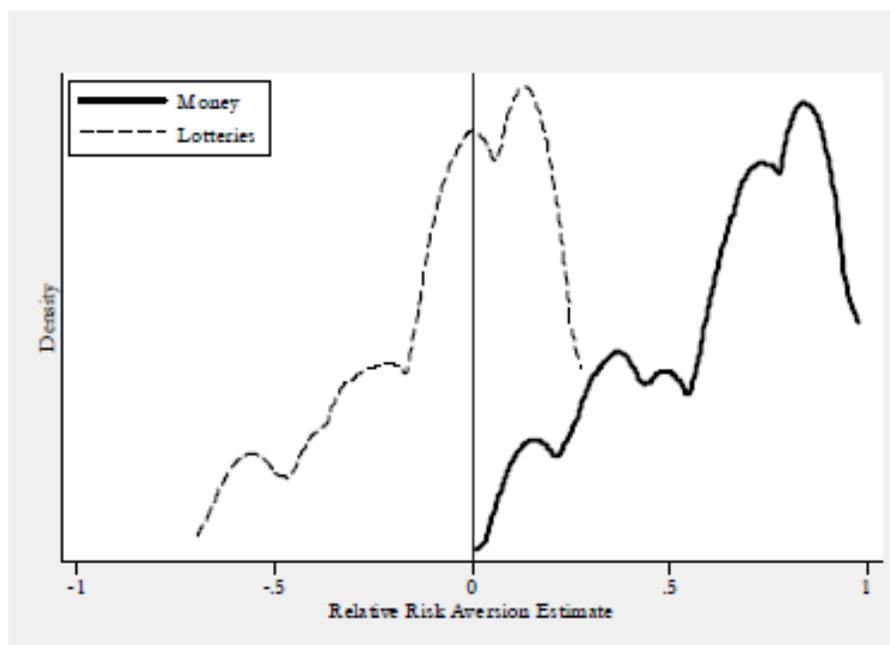
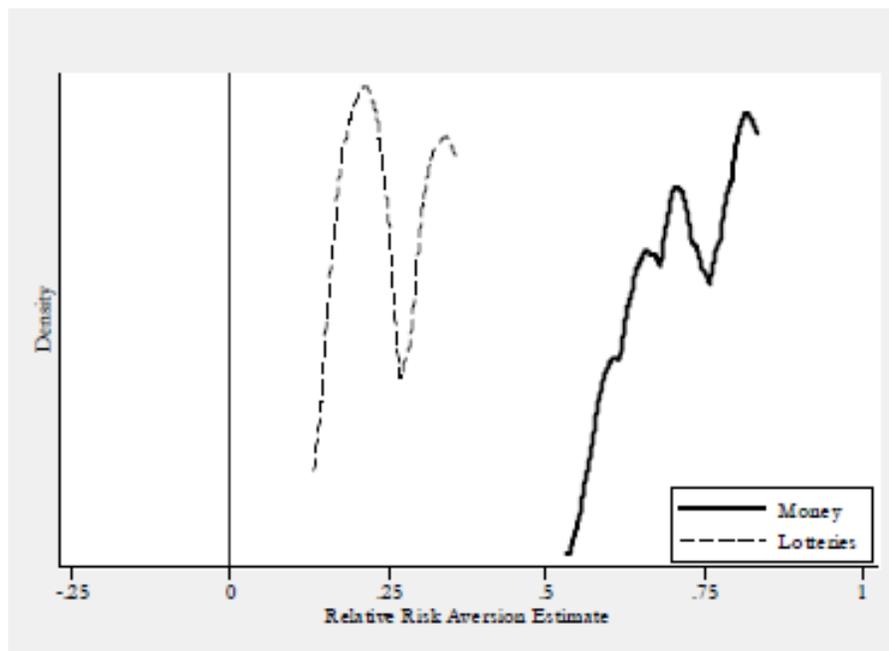


Figure 4.5: Estimated Risk Attitudes in Treatments C and D



## 4.6 Conclusions

Our results clearly show that the binary lottery procedure works for samples of university level students in the simplest possible environment, where we can be certain that there are no contaminating factors and the theory to be tested requires no auxiliary assumptions.

This does not automatically make the lottery procedure useful for samples from different populations. Nor does it automatically mean that it applies in all settings, since it is often the “contaminating factor,” such as strategic behavior, that is precisely the domain where we would like it to work. But there are many circumstances where one can implement the environment considered here.

We find that the lottery procedure works robustly to induce risk neutrality when subjects are given one task, and that it works well when subjects are given more than one task. The

extent to which the procedure works is certainly diminished as one moves from environments with one task to environments with many tasks, but there is always a statistically significant reduction in risk aversion, and in neither case can one reject the hypothesis that the procedure induced risk neutral behavior as advertized.

Our results should encourage efforts to actively try to find procedures that can identify and increase the sub-sample of subjects for whom the lottery procedure does induce linear utility, and the populations for which it appears to work reliably.<sup>22</sup> Even with a given population, it is logically possible that the procedure “works as advertized” for some subjects, just not all, or even for a majority. There can still be value in identifying those subjects. Moreover, if simple treatments can increase that fraction, or just improve the statistical identification of that fraction, then we might discover a “best practice” variant of the basic lottery procedure. Although the variants we considered in our design did not increase the fraction of risk neutral choices significantly, they could play a behavioral role in other populations.

---

<sup>22</sup>For example, Hossain and Okui (2011) evaluate the procedure in the context of eliciting the probability of a binary event.

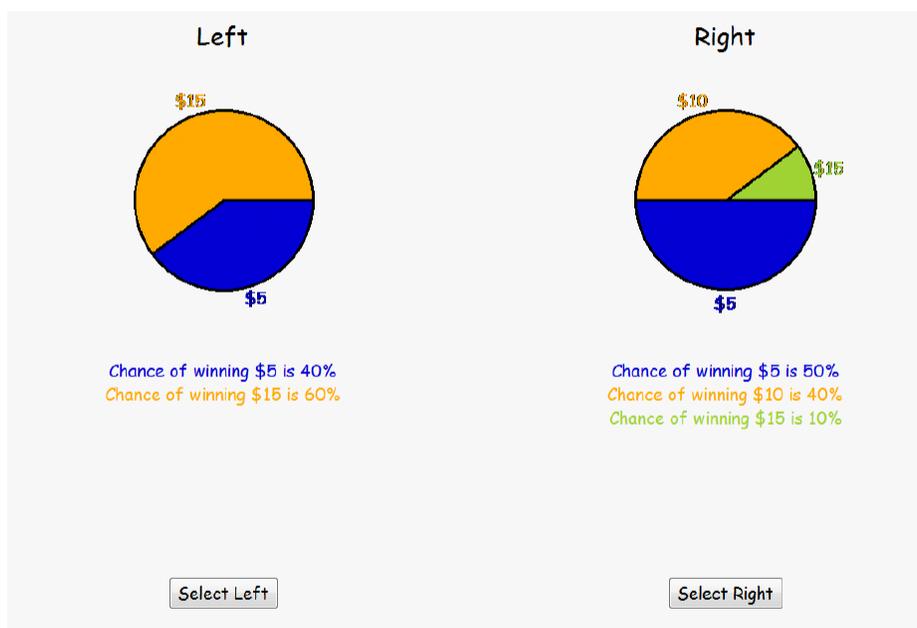
## 4.7 Appendix A. Instructions

*Treatment A*

### Choices Over Risky Prospects

This is a task where you will choose between prospects with varying prizes and chances of winning. You will be presented with one pair of prospects where you will choose one of them. You should choose the prospect you prefer to play. You will actually get the chance to play the prospect you choose, and you will be paid according to the outcome of that prospect, so you should think carefully about which prospect you prefer.

Here is an example of what the computer display of a pair of prospects will look like.



The outcome of the prospects will be determined by the draw of a random number between 1 and 100. Each number between, and including, 1 and 100 is equally likely to occur. In fact, you will be able to draw the number yourself using two 10-sided dice.

In the above example the left prospect pays five dollars (\$5) if the number drawn is between 1 and 40, and pays fifteen dollars (\$15) if the number is between 41 and 100. The blue color in the pie

chart corresponds to 40% of the area and illustrates the chances that the number drawn will be between 1 and 40 and your prize will be \$5. The orange area in the pie chart corresponds to 60% of the area and illustrates the chances that the number drawn will be between 41 and 100 and your prize will be \$15.

Now look at the pie in the chart on the right. It pays five dollars (\$5) if the number drawn is between 1 and 50, ten dollars (\$10) if the number is between 51 and 90, and fifteen dollars (\$15) if the number is between 91 and 100. As with the prospect on the left, the pie slices represent the fraction of the possible numbers which yield each payoff. For example, the size of the \$15 pie slice is 10% of the total pie.

The pair of prospects you choose from is shown on a screen on the computer. On that screen, you should indicate which prospect you prefer to play by clicking on one of the buttons beneath the prospects.

After you have made your choice, raise your hand and an experimenter will come over. It is certain that your one choice will be played out for real. You will roll the two ten-sided dice to determine the outcome of the prospect you chose.

For instance, suppose you picked the prospect on the left in the above example. If the random number was 37, you would win \$5; if it was 93, you would get \$15. If you picked the prospect on the right and drew the number 37, you would get \$5; if it was 93, you would get \$15.

Therefore, your payoff is determined by two things:

- by which prospect you selected, the left or the right; and
- by the outcome of that prospect when you roll the two 10-sided dice.

Which prospects you prefer is a matter of personal taste. The people next to you may be presented with a different prospect, and may have different preferences, so their responses should not matter to you. Please work silently, and make your choices by thinking carefully about the prospect you are presented with.

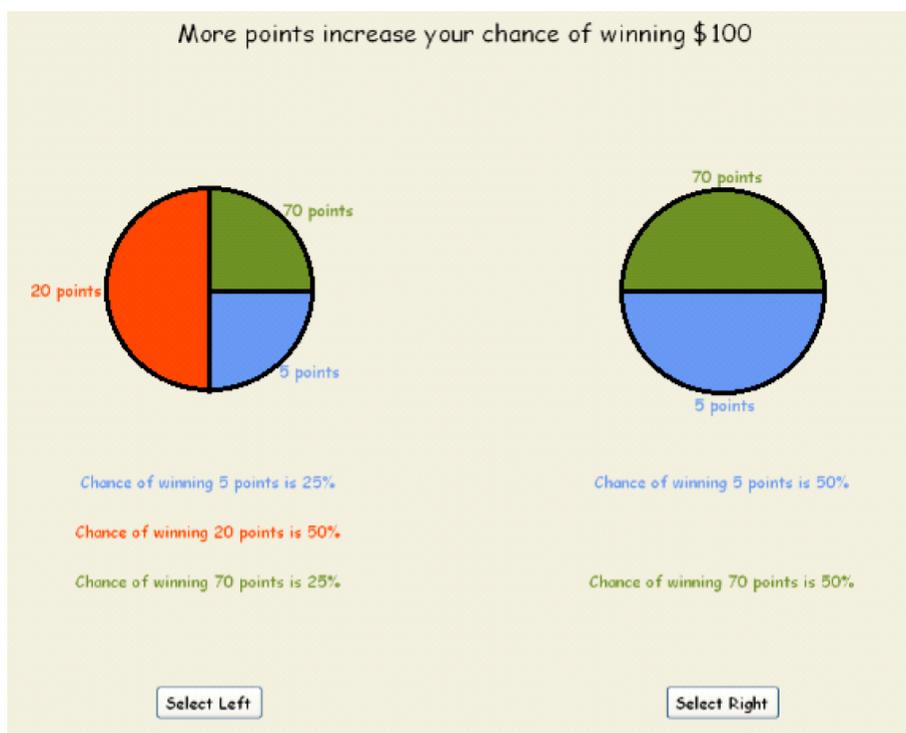
All payoffs are in cash, and are in addition to the \$7.50 show-up fee that you receive just for being here. The only other task today is for you to answer some demographic questions. Your answers to those questions will not affect your payoffs.

#### *Treatment B*

### **Choices Over Risky Prospects**

This is a task where you will choose between prospects with varying chances of winning either a high amount or a low amount. You will be presented with one pair of prospects where you will choose one of them. You should choose the prospect you prefer to play. You will actually get the chance to play the prospect you choose, and you will be paid according to the final outcome of that prospect, so you should think carefully about which prospect you prefer.

Here is an example of what the computer display of a pair of prospects will look like.



You earn points in this task. We explain below how points are converted to cash payoffs.

The outcome of the prospects will be determined by the draw of two random numbers between 1 and 100. The first random number drawn determines the number of points you earn in the chosen prospect, and the second random number determines whether you win the high or the low amount according to the points earned. The high amount is \$100 and the low amount is \$0. Each random number between, and including, 1 and 100 is equally likely to occur. In fact, you will be able to draw the two random numbers yourself by rolling two 10-sided dice twice.

The payoffs in each prospect are points that give you the chance of winning the \$100 high amount. The more points you earn, the greater your chance of winning \$100. In the left prospect of the above example you earn five points (5) if the outcome of the first dice roll is between 1 and 25, twenty points (20) if the outcome of the dice roll is between 26 and 75, and seventy points (70) if the outcome of the roll is between 76 and 100. The blue color in the pie chart corresponds to 25% of the area and illustrates the chances that the number drawn will be between 1 and 25 and your prize will

be 5 points. The orange area in the pie chart corresponds to 50% of the area and illustrates the chances that the number drawn will be between 26 and 75 and your prize will be 20 points. Finally, the green area in the pie chart corresponds to the remaining 25% of the area and illustrates that the number drawn will be between 76 and 100 and your prize is 70 points.

Now look at the pie in the chart on the right. You earn five points (5) if the first number drawn is between 1 and 50 and seventy points (70) if the number is between 51 and 100. As with the prospect on the left, the pie slices represent the fraction of the possible numbers which yield each payoff. For example, the size of the 5 points pie slice is 50% of the total pie.

Every point that you earn gives you greater chance of being paid for this task. If you earn 70 points then you have a 70% chance of being paid \$100. If you earn 20 points then you have a 20% chance of being paid \$100. After you determine the number of points that you earn by rolling the two 10-sided dice once, you will then roll the same dice for a second time to determine if you get \$100 or \$0. If your second roll is a number that is less than or equal to the number of points that you earned, you win \$100. If the second roll is a number that is greater than the number of points that you earned, you get \$0. If you do not win \$100 you receive nothing from this task, but of course you get to keep your show-up fee. Again, the more points you earn the greater your chance of winning \$100 in this task.

The pair of prospects you choose from is shown on a screen on the computer. On that screen, you should indicate which prospect you prefer to play by clicking on one of the buttons beneath the prospects.

After you have made your choice, raise your hand and an experimenter will come over. It is certain that your one choice will be played out for real. You will then roll the two ten-sided dice twice: first to determine the number of points you win, and then to determine whether you win \$100 or \$0 according to the points earned.

For instance, suppose you picked the prospect on the left in the above example. If the outcome of the first dice roll was 17, you would win 5 points. This means that your chance of winning \$100 is 5%. Now, if the outcome of the second dice roll was 1, which is less than the number of points you earned, you would win \$100. If the second roll was 75 instead, you would earn nothing because the outcome of this roll is greater than the number of points you earned.

Here is another example. If you picked the prospect on the right and the outcome of the first dice roll was 60, then you would earn 70 points. This means that your probability of winning \$100 would be 70%. If the outcome of the second dice roll was 1, then you would earn \$100; but if the second roll was 75 instead you would earn \$0 in this task.

Therefore, your payoff is determined by three things:

- the prospect you selected, the left or the right;
- the outcome of the first roll of the two 10-sided dice which determines the number of points you earn in your chosen prospect; and
- the outcome of the second roll of the two 10-sided dice which will be compared with your earned points to determine whether you earn \$0 or \$100.

Which prospect you prefer is a matter of personal taste. The people next to you may be presented with a different pair of prospects, so their responses should not matter to you. Please work silently, and make your choices by thinking carefully about the pair of prospects you are presented with.

All payoffs are in cash, and are in addition to the \$7.50 show-up fee that you receive just for being here. The only other task today is for you to answer some demographic questions. Your answers to those questions will not affect your payoffs.

### *Treatment C*

#### **Choices Over Risky Prospects**

This is a task where you will choose between prospects with varying prizes and chances of winning. You will be presented with a series of pairs of prospects where you will choose one of them. There are 30 pairs in the series. For each pair of prospects, you should choose the prospect you prefer to play. You will actually get the chance to play one of the prospects you choose, and you will be paid according to the outcome of that prospect, so you should think carefully about which prospect you prefer.

Here is an example of what the computer display of such a pair of prospects will look like.

#### SAME DISPLAY AS FOR TREATMENT A

The outcome of the prospects will be determined by the draw of a random number between 1 and 100. Each number between, and including, 1 and 100 is equally likely to occur. In fact, you will be able to draw the number yourself using two 10-sided dice.

In the above example the left prospect pays five dollars (\$5) if the number drawn is between 1 and 40, and pays fifteen dollars (\$15) if the number is between 41 and 100. The blue color in the pie chart corresponds to 40% of the area and illustrates the chances that the number drawn will be between 1 and 40 and your prize will be \$5. The orange area in the pie chart corresponds to 60% of the area and illustrates the chances that the number drawn will be between 41 and 100 and your prize will be \$15.

Now look at the pie in the chart on the right. It pays five dollars (\$5) if the number drawn is between 1 and 50, ten dollars (\$10) if the number is between 51 and 90, and fifteen dollars (\$15) if the number is between 91 and 100. As with the prospect on the left, the pie slices represent the fraction of the possible numbers which yield each payoff. For example, the size of the \$15 pie slice is 10% of the total pie.

Each pair of prospects is shown on a separate screen on the computer. On each screen, you should indicate which prospect you prefer to play by clicking on one of the buttons beneath the prospects.

After you have worked through all of the pairs of prospects, raise your hand and an

experimenter will come over. You will then roll a 30-sided die to determine which pair of prospects will be played out. Since there is a chance that any of your 30 choices could be played out for real, you should approach each pair of prospects as if it is the one that you will play out. Finally, you will roll the two ten-sided dice to determine the outcome of the prospect you chose.

For instance, suppose you picked the prospect on the left in the above example. If the random number was 37, you would win \$5; if it was 93, you would get \$15. If you picked the prospect on the right and drew the number 37, you would get \$5; if it was 93, you would get \$15.

Therefore, your payoff is determined by three things:

- by which prospect you selected, the left or the right, for each of these 30 pairs;
- by which prospect pair is chosen to be played out in the series of 30 such pairs using the 30-sided die; and
- by the outcome of that prospect when you roll the two 10-sided dice.

Which prospects you prefer is a matter of personal taste. The people next to you may be presented with different prospects, and may have different preferences, so their responses should not matter to you. Please work silently, and make your choices by thinking carefully about each prospect.

All payoffs are in cash, and are in addition to the \$7.50 show-up fee that you receive just for being here. The only other task today is for you to answer some demographic questions. Your answers to those questions will not affect your payoffs.

*Treatment D*

### **Choices Over Risky Prospects**

This is a task where you will choose between prospects with varying chances of winning either a high amount or a low amount. You will be presented with a series of pairs of prospects where you will choose one of them. There are 24 pairs in the series. For each pair of prospects, you should choose the prospect you prefer to play. You will actually get the chance to play one of the prospects you choose, and you will be paid according to the final outcome of that prospect, so you should think carefully about which prospect you prefer.

Here is an example of what the computer display of a pair of prospects will look like.

SAME DISPLAY AS FOR TREATMENT B

You earn points in this task. We explain below how points are converted to cash payoffs.

The outcome of the prospects will be determined by the draw of two random numbers between 1 and 100. The first random number drawn determines the number of points you earn in the chosen prospect, and the second random number determines whether you win the high or the low amount according to the points earned. The high amount is \$100 and the low amount is \$0. Each random number between, and including, 1 and 100 is equally likely to occur. In fact, you will be able to draw the two random numbers yourself by rolling two 10-sided dice twice.

The payoffs in each prospect are points that give you the chance of winning the \$100 high amount. The more points you earn, the greater your chance of winning \$100. In the left prospect of the above example you earn five points (5) if the outcome of the first dice roll is between 1 and 25, twenty points (20) if the outcome of the dice roll is between 26 and 75, and seventy points (70) if the outcome of the roll is between 76 and 100. The blue color in the pie chart corresponds to 25% of the area and illustrates the chances that the number drawn will be between 1 and 25 and your prize will be 5 points. The orange area in the pie chart corresponds to 50% of the area and illustrates the chances that the number drawn will be between 26 and 75 and your prize will be 20 points. Finally, the green area in the pie chart corresponds to the remaining 25% of the area and illustrates that the number drawn will be between 76 and 100 and your prize is 70 points.

Now look at the pie in the chart on the right. You earn five points (5) if the first number drawn is between 1 and 50 and seventy points (70) if the number is between 51 and 100. As with the prospect on the left, the pie slices represent the fraction of the possible numbers which yield each payoff. For example, the size of the 5 points pie slice is 50% of the total pie.

Every point that you earn gives you greater chance of being paid for this task. If you earn 70 points then you have a 70% chance of being paid \$100. If you earn 20 points then you have a 20% chance of being paid \$100. After you determine the number of points that you earn by rolling the two 10-sided dice once, you will then roll the same dice for a second time to determine if you get \$100 or \$0. If your second roll is a number that is less or equal to the number of points that you earned, you win \$100. If the second roll is a number that is greater than the number of points that you earned, you get \$0. If you do not win \$100 you receive nothing from this task, but of course you get to keep your show-up fee. Again, the more points you earn the greater your chance of winning \$100 in this task.

Each pair of prospects is shown on a separate screen on the computer. On each screen, you should indicate which prospect you prefer to play by clicking on one of the buttons beneath the prospects.

After you have worked through all of the pairs of prospects, raise your hand and an experimenter will come over. You will then roll a 30-sided die until a number between 1 and 24 comes up to determine which pair of prospects will be played out for real. Since there is a chance that any of your 24 choices could be played out for real, you should approach each pair of prospects as if it is the one that you will play out. Finally, you will then roll the two ten-sided dice twice: first to determine the number of points you win, and then to determine whether you win \$100 or \$0 according to the points earned.

For instance, suppose you picked the prospect on the left in the above example. If the outcome of the first dice roll was 17, you would win 5 points. This means that your chance of winning \$100 is 5%. Now, if the outcome of the second dice roll was 1, which is less than the number of points you earned, you would win \$100. If the second roll was 75 instead, you would earn nothing because the outcome of this roll is greater than the number of points you earned.

Here is another example. If you picked the prospect on the right and the outcome of the first dice roll was 60, then you would earn 70 points. This means that your probability of winning \$100 would be 70%. If the outcome of the second dice roll was 1, then you would earn \$100; but if the

second roll was 75 instead you would earn \$0 in this task.

Therefore, your payoff is determined by four things:

- the prospect you selected, the left or the right, for each of these 24 pairs;
- the prospect pair that is chosen to be played out in the series of 24 such pairs using the 30-sided die;
- the outcome of the first roll of the two 10-sided dice which determines the number of points you earn in your chosen prospect; and
- the outcome of the second roll of the two 10-sided dice which will be compared with your earned points to determine whether you earn \$0 or \$100.

Which prospect you prefer is a matter of personal taste. The people next to you may be presented with different prospects, so their responses should not matter to you. Please work silently, and make your choices by thinking carefully about the pair of prospects you are presented with.

All payoffs are in cash, and are in addition to the \$7.50 show-up fee that you receive just for being here. The only other task today is for you to answer some demographic questions. Your answers to those questions will not affect your payoffs.

*Treatment E*

SAME INSTRUCTIONS AS FOR TREATMENT B, WITH ADDITIONS PRESENTED IN THE TEXT AND USING DISPLAY IN FIGURE 3.

*Treatment F*

SAME INSTRUCTIONS AS FOR TREATMENT E, WITH ADDITIONS PRESENTED IN THE TEXT AND USING DISPLAY IN FIGURE 3.

## 4.8 Appendix B. Parameters of Experiments

Table 4.3: Battery of Monetary Lotteries

Pair	Context	Prizes			“Safe” Lottery Probabilities			“Risky” Lottery Probabilities			EV Safe	EV Risky
		Low	Middle	High	Low	Middle	High	Low	Middle	High		
1	5	\$5	\$20	\$70	0	0.75	0.25	0.25	0	0.75	\$32.50	\$53.75
2	5	\$5	\$20	\$70	0.25	0.5	0.25	0.5	0	0.5	\$28.75	\$37.50
3	5	\$5	\$20	\$70	0	0.5	0.5	0.25	0	0.75	\$45.00	\$53.75
4	6	\$5	\$35	\$70	0	1	0	0.25	0	0.75	\$35.00	\$53.75
5	6	\$5	\$35	\$70	0.25	0.75	0	0.5	0	0.5	\$27.50	\$37.50
6	6	\$5	\$35	\$70	0	0.75	0.25	0.25	0	0.75	\$43.75	\$53.75
7	6	\$5	\$35	\$70	0	0.5	0.5	0.25	0	0.75	\$52.50	\$53.75
8	6	\$5	\$35	\$70	0	0.75	0.25	0.5	0	0.5	\$43.75	\$37.50
9	9	\$10	\$35	\$70	0	1	0	0.25	0	0.75	\$35.00	\$55.00
10	9	\$10	\$35	\$70	0.25	0.75	0	0.5	0	0.5	\$28.75	\$40.00
11	9	\$10	\$35	\$70	0	0.5	0.5	0.25	0	0.75	\$52.50	\$55.00
12	9	\$10	\$35	\$70	0	0.75	0.25	0.5	0	0.5	\$43.75	\$40.00
13	10	\$20	\$35	\$70	0	1	0	0.25	0	0.75	\$35.00	\$57.50
14	10	\$20	\$35	\$70	0.25	0.75	0	0.5	0	0.5	\$31.25	\$45.00
15	10	\$20	\$35	\$70	0	0.75	0.25	0.25	0	0.75	\$43.75	\$57.50
16	10	\$20	\$35	\$70	0	1	0	0.5	0	0.5	\$35.00	\$45.00
17	10	\$20	\$35	\$70	0.5	0.5	0	0.75	0	0.25	\$27.50	\$32.50
18	10	\$20	\$35	\$70	0	1	0	0.25	0.5	0.25	\$35.00	\$40.00
19	10	\$20	\$35	\$70	0.25	0.5	0.25	0.5	0	0.5	\$40.00	\$45.00
20	10	\$20	\$35	\$70	0	0.5	0.5	0.25	0	0.75	\$52.50	\$57.50
21	10	\$20	\$35	\$70	0	1	0	0.5	0.25	0.25	\$35.00	\$36.25
22	10	\$20	\$35	\$70	0.25	0.75	0	0.75	0	0.25	\$31.25	\$32.50
23	10	\$20	\$35	\$70	0	0.75	0.25	0.5	0	0.5	\$43.75	\$45.00
24	10	\$20	\$35	\$70	0	1	0	0.75	0	0.25	\$35.00	\$32.50

Table 4.4: Battery of Binary Lotteries

Pair	Final Prizes Lowest Highest		Left Lottery						Right Lottery						EV Left	EV Right							
			Prizes in Points in the Initial Lotteries			Initial Lottery Probabilities			Second Lottery Probability of Highest Prize			Actuarially-Equivalent Lottery Probabilities of Final Prizes		Initial Lottery Probabilities			Second Lottery Probability of Highest Prize			Actuarially-Equivalent Lottery Probabilities of Final Prizes			
			Low	Middle	High	Low	Middle	High	Low	Middle	High	Lowest	Highest	Low			Middle	High	Low	Middle	High	Lowest	Highest
1	\$0	\$100	5	20	70	0	0.75	0.25	0.05	0.2	0.7	0.675	0.325	0.25	0	0.75	0.05	0.2	0.7	0.463	0.538	\$32.50	\$53.75
2	\$0	\$100	5	20	70	0.25	0.5	0.25	0.05	0.2	0.7	0.713	0.288	0.5	0	0.5	0.05	0.2	0.7	0.625	0.375	\$28.75	\$37.50
3	\$0	\$100	5	20	70	0	0.5	0.5	0.05	0.2	0.7	0.550	0.450	0.25	0	0.75	0.05	0.2	0.7	0.463	0.538	\$45.00	\$53.75
4	\$0	\$100	5	35	70	0	1	0	0.05	0.35	0.7	0.65	0.35	0.25	0	0.75	0.05	0.35	0.7	0.463	0.538	\$35.00	\$53.75
5	\$0	\$100	5	35	70	0.25	0.75	0	0.05	0.35	0.7	0.725	0.275	0.5	0	0.5	0.05	0.35	0.7	0.625	0.375	\$27.50	\$37.50
6	\$0	\$100	5	35	70	0	0.75	0.25	0.05	0.35	0.7	0.563	0.438	0.25	0	0.75	0.05	0.35	0.7	0.463	0.538	\$43.75	\$53.75
7	\$0	\$100	5	35	70	0	0.5	0.5	0.05	0.35	0.7	0.475	0.525	0.25	0	0.75	0.05	0.35	0.7	0.463	0.538	\$52.50	\$53.75
8	\$0	\$100	5	35	70	0	0.75	0.25	0.05	0.35	0.7	0.563	0.438	0.5	0	0.5	0.05	0.35	0.7	0.625	0.375	\$43.75	\$37.50
9	\$0	\$100	10	35	70	0	1	0	0.1	0.35	0.7	0.65	0.35	0.25	0	0.75	0.1	0.35	0.7	0.45	0.55	\$35.00	\$55.00
10	\$0	\$100	10	35	70	0.25	0.75	0	0.1	0.35	0.7	0.713	0.288	0.5	0	0.5	0.1	0.35	0.7	0.600	0.400	\$28.75	\$40.00
11	\$0	\$100	10	35	70	0	0.5	0.5	0.1	0.35	0.7	0.475	0.525	0.25	0	0.75	0.1	0.35	0.7	0.450	0.550	\$52.50	\$55.00
12	\$0	\$100	10	35	70	0	0.75	0.25	0.1	0.35	0.7	0.563	0.438	0.5	0	0.5	0.1	0.35	0.7	0.600	0.400	\$43.75	\$40.00
13	\$0	\$100	20	35	70	0	1	0	0.2	0.35	0.7	0.65	0.35	0.25	0	0.75	0.2	0.35	0.7	0.425	0.575	\$35.00	\$57.50
14	\$0	\$100	20	35	70	0.25	0.75	0	0.2	0.35	0.7	0.688	0.313	0.5	0	0.5	0.2	0.35	0.7	0.550	0.450	\$31.25	\$45.00
15	\$0	\$100	20	35	70	0	0.75	0.25	0.2	0.35	0.7	0.563	0.438	0.25	0	0.75	0.2	0.35	0.7	0.425	0.575	\$43.75	\$57.50
16	\$0	\$100	20	35	70	0	1	0	0.2	0.35	0.7	0.650	0.350	0.5	0	0.5	0.2	0.35	0.7	0.550	0.450	\$35.00	\$45.00
17	\$0	\$100	20	35	70	0.5	0.5	0	0.2	0.35	0.7	0.725	0.275	0.75	0	0.25	0.2	0.35	0.7	0.675	0.325	\$27.50	\$32.50
18	\$0	\$100	20	35	70	0	1	0	0.2	0.35	0.7	0.650	0.350	0.25	0.5	0.25	0.2	0.35	0.7	0.600	0.400	\$35.00	\$40.00
19	\$0	\$100	20	35	70	0.25	0.5	0.25	0.2	0.35	0.7	0.600	0.400	0.5	0	0.5	0.2	0.35	0.7	0.550	0.45	\$40.00	\$45.00
20	\$0	\$100	20	35	70	0	0.5	0.5	0.2	0.35	0.7	0.475	0.525	0.25	0	0.75	0.2	0.35	0.7	0.425	0.575	\$52.50	\$57.50
21	\$0	\$100	20	35	70	0	1	0	0.2	0.35	0.7	0.650	0.350	0.5	0.25	0.25	0.2	0.35	0.7	0.638	0.363	\$35.00	\$36.25
22	\$0	\$100	20	35	70	0.25	0.75	0	0.2	0.35	0.7	0.688	0.313	0.75	0	0.25	0.2	0.35	0.7	0.675	0.325	\$31.25	\$32.50
23	\$0	\$100	20	35	70	0	0.75	0.25	0.2	0.35	0.7	0.563	0.438	0.5	0	0.5	0.2	0.35	0.7	0.550	0.450	\$43.75	\$45.00
24	\$0	\$100	20	35	70	0	1	0	0.2	0.35	0.7	0.650	0.350	0.75	0	0.25	0.2	0.35	0.7	0.675	0.325	\$35.00	\$32.50

## 4.9 Appendix C. Structural Estimation of Risk

### Preferences

Assume that utility of income is defined by

$$U(x) = x^{(1-r)}/(1-r) \quad (4.1)$$

where  $x$  is the lottery prize and  $r \neq 1$  is a parameter to be estimated. For  $r = 1$  assume  $U(x) = \ln(x)$  if needed. Thus  $r$  is the coefficient of CRRA:  $r = 0$  corresponds to risk neutrality,  $r < 0$  to risk loving, and  $r > 0$  to risk aversion. Let there be  $J$  possible outcomes in a lottery, and denote outcome  $j \in J$  as  $x_j$ . Under EUT the probabilities for each outcome  $x_j$ ,  $p(x_j)$ , are those that are induced by the experimenter, so expected utility is simply the probability weighted utility of each outcome in each lottery  $i$ :

$$EU_i = \sum_{i=1,J} [p(x_i) \times U(x_i)] \quad (4.2)$$

The EU for each lottery pair is calculated for a candidate estimate of  $r$ , and the index

$$\nabla EU = EU_R - EU_L \quad (4.3)$$

is calculated, where  $EU_L$  is the “left” lottery and  $EU_R$  is the “right” lottery as presented to subjects. This latent index, based on latent preferences, is then linked to observed choices using a standard cumulative normal distribution function  $\Phi(\nabla EU)$ . This “probit”

function takes any argument between  $\pm\infty$  and transforms it into a number between 0 and 1. Thus we have the probit link function,

$$\text{prob}(\text{choose lottery R}) = \Phi(\nabla EU) \quad (4.4)$$

Even though this “link function” is common in econometrics texts, it forms the critical statistical link between observed binary choices, the latent structure generating the index  $\nabla EU$ , and the probability of that index being observed. The index defined by equation 4.3 is linked to the observed choices by specifying that the R lottery is chosen when  $\Phi(\nabla EU) > 0.5$ , which is implied by equation 4.4.

The likelihood of the observed responses, conditional on the EUT and CRRA specifications being true, depends on the estimates of  $r$  given the above statistical specification and the observed choices. The “statistical specification” here includes assuming some functional form for the cumulative density function (CDF). The conditional log-likelihood is then

$$\ln L(r; y, X) = \sum [( \ln \Phi(\nabla EU) \times \mathbf{I}(y_i = 1) ) + ( \ln(1 - \Phi(\nabla EU)) \times \mathbf{I}(y_i = -1) )] \quad (4.5)$$

where  $\mathbf{I}(\cdot)$  is the indicator function,  $y_i = 1(-1)$  denotes the choice of the right (left) lottery in risk aversion task  $i$ , and  $X$  is a vector of individual characteristics reflecting age, sex, race, and so on.

Harrison and Rutström (2008, Appendix F) review procedures that can be used to estimate structural models of this kind, as well as more complex non-EUT models, with the goal of illustrating how to write explicit maximum likelihood (ML) routines that are specific to

different structural choice models. It is a simple matter to correct for multiple responses from the same subject (“clustering”), if needed.

It is also a simple matter to generalize this ML analysis to allow the core parameter  $r$  to be a linear function of observable characteristics of the individual or task. We extend the model to be  $r = r_0 + R \times X$ , where  $r_0$  is a fixed parameter and  $R$  is a vector of effects associated with each characteristic in the variable vector  $X$ . In effect, the unconditional model assumes  $r = r_0$  and estimates  $r_0$ . This extension significantly enhances the attraction of structural ML estimation, particularly for responses pooled over different subjects and treatments, since one can condition estimates on observable characteristics of the task or subject.

An important extension of the core model is to allow for subjects to make some *behavioral* errors. The notion of error is one that has already been encountered in the form of the statistical assumption that the probability of choosing a lottery is not 1 when the EU of that lottery exceeds the EU of the other lottery. This assumption is clear in the use of a non-degenerate link function between the latent index  $\nabla EU$  and the probability of picking one or other lottery; in the case of the normal CDF, this link function is  $\Phi(\nabla EU)$ . If there were no errors from the perspective of EUT, this function would be a step function: zero for all values of  $\nabla EU < 0$ , anywhere between 0 and 1 for  $\nabla EU = 0$ , and 1 for all values of  $\nabla EU > 0$ . We employ the error specification originally due to Fechner and popularized by Hey and Orme (1994). This error specification posits the latent index

$$\nabla EU = (EU_R - EU_L)/\mu \tag{4.6}$$

instead of equation 4.3, where  $\mu$  is a structural “noise parameter” used to allow some errors from the perspective of the deterministic EUT model. This is just one of several different types of error story that could be used, and Wilcox (2008) provides a masterful review of the implications of the alternatives.<sup>23</sup> As  $\mu \rightarrow 0$  this specification collapses to the deterministic choice EUT model, where the choice is strictly determined by the EU of the two lotteries; but as  $\mu$  gets larger and larger the choice essentially becomes random. When  $\mu = 1$  this specification collapses to equation 4.3, where the probability of picking one lottery is given by the ratio of the EU of one lottery to the sum of the EU of both lotteries. Thus  $\mu$  can be viewed as a parameter that flattens out the link functions as it gets larger. An important contribution to the characterization of behavioral errors is the “contextual error” specification proposed by Wilcox (2011). It is designed to allow robust inferences about the primitive “more stochastically risk averse than,” and posits the latent index

$$\nabla EU = (EU_R - EU_L)/\mu/\nu \quad (4.7)$$

instead of equation 4.6, where  $\nu$  is a new, normalizing term for each lottery pair L and R. The normalizing term  $\nu$  is defined as the maximum utility over all prizes in this lottery pair minus the minimum utility over all prizes in this lottery pair. The value of  $\nu$  varies, in principle, from lottery choice pair to lottery choice pair: hence it is said to be “contextual.” For the Fechner specification, dividing by  $\nu$  ensures that the *normalized* EU difference

---

<sup>23</sup>Some specifications place the error at the final choice between one lottery or after the subject has decided which one has the higher expected utility; some place the error earlier, on the comparison of preferences leading to the choice; and some place the error even earlier, on the determination of the expected utility of each lottery.

$[(EU_R - EU_L)/\nu]$  remains in the unit interval for each lottery pair. The term  $\nu$  does not need to be estimated in addition to the utility function parameters and the parameter for the behavioral error term, since it is given by the data and the assumed values of those estimated parameters.

The specification employed here is the CRRA utility function from equation 4.1, the Fechner error specification using contextual utility from equation 4.7, and the link function using the normal CDF from 4.4. The log-likelihood is then

$$\ln L(r, \mu; y, X) = \sum [(\ln \Phi(\nabla EU) \times \mathbf{I}(y_i = 1)) + (\ln(1 - \Phi(\nabla EU)) \times \mathbf{I}(y_i = -1))] \quad (4.8)$$

and the parameters to be estimated are  $r$  and  $\mu$  given observed data on the binary choices  $y$  and the lottery parameters in  $X$ .

Table 4.5: Estimation Results for Treatments A and B

```

Log likelihood = -117.39741
Number of obs   =      218
LR chi2(2)      =       4.06
Prob > chi2     =      0.1316

```

	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
-----+-----						
r						
blp	-.7015267	.3317442	-2.11	0.034	-1.351733	-.0513201
female	.2341047	.2037852	1.15	0.251	-.165307	.6335164
sophomore	-.1999455	.2006873	-1.00	0.319	-.5932854	.1933943
senior	-.224371	.2281513	-0.98	0.325	-.6715393	.2227973
asian	-.3286855	.2579515	-1.27	0.203	-.8342612	.1768902
white	-.513592	.362524	-1.42	0.157	-1.224126	.196942
_cons	.7439132	.1839053	4.05	0.000	.3834655	1.104361
-----+-----						
mu						
_cons	.1417281	.025342	5.59	0.000	.0920587	.1913976
-----+-----						

```
. lincom [r]_cons + [r]blp
```

```
( 1) [r]blp + [r]_cons = 0
```

	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
-----+-----						
(1)	.0423864	.3356574	0.13	0.900	-.6154901	.7002629
-----+-----						

```
. predictnl r_estimate = xb(r) if e(sample)
```

```
. bysort blp: summ r_estimate if e(sample)
```

```
-> blp = 0
```

Variable	Obs	Mean	Std. Dev.	Min	Max
r_estimate	149	.6261916	.2954439	.0059502	.9780179

```
-> blp = 1
```

Variable	Obs	Mean	Std. Dev.	Min	Max
r_estimate	69	-.0772987	.2888739	-.6955765	.2764911

Table 4.6: Estimation Results for Treatments A, B, E and F

```

Log likelihood = -154.36753
Number of obs   =      290
LR chi2(2)      =      0.66
Prob > chi2     =      0.7201

```

	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
-----						
r						
blp	-.6757346	.3085369	-2.19	0.029	-1.280456	-.0710135
ctalk	.438053	.5688851	0.77	0.441	-.6769413	1.553047
evinfo	-.174793	.5359854	-0.33	0.744	-1.225305	.875719
female	.1367566	.1755596	0.78	0.436	-.2073339	.4808471
sophomore	-.2348606	.1868328	-1.26	0.209	-.6010462	.131325
senior	-.2005559	.2006742	-1.00	0.318	-.5938702	.1927584
asian	-.2721212	.2299213	-1.18	0.237	-.7227587	.1785163
white	-.4757928	.28915	-1.65	0.100	-1.042516	.0909307
_cons	.8005215	.1649517	4.85	0.000	.477222	1.123821
-----						
mu						
_cons	.1332112	.0204761	6.51	0.000	.0930787	.1733437
-----						

```
. test ctalk evinfo
```

```
( 1) [r]ctalk = 0
( 2) [r]evinfo = 0
```

```
      chi2( 2) =    0.66
      Prob > chi2 =  0.7191
```

```
. test blp ctalk evinfo
```

```
( 1) [r]blp = 0
( 2) [r]ctalk = 0
( 3) [r]evinfo = 0
```

```
      chi2( 3) =    7.67
      Prob > chi2 =  0.0534
```

```
. bysort blp: summ r_estimate if e(sample)
```

```
-> blp = 0
```

Variable	Obs	Mean	Std. Dev.	Min	Max
r_estimate	149	.6377861	.2530015	.0898681	.9372781

```
-> blp = 1
```

Variable	Obs	Mean	Std. Dev.	Min	Max
r_estimate	141	-.0506123	.2912195	-.7263549	.5248035

Table 4.7: Estimation Results for Treatments C and D

```

Log pseudolikelihood = -4529.7244
Number of obs = 7590
Wald chi2(6) = 16.02
Prob > chi2 = 0.0137

```

(Std. Err. adjusted for 309 clusters in id)

	Coef.	Robust Std. Err.	z	P> z	[95% Conf. Interval]	
-----						
r						
blp	-.4760487	.1585904	-3.00	0.003	-.7868801	-.1652173
female	.1418673	.0693712	2.05	0.041	.0059023	.2778323
sophomore	.0168658	.0812499	0.21	0.836	-.1423811	.1761126
senior	-.0787157	.0742185	-1.06	0.289	-.2241813	.0667499
asian	-.0653329	.0812816	-0.80	0.422	-.2246419	.0939761
white	-.0040662	.0960593	-0.04	0.966	-.192339	.1842066
_cons	.6733916	.0867036	7.77	0.000	.5034556	.8433275
-----						
mu						
_cons	.1639391	.0069316	23.65	0.000	.1503535	.1775247
-----						

```
. lincom [r]_cons + [r]blp
```

```
( 1) [r]blp + [r]_cons = 0
```

	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
-----						
(1)	.1973428	.1701349	1.16	0.246	-.1361155	.5308011
-----						

```
. predictnl r_estimatel = xb(r) if e(sample)
```

```
. bysort blp: summ r_estimatel if e(sample)
```

```
-> blp = 0
```

Variable	Obs	Mean	Std. Dev.	Min	Max
r_estimatel	6654	.7278554	.0835576	.5293429	.8321247

```
-> blp = 1
```

Variable	Obs	Mean	Std. Dev.	Min	Max
r_estimatel	936	.263594	.0716006	.1320099	.3560759

## Chapter 5

# Eliciting Subjective Probabilities with Binary Lotteries

*by Glenn W. Harrison, Jimmy Martínez-Correa and Todd J. Swarthout*

### **Abstract**

We evaluate the binary lottery procedure for inducing risk neutral behavior in a subjective belief elicitation task. Harrison, Martínez-Correa and Swarthout (2012a) found that the binary lottery procedure works robustly to induce risk neutrality when subjects are given one risk task defined over objective probabilities. Using individuals sampled from the same pool of subjects, we find evidence which supports the hypothesis that the binary lottery procedure induces linear utility in a subjective probability elicitation task that uses the Quadratic Scoring Rule. We also show that the binary lottery procedure can also induce direct revelation of subjective probabilities in subjects with certain Non-Expected Utility preference representations.

## 5.1 Introduction

The notion of subjective probabilities as prices at which one is willing to trade is due to de Finetti (1937, 1970) and Savage (1971), who propose bets as one operational procedure to both define and elicit subjective probabilities. Their central point is that subjective probabilities of events are marginal rates of substitution between contingent claims that obey certain rules of coherence.

This literature relies on the assumption that subjects behave as if they are *risk neutral*, which is considered plausible *a priori* if the stakes used in the elicitation procedure are sufficiently small. However, there is systematic evidence that, with the small stakes normally used in the laboratory, subjects behave as if they are risk averse. Therefore, it is intuitively obvious, and also well known in the literature (e.g., Winkler and Murphy, 1970, and Kadane and Winkler, 1988), that risk attitudes will affect the incentive to directly report one's subjective probability.

Consider a scoring rule to elicit subjective probabilities. A sufficiently risk averse agent is going to be drawn to a report of 0.5, since this equalizes earnings under each state of nature, at least for the most popular scoring rules. Varying degrees of risk aversion will cause varying distortions in reports from true, latent subjective probabilities. If we knew the form of the utility function of the subjects, and their degree of risk aversion, we could infer back from any report what subjective probability they must have had. The need to do this jointly is in fact central to the operational *definition* of subjective probability provided by Savage (1972): under certain postulates, he showed that there existed a

subjective probability and a utility function that could rationalize observable choices.

Andersen et al. (2010) illustrate how joint estimation of risk attitudes and subjective beliefs, using structural maximum likelihood, can be used to make the necessary calibration to recover the latent subjective probability.

An alternative and operationally meaningful approach is to use proper scoring rules combined with the Binary Lottery Procedure (BLP) to induce linear utility in subjects.

The theoretical prediction is that, under certain conditions, this approach allows the researcher to directly elicit the subjective probability without further statistical corrections for risk attitudes.

Procedures to induce linear utility functions have a long history, with the major contributions being Smith (1961), Roth and Malouf (1979) and Berg et al. (1986). The consensus appears to be that these “binary lottery procedures” may be fine in theory, but behaviorally they just do not work as advertised. However, Harrison et al. (2012a) show that the BLP works for samples of university level students in the simplest possible environment, where one can avoid contaminating factors such as strategic equilibrium concepts or traditionally used payment protocols. They find that the BLP works robustly to induce risk neutrality when subjects are given one simple binary lottery choice, and that it works well when subjects are given more than one of these binary choices. This does not imply that BLP is useful for samples from different populations or that it can be systematically apply to any setting, since it is often the “contaminating factors” of interest in some designs that can dilute the power of BLP to induce risk neutrality.

We study if the ability of the BLP to induce risk neutrality in simple binary choices defined

over objective probabilities found by Harrison et al. (2012a) also extends to subjective probability elicitation tasks. In particular, we study the ability of the Quadratic Scoring Rule (QSR), combined with the BLP, to directly elicit subjective probabilities without having to control for risk attitudes. The first statements of this mechanism, joining the QSR and the BLP, appear to be Allen (1987) and McKelvy and Page (1990). Schlag and van der Weele (2009) and Hossain and Okui (2011) examine the same extension of the QSR, along with certain generalizations, calling it a “randomized QSR” and “probabilistic scoring rule” respectively.

There exist other mechanisms that will elicit subjective probabilities without requiring that one corrects for risk attitudes, such as the procedures proposed by Grether (1992), Köszegi and Rabin (2008, p. 199), Offerman et al. (2009), Karni (2009) and Holt and Smith (2009). Thus there is no shortage of theoretical procedures to elicit subjective probabilities, and the issue becomes which generates them most reliably from a behavioral perspective. For example, Trautmann and van de Kuilen (2011) compare several incentivized procedures in the context of eliciting own-beliefs in a two-person game, and find few differences between the procedures. That elicitation context, while important, is complex, as stressed by Rutström and Wilcox (2009).

Two aspects of elicitation concern us in operationalizing these procedures. The first is the use of procedures that assume the validity of the Becker, De Groot and Marschak (1964) procedure for eliciting certainty-equivalents; Harrison (1992) explains the concerns. The second is the use of payment protocols over multiple choices that assume the validity of the (compound or mixture) independence axiom; Harrison and Swarthout (2012) explain the

concerns. We avoid both of these in our tests.

Using non-parametric statistical tests we find evidence that the BLP mitigates the distortion in reports introduced by risk aversion. We cannot claim that the BLP provides incentives to subjects to truthfully report their true, latent subjective probabilities: this would require econometric estimations of subjective probabilities, which we leave for future work. However, we can claim that inferred subjective probabilities under the BLP robustly shift in the direction predicted under the assumption that subjects are risk averse and that the BLP reduces the contaminating factor of risk aversion. We can only “reduce in the predicted direction” rather than “remove,” since we cannot know a priori what the true, latent subjective probability actually is.<sup>1</sup>

In section 5.2 we review the theory of scoring rules and explain the benefits of using the BLP in belief elicitation tasks. In section 5.3 we present our experimental design, in section 5.4 we evaluate the results, and in section 5.5 we conclude.

## 5.2 Theoretical Issues

### 5.2.1 Binary Scoring Rules for Subjective Probabilities

A binary scoring rule is defined over some binary event, which is either true or false. A binary scoring rule asks the subject to make some report  $\theta$ , and then defines how an elicitor pays a subject depending on their report and the outcome of the event. This

---

<sup>1</sup>In this respect tests of the BLP with lotteries defined over objective probabilities are much easier. Hossain and Okui (2011, §3.1) use the QSR with and without the BLP to test if elicited beliefs are the same as objective probabilities when one uses the BLP, and find that they are indeed closer when one uses the BLP.

framework for eliciting subjective probabilities can be formally viewed from the perspective of a trading game between two agents: you give me a report, and I agree to pay you \$X if one outcome occurs and \$Y if the other outcome occurs. The scoring rule defines the terms of the exchange quantitatively, explaining how the elicitor converts the report from the subject into a lottery defined over the subjective probability of the subject. We use the terminology “report” because we want to view this formally as a mechanism, and want to emphasize the idea that the report may or may not be the subjective probability  $\pi$  of the subject. When the report is equal to subjective probability of the individual, we say that the scoring rule is a direct mechanism, following standard methodology.

The popular Quadratic Scoring Rule (QSR) for binary events is defined in terms of two positive parameters,  $\alpha$  and  $\beta$ , that determine a fixed reward the subject gets and a penalty for error. Assume that the possible outcomes are A or B, where B is the complement of A, that  $\theta$  is the reported probability for A, and that  $\Theta$  is the true binary-valued outcome for A. Hence  $\Theta = 1$  if A occurs, and  $\Theta = 0$  if it does not occur (and thus B occurs instead). The subject is paid  $S(\theta|A \text{ occurs}) = \alpha - \beta(\Theta - \theta)^2 = \alpha - \beta(1 - \theta)^2$  if event A occurs and  $S(\theta|B \text{ occurs}) = \alpha - \beta(\Theta - \theta)^2 = \alpha - \beta(0 - \theta)^2$  if B occurs. The score or payment penalizes the subject by the squared deviation of the report from the true binary-valued outcome,  $\Theta$ , which is 1 and 0 respectively for A and B occurring. An omniscient seer would obviously set  $\theta = \Theta$ . The fixed reward is a convenience<sup>2</sup> to ensure that subjects are willing to play this trading game, and the penalty function accentuates the penalty from not responding what the subject thinks an omniscient seer would respond. It can be shown that a risk neutral

---

<sup>2</sup>In the language of mechanism design, it can be chosen to satisfy the participation constraint. This requires that  $\theta > \beta$ .

decision maker will report his subjective probability truthfully. For example, assume  $\alpha = 1$  and  $\beta = 1$  so that the subject could earn up to 1 or as little as 0. If they reported 1 they earned 1 if event A occurred or 0 if event B occurred; if they reported 0.75 they earned 0.9375 or 0.4375; and if they reported 0.5 they earned 0.75 no matter what event occurred.

### **5.2.2 Subjective Belief Elicitation with Scoring Rules and the Binary Lottery Procedure**

Our strategy is to rely on the BLP to induce linear utility functions in subjects, which implies that the QSR should provide incentives to subjects to report their subjective probabilities truthfully. The central insight is to define the payoffs in the QSR as points that define the probability of winning either a high or a low amount of money in some subsequent binary lottery. We explain below the conditions under which this combination of BLP and the QSR provides incentives to individuals to directly report “truthfully” their unobserved subjective probabilities.

For example, set the high and the low payoff of this binary lottery to be \$50 and \$0. In theory the BLP induces subjects to report the true subjective probability of some event independently of the shape of the utility function and, under some weak conditions, independently of the shape of the probability weighting function. For exposition purposes, take first the case of a Subjective Expected Utility (SEU) maximizer that is given a QSR task defined over points using the BLP to convert those points into money.

Assume that there are two events: a ball drawn from a Bingo Cage is either Red (R) or White (W). A subject betting on event R might estimate that it happens with subjective

probability  $R$ , and that  $W$  will happen with subjective probability  $\pi_W = (1 - \pi_R)$ .

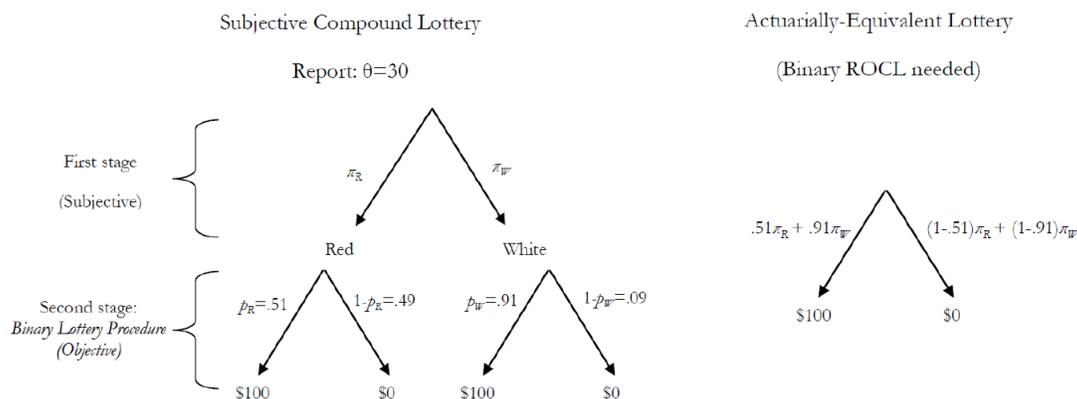
Additionally, set the parameters of the QSR to be  $\alpha = \beta = 100$ .

If event  $R$  is realized and a subject reports  $\theta$ , he wins an amount of points defined by  $S(\theta|R) = 100 - 1(1 - \theta/100)^2$ . For simplicity, the report can be any number between 1 and 100, although for practical purposes we can think of reports in increments of single percentage points. Similarly, if event  $W$  is realized and a subject reports  $\theta$ , he wins an amount of points given by  $S(\theta|W) = 100 - 1(1 - \theta/100)^2$ . Suppose a subject reports  $\theta = 30$ . This implies that he would win 51 points if event  $R$  is realized and 91 points if event  $W$  is realized. The BLP implies that a subject would then play a binary lottery where the probabilities of winning are defined by the points earned. If the realized event is  $R$ , then the individual would play a lottery that pays \$50 with 51% and \$0 with 49%. Define  $p_R(\theta) = S(\theta|R)/100$  as the objective probability of winning \$50 in the binary lottery induced by the points earned in the scoring rule task when the report is equal to  $\theta$  and event  $R$  is realized. The objective probability  $p_W(\theta) = S(\theta|W)/100$  is similarly defined for event  $W$ . In the example above,  $p_R(30) = 51\%$  and  $p_W(30) = 91\%$ . Figure 5.1 represents graphically the subjective compound lottery and the actuarially equivalent simple lottery induced by report  $\theta = 30$ .

In the QSR a subject may choose among 101 possible Subjective Compound Lotteries (SCL) of the type depicted in Figure 5.1, because the possible reports are integer numbers between 0 and 100. In these SCL, the first stage involves subjective probabilities while the second stage involves objective probabilities defined by the points earned in the first. The structure of these 101 SCL are explicitly described below:

1. SCL( $\theta=0$ ): pays simple lottery ( $p_R(0)$ , \$50;  $1-p_R(0)$ , \$0) with subjective probability  $\pi_R$ , and pays simple lottery ( $p_W(0)$ , \$50;  $1-p_W(0)$ , \$0) with subjective probability  $\pi_W = (1-\pi_R)$
2. SCL( $\theta=1$ ): pays simple lottery ( $p_R(1)$ , \$50;  $1-p_R(1)$ , \$0) with subjective probability  $\pi_R$ , and pays simple lottery ( $p_W(1)$ , \$50;  $1-p_W(1)$ , \$0) with subjective probability  $\pi_W = (1-\pi_R)$
- .
- .
100. SCL( $\theta=99$ ): pays simple lottery ( $p_R(99)$ , \$50;  $1-p_R(99)$ , \$0) with subjective probability  $\pi_R$ , and pays simple lottery ( $p_W(99)$ , \$50;  $1-p_W(99)$ , \$0) with subjective probability  $\pi_W = (1-\pi_R)$
101. SCL( $\theta=100$ ): pays simple lottery ( $p_R(100)$ , \$50;  $1-p_R(100)$ , \$0) with subjective probability  $\pi_R$ , and pays simple lottery ( $p_W(100)$ , \$50;  $1-p_W(100)$ , \$0) with subjective probability  $\pi_W = (1-\pi_R)$

Figure 5.1: Binary Scoring Rule Using the Binary Lottery Procedure



If the subject maximizes SEU, and therefore satisfies the Reduction of Compound Lotteries (ROCL) axiom, the valuation of each report  $\theta$  will be given by

$$\begin{aligned}
 SEU(\theta) = & \pi_R \times p_R(\theta) \times U(\$50) + (1 - p_R(\theta)) \times U(\$0) \\
 & + (1 - \pi_R) \times p_W(\theta) \times U(\$50) + (1 - p_W(\theta)) \times U(\$0)
 \end{aligned} \tag{5.1}$$

and the subject chooses the report  $\theta^*$  that maximizes 5.2.2 conditional on the subjective

belief  $\pi_R$ . Because  $U(\cdot)$  is unique up to an affine positive transformation under SEU we can normalize  $U(\$50) = 1$  and  $U(\$0) = 0$ . Thus the  $SEU(\theta)$  in 5.2.2 can be simplified to

$$SEU(\theta) = \pi_R \times p_R(\theta) + (1 - \pi_R) \times p_W(\theta) = Q(\theta) \quad (5.2)$$

We rename  $SEU(\theta)$  as  $Q(\theta)$  to emphasize that the subject's valuation of the SCL induced by  $\theta$  can be interpreted as the subjective average probability  $Q(\theta)$  of winning the high \$50 amount in the binary lottery. For the sake of the argument assume that the state space of the report is a continuum such that  $\theta \in [0, 100]$ , therefore a SEU maximizer would make a report  $\theta^*$  that maximizes the subjective expected probability  $Q(\cdot)$  of winning the binary lottery. Taking the first order condition with respect to report  $\theta$  we obtain

$$\begin{aligned} SEU'(\theta) = Q'(\theta) &= \pi_R \times p'_R(\theta) + (1 - \pi_R) \times p'_W(\theta) = 0 \\ &= \pi_R \times [2 \times (1 - \theta/100)] + (1 - \pi_R) \times [2 \times (0 - \theta/100)] = 0 \end{aligned} \quad (5.3)$$

The report that maximizes 5.2.2 is  $\theta^* = \pi_R \times 100$ , which implies that the QSR combined with the BLP provides incentives to report the true subjective probability directly. The existence of a unique maximum is guaranteed because the function  $Q(\cdot)$  is strictly concave in  $\theta$  given that it is a linear combination of two strictly concave functions,  $p_R(\theta)$  and  $p_W(\theta)$ . Note that the strict concavity of these functions is determined by the QSR because these objective probabilities are a function of the scoring rule.<sup>3</sup>

For comparison purposes suppose a simple QSR with payouts defined directly in dollars.

---

<sup>3</sup>In the present example,  $p_R(\theta)$  and  $p_W(\theta)$  are strictly concave because  $p''_R(\theta) = p''_W(\theta) = -2/100 < 0$ .

Refer to the score in this case as  $\$S(\cdot)$ . The subject would choose a report  $\theta$  that maximizes the following valuation

$$SEU(\theta, U(\cdot)) = \pi_R \times U(\$S(\theta|R)) + (1 - \pi_R) \times U(\$S(\theta|W)) \quad (5.4)$$

Assume a simple power function with risk aversion parameter equal to 0.57 and a subjective probability of event R of  $\pi_R = 0.3$ . In this case the optimal report would be  $\theta^* = 34$ . Conversely, if  $\pi_R = 0.7$ , the optimal report would be  $\theta^* = 66$ . Notice that a sufficiently risk averse individual would be drawn to make a report of 50, independent of his subjective probability, because this report provides the same payoff under each event. A proper scoring rule provides incentives to subjects to optimally choose one distinct report. The uniqueness of the optimal report can be achieved by guaranteeing that the scoring rule induces strict concavity in the subject's valuation of choices in the belief elicitation task. In the case of the scoring rule without BLP, the subject's valuation  $SEU(\theta, U(\cdot))$  is a concave function of  $\theta$ , assuming only weak monotonicity condition on  $U(\cdot)$ . Under SEU this concavity is immediately guaranteed by the concavity of the utility function.<sup>4</sup>

---

<sup>4</sup>A Linear Scoring Rule (LSR) defines the scores for events A and B as  $S(\theta|A \text{ occurs}) = \alpha - \beta|1 - \theta|$  if event A occurs and  $S(\theta|B \text{ occurs}) = \alpha - \beta|0 - \theta|$  if B occurs. Therefore a LSR also results in a subject's valuation that is concave in the report if the utility function is concave. Andersen et al. (2010) show how one can infer true subjective probabilities with the LSR if one also knows the risk attitudes of subjects. However, if subjects are risk neutral the LSR does not allow one to directly identify subjective probabilities from reports because the optimal report would be either 0 or 100, depending on whether the true latent subjective probability was less than 0.5 or greater than 0.5, respectively. This result is immediately relevant if one wants to induce risk neutrality with the BLP. Consequently, if we rely on risk neutrality being induced by the BLP, any scoring rule that we use must be concave in the report that subjects can make for all weakly concave utility functions, and not just for strictly concave utility functions.

### 5.2.3 Non-Expected Utility Theory Preference Representations

The incentive-compatibility of the QSR is normally developed theoretically in the context of Expected Utility Theory (EUT), and specifically SEU, whether or not one assumes the BLP. When implemented with the BLP, can the QSR be a proper scoring rule for subjects that have non-EUT preference representations? The answer to this question depends on the specific non-EUT preference representation.

A subject that follows the Rank-Dependent Utility model (RDU) will report his subjective probability under relatively weak conditions. Assume that the decision maker is a RDU maximizer with a strictly increasing probability weighting function. Then the higher prize receives decision weight  $w(p)$ , where  $p$  is the probability of the higher prize, and the lower prize receives decision weight  $1 - w(p)$ . SEU is violated in this case, but none of the axioms needed for the BLP to induce linear utility are violated. For the BLP to directly elicit the subjective probability from a RDU maximizer we need the following assumptions to hold:

(1) uniqueness of  $U(\cdot)$  up to an affine positive transformation and  $U(\cdot)$  increasing, (2) probabilistic sophistication as defined by Machina and Schmeidler (1992, 1995), (3) ROCL for binary lotteries, (4) the probability weighting function is strictly increasing, and (5) the scoring rule must be strictly concave. We formally derive below the conditions under which a non-EUT subject would optimally report his true subjective belief.

An individual with RDU preferences will have a QSR valuation of the subjective compound lottery induced by a report  $\theta$  given by

$$RDU(\theta) = w(Q(\theta)) \times U(\$50) + (1 - w(Q(\theta))) \times U(\$0) \quad (5.5)$$

Binary ROCL implies that the probability weighting is done on the reduced compound probability  $Q(\theta)$ . Since  $U(\cdot)$  is unique up to an affine positive transformation in the RDU model, we can also normalize  $U(\$50) = 1$  and  $U(\$0) = 0$  and the valuation of the individual becomes  $RDU(\theta) = w(q(\theta))$ . An RDU maximizer and a SEU maximizer, each with subjective probability  $\pi_R = 0.3$  for example, would have incentives to make exactly the same optimal report,  $\theta^* = \pi_R \times 100 = 30$ . This is easily seen by taking the first order condition on the subject's valuation of report  $\theta$ ,

$$RDU'(\theta) = w'(Q(\theta)) \times Q'(\theta) = 0, \quad (5.6)$$

which is satisfied when  $Q'(\theta) = 0$ , exactly equal to the first order condition of an SEU maximizer, because the probability weighting function is assumed to be strictly increasing (i.e.,  $w'(Q(\theta)) > 0$ ). Therefore the RDU maximizer would optimally make the same report as a SEU maximizer with the same beliefs, and both would have incentives to directly report their true subjective probability. To guarantee the uniqueness of the optimal report we rely on two assumptions: (1) that the scoring rule is strictly concave because  $Q(\theta)$  is a linear combination of strictly concave functions that depend on the scoring rule (i.e.,  $p_R(\theta) = S(\theta|R)/100$  and  $p_W(\theta) = S(\theta|W)/100$ ); and (2) the probability weighting function must be strictly increasing. This guarantees that  $w(Q(\theta))$  is strictly quasi-concave, which implies that there is a unique global maximum.

**Proposition 1.** If  $F(\cdot)$  is strictly increasing and  $f(\cdot)$  is strictly concave, then  $F(f(\cdot))$  is strictly quasi-concave.

We want to show this result so the uniqueness of the optimal report is guaranteed in the scoring rule task with certain non-EUT preference representations.

*Proof.* Suppose  $F$  is strictly increasing and  $f$  is strictly concave. We define  $g(\cdot)$  to be strictly quasi-concave if  $g(\lambda x + (1 - \lambda)x^*) > \min\{g(x), g(x^*)\}$  for  $\lambda \in (0,1)$ . Since  $f(\cdot)$  is strictly concave then  $f(\lambda x + (1 - \lambda)x^*) > \lambda f(x) + (1 - \lambda)f(x^*)$ . Since  $F$  is strictly increasing,  $F(f(\lambda x + (1 - \lambda)x^*)) > F(\lambda f(x) + (1 - \lambda)f(x^*))$ . Because  $\lambda \in (0,1)$ ,  $\lambda f(x) + (1 - \lambda)f(x^*) > \min\{f(x), f(x^*)\}$ . Since  $F$  is increasing,  $F(\lambda f(x) + (1 - \lambda)f(x^*)) > \min\{F(f(x)), F(f(x^*))\}$ . This implies that  $F(f(\lambda x + (1 - \lambda)x^*)) > F(\lambda f(x) + (1 - \lambda)f(x^*)) > \min\{F(f(x)), F(f(x^*))\}$  and thus  $F(f(\lambda x + (1 - \lambda)x^*)) > \min\{F(f(x)), F(f(x^*))\}$  for any  $\lambda \in (0,1)$ . This is the definition of a strictly quasiconcave function, so  $F(f(\cdot))$  is a strictly quasi-concave function.

Hossain and Okui (2011) independently prove the same result with respect to RDU, but our proof is arguably more instructive because it points to a general mechanism-design principle to show how to make the incentives for the scoring rule more powerful. By recognizing that the strict quasiconcavity of  $w(Q(\theta))$  is needed to ensure that the scoring rule is proper, we can identify ways to design scoring rules that have better properties in certain regions of  $\theta$ . This issue is an important one when trying to elicit subjective beliefs with respect to extremely small probabilities, as occurs almost all of the time when designing insurance contracts for low-probability, but high cost, events.

The approach we consider also works for subjects that follow Cumulative Prospect Theory (CPT), with just one additional restriction on top of the restriction needed for RDU.

Hossain and Okui (2011) demonstrate that a subject that follows CPT will directly report

his subjective probability if the probability weighting function is strictly increasing.<sup>5</sup> This requirement is weak, although not met by some popular probability weighting functions, such as the “inverse-S” function for extreme parameter values.

## 5.3 Experiments

### 5.3.1 Experimental Design

We implement two between-subjects treatments that were presented with the same stimuli in any given session. First, in **treatment M** subjects were presented with only one belief elicitation question that uses the QSR where the scores are money amounts. Second, in **treatment P** subjects were also presented with only one belief elicitation question that also uses the QSR, but with the scores denominated in points that subsequently determine the objective probability of winning a binary lottery. In both treatments subjects are presented first with some relevant information to complete the belief elicitation task and after that the choice is made. Immediately subjects completed a demographic survey, but they were explicitly told that their responses would not affect any other earnings. Then subjects received monetary compensation according to a chance realization of the binary event and their report in the belief elicitation. There were no other salient tasks, before or after a subject’s choices, affecting the outcome. Table 5.1 summarizes our experimental design for each of four sessions, and the sample size of subjects in each treatment per

---

<sup>5</sup>They also state that one needs the subject to treat the low payoff in the BLP as a loss and the high payoff as a gain, but this does not appear necessary if one just assumes non-satiation in the gain domain (or across domains).

session.

Table 5.1: Experimental Design

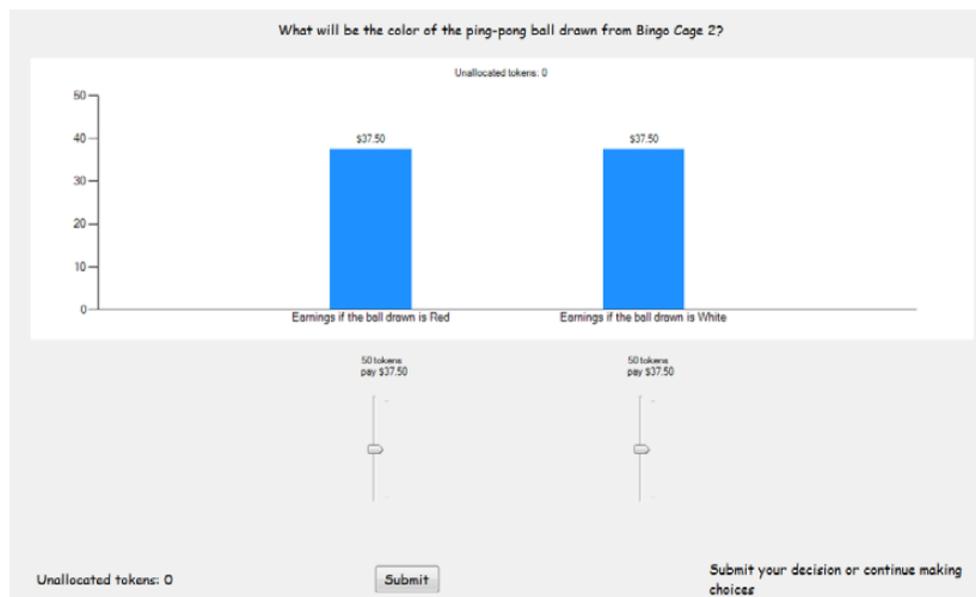
	Treatment M	Treatment P	Total
Session 1	17	18	35
Session 2	16	17	33
Session 3	17	16	33
Session 4	18	19	37
Total	68	70	138

We used software we created in *Visual Basic .NET* to present the QSR to subjects and record their choices. Figures 5.2 and 5.3 illustrate the scoring rule task faced by subjects in treatments M and B, respectively, which are variants on the “slider interface” proposed by Andersen et al. (2010). Subjects can move one or other of two sliders, and the other slider changes automatically so that 100 tokens are allocated. The main difference between the figures is that the payoffs of the scoring rule are denominated in dollars in the case of Figure 5.2, and determined in points in Figure 5.2. Subjects can earn up to \$50 in treatment M and either \$50 or \$0 in treatment P.

Our experiment elicits beliefs from subjects in each treatment over the composition of a Bingo cage that contained red and white Ping-Pong balls with a composition that was not known with certainty to subjects before they made their choices. We explain below the procedure used to determine the composition of the Bingo cage. This procedure was intended to avoid trust issues between the experimental subjects and the experimenter, an important source of potential confounds in tasks that involve belief elicitation.

The general procedures during an experimental session were as follows. Upon arrival at the

Figure 5.2: Subject Display for Treatment M



laboratory, each subject drew a number from a box which determined random seating position within the laboratory. After being seated and signing the informed consent document, subjects were given printed introductory instructions and allowed sufficient time to read these instructions.<sup>6</sup> Then a Verifier was selected at random among subjects solely for the purpose of verifying that the procedures of the experiment were carried out according to the instructions. The Verifier was paid a fixed amount for this task. Each subject was assigned to one of two groups depending on whether the seating number was even or odd, where each group corresponded to one of the treatments. One of the groups was then asked to leave and wait outside the room for a few minutes, always under the supervision of an experimenter. The other group remained in the laboratory and went over the main instructions with the experimenter. Simultaneously, subjects waiting outside were given instructions to read individually. Then the groups swapped places and the

<sup>6</sup>Section 5.6 provides complete subject instructions.

Figure 5.3: Subject Display for Treatment P



experimenter read the main instructions designed for the other group. Once all instructions were finished, and both groups were together in the room again, we proceeded with the rest of the experiment.

The reason for these procedures is to ensure that M and P subjects would confront the same stimuli, defined below. We did not want to explain P-specific instructions in the presence of M subjects and tell them not to pay attention, and vice versa. Subjects were told the reason for this step using the following language:

Part of this experiment is to test different computer screens. Therefore, we will divide you into two groups, and each group will be presented with a slightly different instructions and computer screens. If you are sitting in a computer station that has an odd number on it, you are part of the Odd group. If you are sitting in a computer station that has an even number on it, you are part of the Even group.

An important reason to assign subjects to treatments according to their station number in

the laboratory is to avoid potential confounds in the results generated by subjects in each treatment having very different visual access to the stimuli. By mapping even or odd station numbers to treatment M or P, we ensure that if there exist any difference in subjects vantage point, this difference was the same across treatments.

We used two bingo cages: Bingo Cage 1 and Bingo Cage 2. Bingo Cage 1 was loaded with balls numbered 1 to 99 in front of everyone.<sup>7</sup> A numbered ball was drawn from Bingo Cage 1, but the draw took place behind a divider. The outcome of this draw was not verified in front of subjects until the very end of the experiment, after their decisions had been made. The number on the chosen ball from Bingo Cage 1 was used to construct Bingo Cage 2 behind the divider. The total number of balls in Bingo Cage 2 was always 100: the number of red balls matched the number drawn from Bingo Cage 1, and the number of white balls was 100 minus the number of red balls. Since the actual composition of Bingo Cage 2 was only revealed and verified in front of everybody at the end of the experiment, the Verifier's role was to confirm that the experimenter constructed Bingo Cage 2 according to the randomly chosen numbered ball. Once Bingo Cage 2 was constructed, the experimenter put the chosen numbered ball in an envelope and affixed it to the front wall of the laboratory. Bingo Cage 2 was then covered and placed on a platform in the front of the room. Bingo Cage 2 was then uncovered for subjects to see, spun for 10 turns, and covered again.

Subjects then made their decisions about the number of red and white balls in Bingo Cage 2. After choices were made and subjects completed a non-salient demographic survey, the

---

<sup>7</sup>When shown in public, Bingo Cage 1 and 2, were always displayed always in front of the laboratory where everyone could see them. We also used a high resolution video camera to display the bingo cages in three flat screen TVs distributed throughout the laboratory, and on the projection screen at the front of the room. Our intention was that everyone had equal chance of observing the bingo cages from a good perspective.

experimenter drew a ball from Bingo Cage 2. The sealed envelope was opened and the chosen numbered ball was shown to everyone, and the experimenter publicly counted the number of red and white balls in Bingo Cage 2. Then an experimenter approached each subject and recorded earnings according to the betting choices made and the ball drawn from Bingo Cage 2. If subjects were part of treatment M, their earnings were determined by the report and the corresponding score in dollars of the QSR. If subjects were in treatment P, the number of points they earned in the belief elicitation task was recorded. Then subjects rolled two 10-sided dice, and if the outcome was less or equal to the number of points earned they won \$50, otherwise they earned \$0 in the task. Finally, subjects left the laboratory and were privately paid their earnings: a \$7.50 participation payment in addition to the monetary outcome of the belief elicitation task. The Verifiers were paid a flat \$25 fee plus the participation fee. Subjects on average earned approximately \$45.6 including the participation payment.

### 5.3.2 Evaluation of Hypothesis

We want to test if the BLP induces linear utility, providing incentives to subjects to report truthfully and directly their underlying subjective probability. In our tests we assume that the distributions of risk attitudes across subjects, and also of subjective probabilities, are the same in treatments M and P. Therefore any observed difference in reports would be a result of BLP affecting subjects' behavior.

There are at least two ways of testing our hypothesis. The first takes advantage of observed behavior in scoring rules of risk averse individuals versus behavior of risk neutral

individuals. If subjects are indeed risk averse and the BLP does induce linear utility, then subjects in treatment M should be drawn on average to make reports closer to 50 than subjects in treatment P. This implies that, depending on the true underlying subjective probability, the average report in treatment M would be smaller or greater than in treatment B in such a manner that the former is always closer to 50. To make this test operational, we calculate a measure of distance between each report and the middle of the report interval. For example, if a subject made a report of 30 for red balls, then the measure of report distance is the absolute value of the difference which is 20 ( $= |30 - 50|$ ). If the underlying subjective probability is close to 50%, there would be an identification problem because subjects in both treatments have strong incentives to make a report close to 50. This might be very likely in situations where Bingo Cage 2 has a composition of red and white balls close to 50/50, which was indeed the case in one of our sessions. Similarly, if the underlying subjective probability is close to 0% or 100% we would have an identification problem. This did not happen in any of the sessions, but was a risk in this design, of course the risk was just wasted subject fees and time.

An ideal test of our hypothesis would involve comparing the reports in treatment P with the underlying subjective probabilities of subjects in treatment M. However, since this is an unobserved, latent variable, we use instead the correct proportion of red balls in the Bingo Cage 2 that subjects actually faced in each session as a proxy for the underlying subjective probability.<sup>8</sup> If the BLP does induce linear utility, subjects in treatment P should make reports on average that are closer to the true number of red balls in Bingo

---

<sup>8</sup>This assumption implies that on average subjects have the right idea about the number of red balls in Bingo Cage 2. In the absence of an estimate, this is a natural proxy for the underlying subjective probability.

Cage 2 than subjects in treatment M. To make this test operational, we use also a measure of distance, but instead of using 50 as a point of reference we use the true number of red balls in the Bingo Cage 2 each subject faced. We also refer to this measure of distance as a report distance. For example, if a subject made a report of 30 and the correct number of red balls in the Bingo Cage 2 he faced was 25, then the measure of report distance is the absolute value of the difference which is 5 ( $= |30 - 25|$ ). The comparison across treatments of this measure of distance is also a test of relative accuracy of reports. Even though it is interesting in its own right, it is not our primary objective to assess perceptual accuracy of subjects.<sup>9</sup>

We pool data across sessions, but also analyze data from individual session. We apply nonparametric tests to the distance measures to test if there is support for our main hypothesis that the BLP induces linear utility in the belief elicitation task.

## 5.4 Results

### 5.4.1 Does the BLP Mitigate the Effects of Risk Aversion?

We find evidence of a treatment effect which supports the hypothesis that the BLP induces linear utility in our belief elicitation tasks. Our sample size is almost evenly distributed among treatments: pooling across sessions, there were 68 subjects in treatment M and 70 in treatment P. Figure 5.4 shows the frequency of reports in each treatment, by session.

---

<sup>9</sup>In fact there might be some visual saliency of red balls that might have induced subjects to make reports higher than if we had used balls of different colors. Again, this is an interesting issue to study but not our primary research purpose.

Figure 5.5 displays, again by session, the estimated densities of the reports, the correct number of red balls in Bingo Cage 2, and the mean report in each treatment. In session 1 the average reports for treatments M and P are 34.2 and 30.8, respectively. Excluding a subject that did an idiosyncratic report of 100 for red balls, the average report for treatment P in session 1 is decreased to 26.8.<sup>10</sup> The average reports from treatments M and P are 59.7 and 65.8 for session 2. The panels for sessions 1 and 2 in Figure 5.5 are illustrative of the treatment effect consistent with risk aversion: the mass of the estimated densities for treatment M is closer to the middle of the report interval than for the case of treatment P. This feature is not readily seen in the case of sessions 3 and 4, but one can use non-parametric statistics to test the statistical significance of this treatment effect across sessions.

We find evidence that subjects in treatment M tend to make reports closer to 50 than subjects in treatment P. Across all sessions, the average of the absolute value of the difference between reports and 50 is 14.2 and 18.7 for treatments M and P, respectively. A one-sided Fisher-Pitman permutation test for difference in means results in a  $p$ -value of 0.02, which suggests that on average subjects in treatment M tend to make reports closer to 50. Session 3 ended up having a Bingo Cage 2 with composition of red and white balls close to 50/50, precisely where we predict this treatment effect test would have low power.

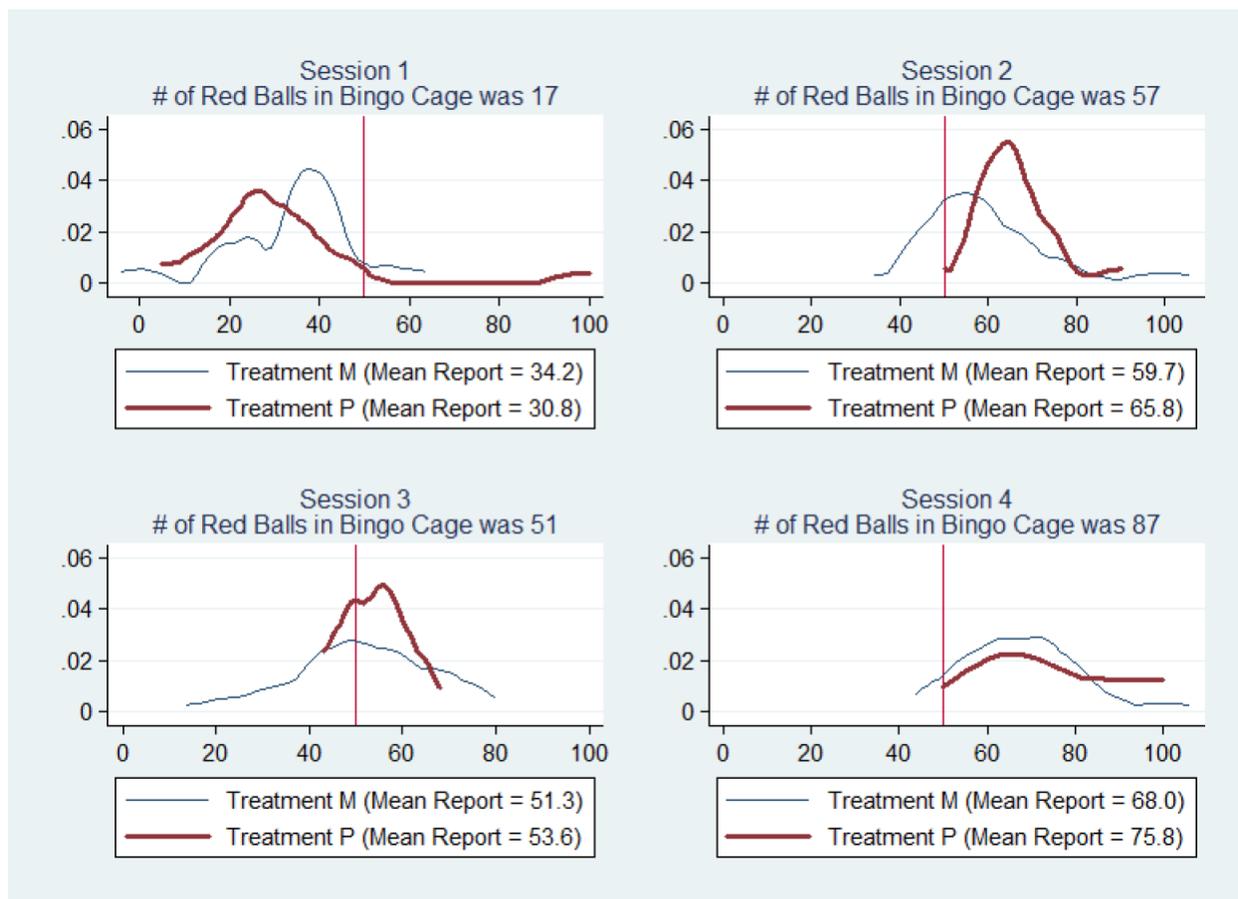
Thus we present non-parametric test results on distributions with and without this session included. Figure 5.6 shows the empirical cumulative distribution of our measure of distance

---

<sup>10</sup>Although this subject's reporting behavior was certainly puzzling and idiosyncratic, it can still be rationalized by non-EU preferences. In particular, an appropriate combination of probability weighting function that violates the assumptions outline in section 1 and subjective beliefs can provide incentives to subjects to make a report close to 100. A more simple explanation is that the subject had a strong preference for color red that was not related to the actual configuration of Bingo Cage 2.



Figure 5.5: Estimated Densities of Reports by Session



### 5.4.2 Does the BLP Improve Accuracy?

We find evidence that subjects from treatment P tend to make reports closer to the correct number of red balls in Bingo Cage 2, supporting the hypothesis that the BLP induces linear utility in our belief elicitation tasks. There were 68 subjects in treatment M whose average report distance was 15.2, while there were 70 subjects in treatment P whose average report distance was 12.8. Figure 5.7 shows, the empirical distribution of the absolute value of differences between reports and the correct number of red balls, pooling over all sessions. Figure 5.7 illustrates how the cumulative distribution of treatment P is

dominated by the distribution of treatment M, which implies that distances are smaller in treatment P. The one-sided Kolmogorov-Smirnov test for two samples results in a  $p$ -value of 0.04, which supports the hypothesis that distances of reports from the correct number of red balls in treatment P are smaller than in treatment M. Under the assumption that the correct number of red balls in Bingo Cage 2 is a good proxy for the average underlying subjective probability, we find that subjects tend to make reports closer to the correct number of red balls. This could be interpreted as better accuracy from the part of subjects in treatment P. However we interpret this observed behavior as a result of the BLP: this procedure induces linear utility in subjects which provides incentives to reveal the underlying subjective probability, thus mitigating the distortion in reports introduced by risk attitudes.

## 5.5 Conclusions

Harrison, Martinez-Correa and Swarthout (2012) found that the binary lottery procedure works robustly to induce risk neutrality when subjects are given one risk task defined over objective probabilities and the evaluation of the hypothesis does not depend on the assumed validity of any strategic equilibrium behavior, or even the customary independence axiom. Using individuals sampled from the same pool of subjects, we find evidence of treatment effects which support the hypothesis that the Binary Lottery Procedure induces linear utility in a belief elicitation task that uses the Quadratic Scoring Rule and that presents only one question to subjects about the composition of a Bingo Cage.

First, we found that subjects that were not exposed to the BLP tend on average to make reports closer to the middle of the report interval, which reduces the uncertainty involved in the belief elicitation task. Second, we also found evidence that subjects that were exposed to the BLP tend to make reports closer to the correct number of red balls in Bingo Cage 2. We interpret both findings as the BLP inducing linear utility in subjects, thus mitigating the distortion in reports introduced by risk attitudes.

An important feature of the BLP is that it theoretically provides incentives for subjects to directly report their underlying subjective probability. This applies for subjects with subjective expected utility representations and, under certain weak conditions, to individuals with certain non- EU preference representations. In particular, the BLP theoretically works for subjects that follows Rank-Dependent Utility theory.

There are several important extensions of our approach left for future work. First, econometric techniques could be used to estimate the underlying subjective probabilities of subjects not exposed to the BLP and compare them to the reports directly elicited from subjects exposed to the BLP. In this manner we can say whether the BLP directly elicits the true subjective probabilities, and not just that it elicits subjective probabilities in the predicted direction compared to the QSR without controls for risk aversion. Second, a natural extension of our approach would be to test if the BLP works in more general scoring rules designed to elicit full distributions for continuous events, such as the generalization of the QSR proposed by Mathieson and Winkler (1976). Third, the procedures developed here can be used to test the validity of the reduction of compound lotteries axiom defined over subjective beliefs. This important extension entails some

subtle extensions in the experimental design, building on the design employed here.

Figure 5.6: Empirical Cumulative Distribution of Distance of Reports from 50

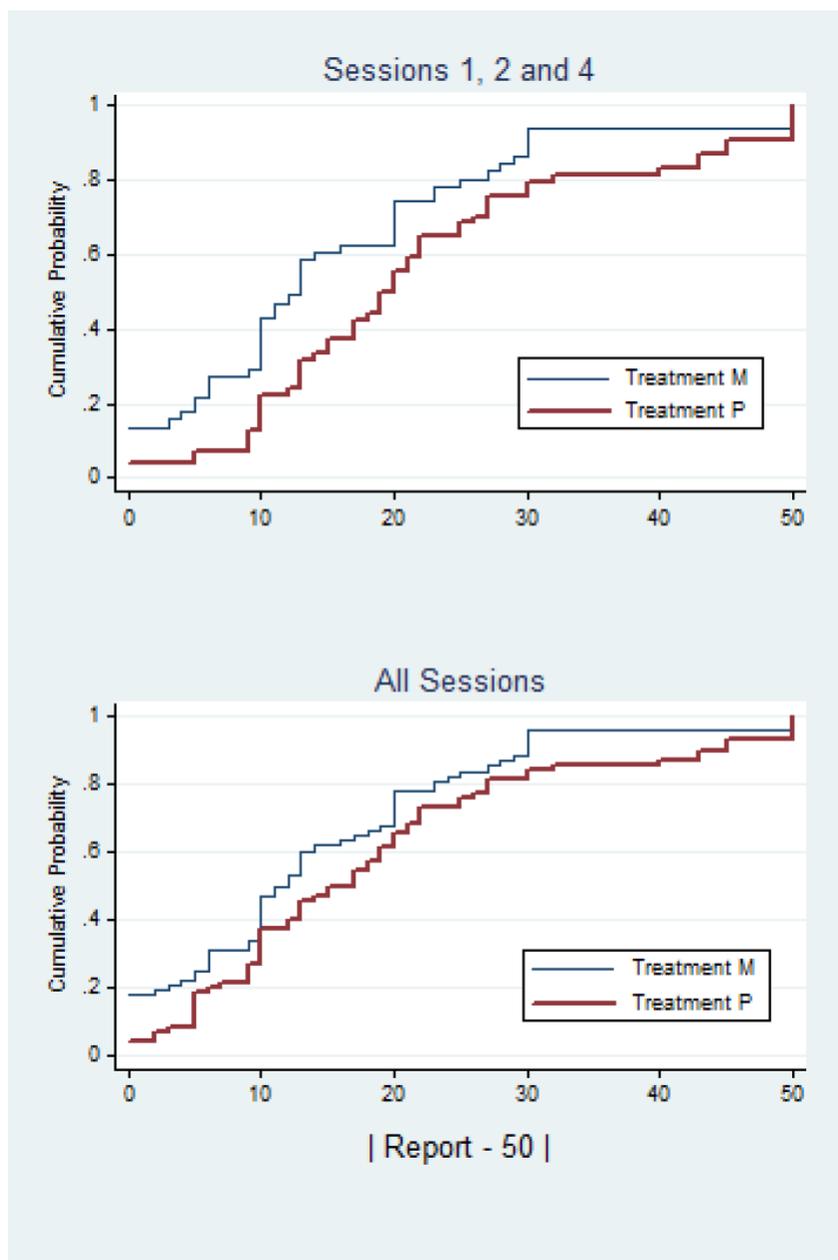
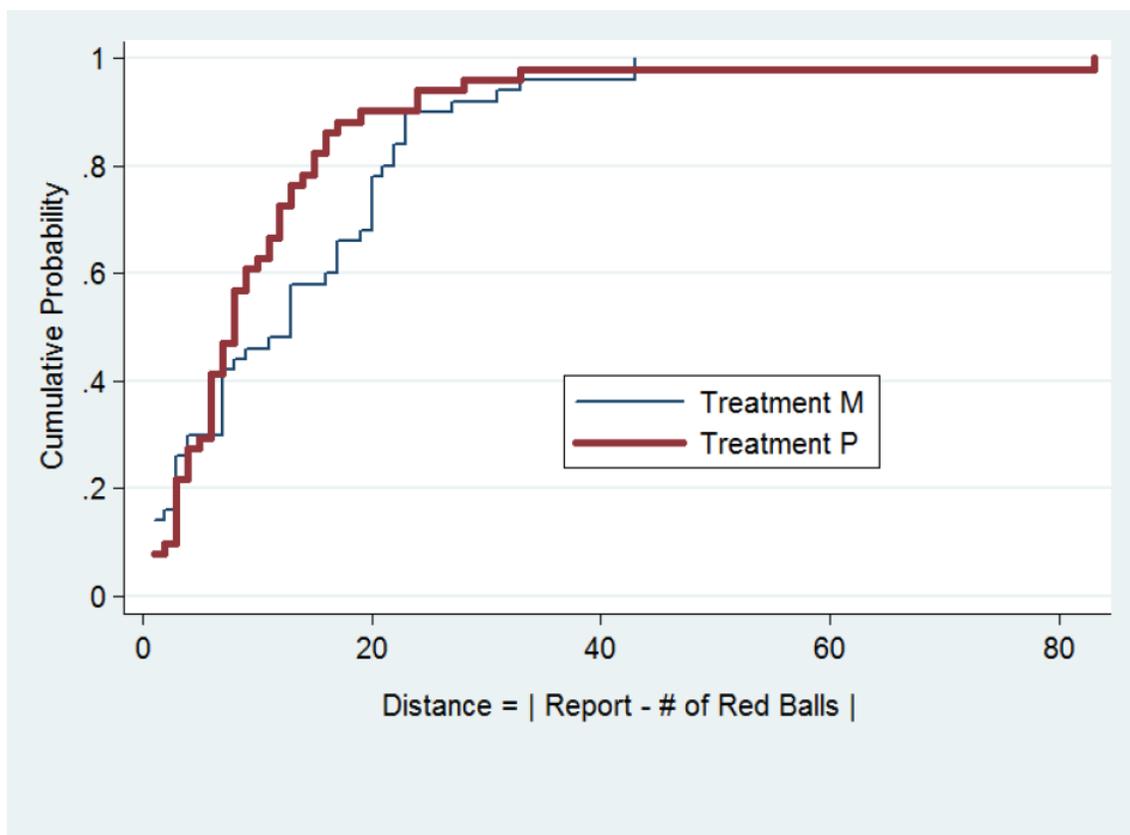


Figure 5.7: Empirical Cumulative Distribution of Distance Pooling Data From All Sessions



## 5.6 Appendix A. Instructions

### *A. Introductory Instructions*

*sub-roc*

#### Introductory Instructions

You are now participating in a decision-making experiment. Based on your decisions in this experiment, you can earn money that will be paid to you in cash today. It is important that you understand all instructions before making your choices in this experiment.

Please turn to silent, and put away, your cell phone, laptop computer, or any other device you may have brought with you. Please do not talk with others seated nearby for the duration of the experiment. If at any point you have a question, please raise your hand and we will answer you as soon as possible.

The experiment consists of one decision-making task and a demographics survey. You have already earned \$7.50 for agreeing to participate in the experiment, which will be paid in cash at the end of the session. In addition to this show-up fee, you may earn considerably more from your choices in the decision-making task. This task and the potential earnings from it will be explained in detail as we proceed through the session.

This experiment requires us to do some things out of your sight. However, at the end of the experiment we will prove to you that we followed the procedures described in the instructions. Additionally, we will select one of you at random solely for the purpose of verifying that the steps of this experiment are done exactly as described in the instructions. As we proceed in the experiment, we will outline clearly the steps that this Verifier has to verify. In a moment we will select the Verifier by drawing a random number and matching the outcome with the appropriate seat number. The Verifier will be paid \$25 for this job on top of the \$7.50 show-up fee, and will not make any decisions in the experiment. The Verifier will join the experimenter, observe the procedures, and confirm that we are following the procedures explained in these instructions. The Verifier must not communicate with anyone in the room except the experimenter. Failure to do so will result in that person losing the promised amount, another person being chosen as Verifier, and a restart of the experiment.

Part of this experiment is to test different computer screens. Therefore, we will divide you into two groups, and each group will be presented with a slightly different instructions and computer screens. If you are sitting in a computer station that has an odd number on it, you are part of the Odd group. If you are sitting in a computer station that has an even number on it, you are part of the Even group.

Once the Verifier is chosen and joins the experimenter at the front of the room, we will hand out the rest of the instructions. We will then have one of the two groups leave the room for a few minutes, so that an experimenter can read the instructions aloud to the remaining group and answer any questions if necessary. Then the groups will swap places and an experimenter will read instructions to the other group and answer any questions if necessary. There will always be some experimenters guiding you to get in or out of the room at the right moment.

Once all instructions are finished, and both groups are together in the room again, we will proceed with the experiment. Please remain silent during the experiment, and simply raise your hand if you have any question so that an experimenter will come to you.

*B. Instructions for Treatment M*

2m

**Your Beliefs**

This is a task where you will be paid according to how accurate your beliefs are about certain things. You will be presented with one and only one question of the type we will explain below. You will actually get the chance to play the question presented to you, so you should think carefully about your answer to the question.

You will make decisions about the color of a ball to be drawn from a bingo cage. This bingo cage will contain 100 balls colored red and white. The exact mix of red and white balls will be unknown to you, but you will receive information about the mixture. The following instructions explain in more detail how this experiment will work.

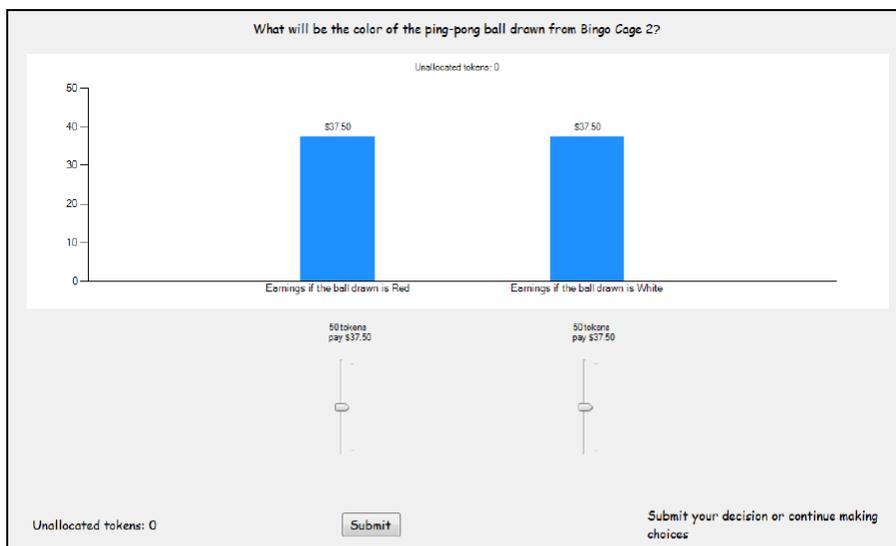
We have selected a Verifier at random solely for the purpose of verifying that we follow the process described in the instructions. When the time comes we will display a summary of the steps the Verifier will have to verify. We remind you that the Verifier must not communicate with anyone in the lab except the experimenter. Failure to do so will result in that person losing the promised amount, another person being chosen as verifier, and a restart of the experiment.

We have two bingo cages: Bingo Cage 1 and Bingo Cage 2. We will load Bingo Cage 1 with balls numbered 1 to 99. You will watch us do this, and be able to verify yourself that Bingo Cage 1 is loaded with the correct numbered balls. We will then draw a numbered ball from Bingo Cage 1. However, the draw of a numbered ball from Bingo Cage 1 will take place behind a divider, and you will not know the outcome of this draw until the very end of the experiment, after you have made your decisions. Any number between 1 and 99 is equally likely.

The number on the chosen ball from Bingo Cage 1 will be used to construct Bingo Cage 2 behind the divider. The total number of balls in Bingo Cage 2 will always be 100: the number of red balls will match the number drawn from Bingo Cage 1, and the number of white balls will be 100 minus the number of red balls. Since the actual composition of the Bingo Cage 2 will only be revealed and verified in front of everybody at the end of the experiment, the Verifier will confirm that the experimenter constructs Bingo Cage 2 according to the randomly chosen numbered ball. Once Bingo Cage 2 is constructed, the experimenter will put the chosen numbered ball in an envelope and affix it to the front wall above the white board.

Next, Bingo Cage 2 will be covered and placed on the platform in the front of the lab. Then, Bingo Cage 2 will be uncovered for you to see and spun for 10 turns. After this, we will again cover Bingo Cage 2. You will then make your decisions about the number of red and white balls in Bingo Cage 2. After you have made your choices, we will draw a ball from Bingo Cage 2 and your winnings will depend on your choices and the outcome of this draw. Finally, the sealed envelope will be opened and we will show the chosen numbered ball to everyone, and we will also publicly count the number of red and white balls in Bingo Cage 2. We go through with this verification process so that you can believe that the experiment will take place exactly as we describe in the instructions.

Now we will explain how you will actually make your choices. To make your choices, you will use a computer screen like the one shown below.

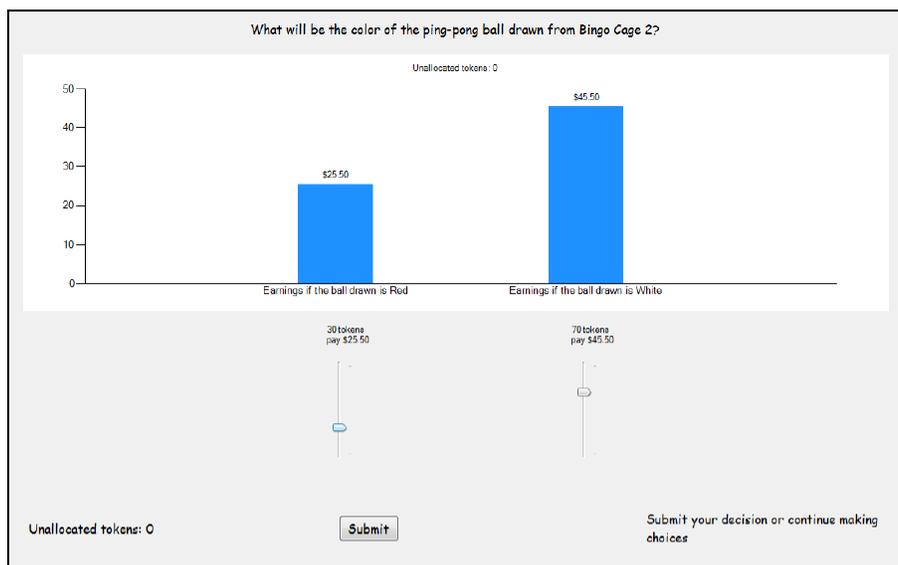


The display on your computer will be larger and easier to read. You have 2 sliders to adjust, shown at the bottom of the screen. Each slider allows you to allocate tokens to reflect your belief about the answer to this question. You must allocate all 100 tokens in order to submit your decision, and we always start with 50 tokens being allocated to each slider. The dollar payoffs shown on the screen only apply when you allocate all 100 tokens. As you allocate tokens, by adjusting sliders, the dollar payoffs displayed on the screen will change. Your earnings are based on the payoffs that are displayed after you have allocated all 100 tokens.

You can earn up to \$50 in this task.

Where you position each slider depends on your beliefs about the color of the Ping-Pong ball to be drawn from the bingo cage. The tokens you allocate to each bar will naturally reflect your beliefs about the number of red and white balls in Bingo Cage 2. The bar on the left depends on your beliefs that the ball to be drawn will be red and the bar on the right depends on your beliefs that the ball to be drawn will be white. Each bar shows the amount of money you earn if the ball drawn from the bingo cage is red or white.

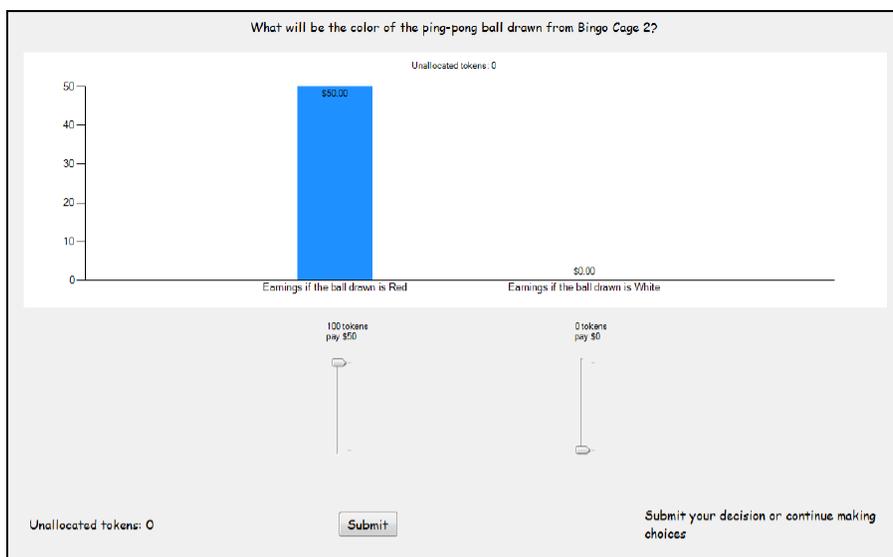
To illustrate how you use these sliders, suppose you think there is a fair chance that there are less red balls than white balls in Bingo Cage 2. Then you might allocate 30 tokens to the first bar, as shown below. Notice that the second bar will be automatically adjusted depending on the number of tokens you allocated on the first bar. Therefore, by allocating 30 tokens to the first bar you are allocating 70 tokens to the second. So you can see that if indeed the ball drawn is red you would now earn \$25.50. If the ball drawn is white instead you would earn \$45.50.



The above pictures show someone who allocated 30 tokens to red Ping-Pong balls and 70 tokens to white Ping-Pong balls. You can adjust this as much as you want to best reflect your personal beliefs about the composition of the bingo cage.

Your earnings depend on your reported beliefs and, of course, the ball drawn. Suppose that a red ball was drawn from Bingo Cage 2 and you reported the beliefs shown above. You would have earned \$25.50.

What if instead you had put all of your eggs in one basket, and allocated all 100 tokens to the draw of a red ball? Then you would have faced the earnings outcomes shown below.



Note the “good news” and “bad news” here. If the chosen ball is red, you can earn the maximum payoff, shown here as \$50. But if a white ball is chosen, then you would have earned nothing in this task.

It is up to you to balance the strength of your personal beliefs with the risk of them being wrong. There are three important points for you to keep in mind when making your decisions:

- **Your belief about the chances of each outcome is a personal judgment that depends on the information you have about the different events.** Remember that you will have the chance to see Bingo Cage 2 being spun for ten turns before it is covered again. This is the information you will have to make your choices.
- **Depending on your choices and the ball drawn from Bingo Cage 2 you can earn up to \$50.**
- **Your choices might also depend on your willingness to take risks or to gamble.** There is no right choice for everyone. For example, in a horse race you might want to bet on the long shot since it will bring you more money if it wins. On the other hand, you might want to bet on the favorite since it is more likely to win something.

The decisions you make are a matter of personal choice. Please work silently, and make your choices by thinking carefully about the task you are presented with.

When you are happy with your decisions, you should click on the **Submit** button and confirm your choices. When everyone is finished we will uncover and spin Bingo Cage 2, and pick one ball at random in front of you. Then an experimenter will come to you and record your earnings according to the color of the ball that was picked and the choices you made.

All payoffs are in cash, and are in addition to the \$7.50 show-up fee that you receive just for being here. The only other task today is for you to answer some demographic questions. Your answers to those questions will not affect your payoffs.

Are there any questions?

*C. Instructions for Treatment P*

2p

**Your Beliefs**

This is a task where you will be paid according to how accurate your beliefs are about certain things. You will be presented with one and only one question of the type we will explain below. You will actually get the chance to play the question presented to you, so you should think carefully about your answer to the question.

You will make decisions about the color of a ball to be drawn from a bingo cage. This bingo cage will contain 100 balls colored red and white. The exact mix of red and white balls will be unknown to you, but you will receive information about the mixture. The following instructions explain in more detail how this experiment will work.

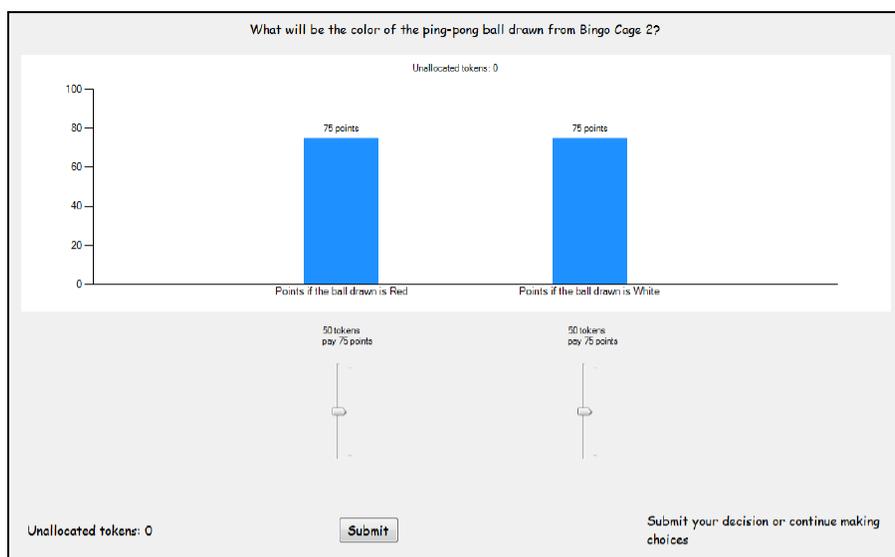
We have selected a Verifier at random solely for the purpose of verifying that we follow the process described in the instructions. When the time comes we will display a summary of the steps the Verifier will have to verify. We remind you that the Verifier must not communicate with anyone in the lab except the experimenter. Failure to do so will result in that person losing the promised amount, another person being chosen as verifier, and a restart of the experiment.

We have two bingo cages: Bingo Cage 1 and Bingo Cage 2. We will load Bingo Cage 1 with balls numbered 1 to 99. You will watch us do this, and be able to verify yourself that Bingo Cage 1 is loaded with the correct numbered balls. We will then draw a numbered ball from Bingo Cage 1. However, the draw of a numbered ball from Bingo Cage 1 will take place behind a divider, and you will not know the outcome of this draw until the very end of the experiment, after you have made your decisions. Any number between 1 and 99 is equally likely.

The number on the chosen ball from Bingo Cage 1 will be used to construct Bingo Cage 2 behind the divider. The total number of balls in Bingo Cage 2 will always be 100: the number of red balls will match the number drawn from Bingo Cage 1, and the number of white balls will be 100 minus the number of red balls. Since the actual composition of the Bingo Cage 2 will only be revealed and verified in front of everybody at the end of the experiment, the Verifier will confirm that the experimenter constructs Bingo Cage 2 according to the randomly chosen numbered ball. Once Bingo Cage 2 is constructed, the experimenter will put the chosen numbered ball in an envelope and affix it to the front wall above the white board.

Next, Bingo Cage 2 will be covered and placed on the platform in the front of the lab. Then, Bingo Cage 2 will be uncovered for you to see and spun for 10 turns. After this, we will again cover Bingo Cage 2. You will then make your decisions about the number of red and white balls in Bingo Cage 2. After you have made your choices, we will draw a ball from Bingo Cage 2 and your winnings will depend on your choices and the outcome of this draw. Finally, the sealed envelope will be opened and we will show the chosen numbered ball to everyone, and we will also publicly count the number of red and white balls in Bingo Cage 2. We go through with this verification process so that you can believe that the experiment will take place exactly as we describe in the instructions.

Now we will explain how you will actually make your choices. To make your choices, you will use a computer screen like the one shown below.



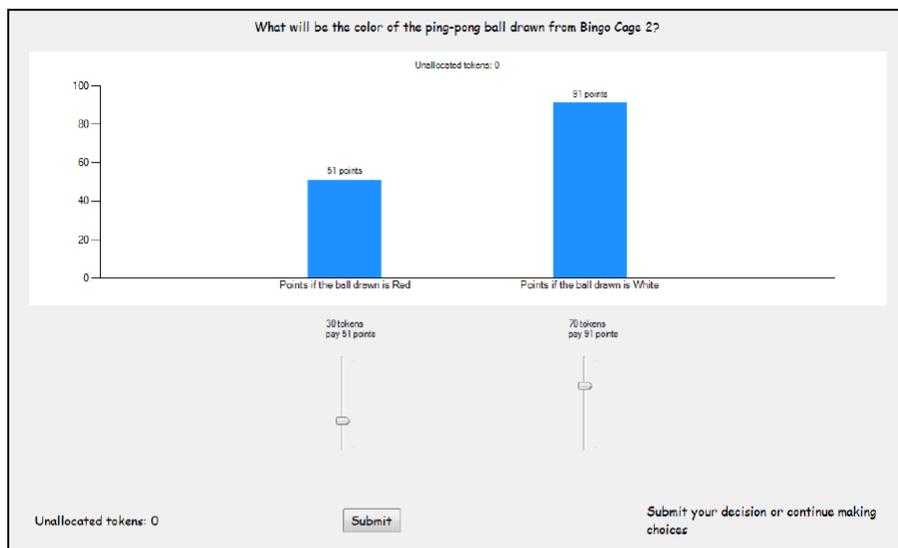
The display on your computer will be larger and easier to read. You have 2 sliders to adjust, shown at the bottom of the screen. Each slider allows you to allocate tokens to reflect your belief about the answer to this question. You must allocate all 100 tokens in order to submit your decision, and we always start with 50 tokens being allocated to each slider. The point payoffs shown on the screen only apply when you allocate all 100 tokens. As you allocate tokens, by adjusting sliders, the point payoffs displayed on the screen will change. Your earnings are based on the payoffs that are displayed after you have allocated all 100 tokens.

You earn points in this task. Every point that you earn gives you a greater chance of being paid \$50. To be paid for this task you will roll two 10-sided dice, with every outcome between 1 and 100 equally likely. If you roll a number that is less than or equal to your earned points, you earn \$50; otherwise you earn \$0.

Where you position each slider depends on your beliefs about the color of the Ping-Pong ball to be drawn from the bingo cage. The tokens you allocate to each bar will naturally reflect your beliefs about the number of red and white balls in Bingo Cage 2. The bar on the left depends on your beliefs that the ball to be drawn will be red and the bar on the right depends on your beliefs that the ball to be drawn will be white. Each bar shows the amount of points you earn if the ball drawn from the bingo cage is red or white.

To illustrate how you use these sliders, suppose you think there is a fair chance that there are less red balls than white balls in Bingo Cage 2. Then you might allocate 30 tokens to the first bar, as

shown below. Notice that the second bar will be automatically adjusted depending on the number of tokens you allocated on the first bar. Therefore, by allocating 30 tokens to the first bar you are allocating 70 tokens to the second. So you can see that if indeed the ball drawn is red you would now earn 51 points. If the ball drawn is white instead you would earn 91 points.

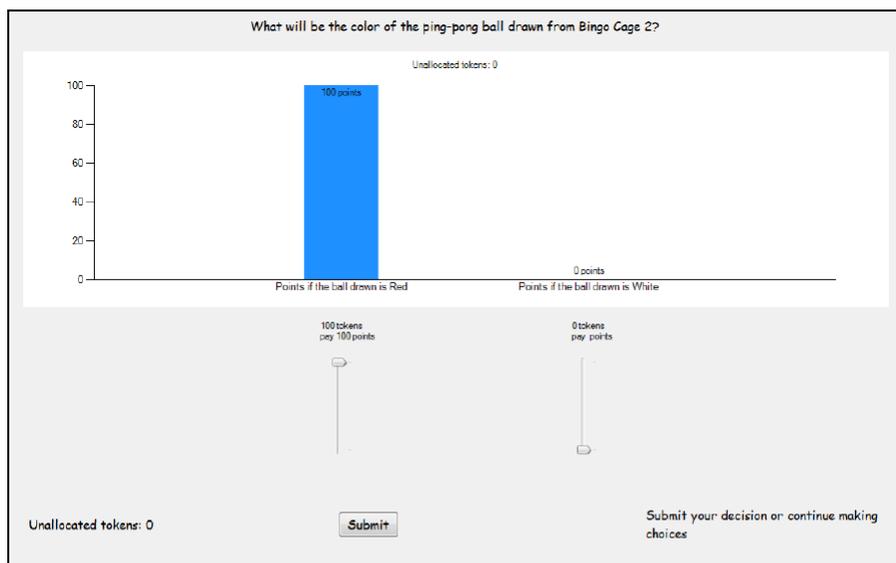


The above pictures show someone who allocated 30 tokens to red Ping-Pong balls and 70 tokens to white Ping-Pong balls. You can adjust this as much as you want to best reflect your personal beliefs about the composition of the bingo cage.

For instance, suppose you allocated your tokens as in the figure shown above. If a red ball is drawn from Bingo Cage 2, then you would earn 51 points. Then suppose that you rolled a 40 with the two 10-sided dice. In this case, you would be paid \$50 since your dice roll is less than or equal to your earned points. However, if your dice roll was some number greater than 51, say 60, you earn \$0. If you earn 100 points then you will earn \$50 for sure, since every outcome of your dice roll would result in a number less than or equal to 100.

If you do not earn \$50 you receive nothing from this task, but of course get to keep your show-up fee. Again, the more points you earn in the correct bar the greater your chance of getting \$50 in this task.

What if instead you had put all of your eggs in one basket, and allocated all 100 tokens to the draw of a red ball? Then you would have faced the earnings outcomes shown below.



Note the “good news” and “bad news” here. If the chosen ball is red, you can earn the maximum payoff, shown here as 100 points. But if a white ball is chosen, then you would have earned nothing in this task.

It is up to you to balance the strength of your personal beliefs with the risk of them being wrong. There are three important points for you to keep in mind when making your decisions:

- **Your belief about the chances of each outcome is a personal judgment that depends on the information you have about the different events.** Remember that you will have the chance to see Bingo Cage 2 being spun for ten turns before it is covered again. This is the information you will have to make your choices.
- **Depending on your choices and the ball drawn from Bingo Cage 2 you can only earn either \$50 or \$0.**
- **More points increase your chance of being paid \$50.** The points you earn will be compared with the outcome of the roll of the two 10-sided dice to determine whether you win \$50 or \$0.

The decisions you make are a matter of personal choice. Please work silently, and make your choices by thinking carefully about the task you are presented with.

When you are happy with your decisions, you should click on the **Submit** button and confirm your choices. When everyone is finished we will uncover and spin Bingo Cage 2, and pick one ball at random in front of you. Then an experimenter will come to you and record your earnings according to the color of the ball that was picked and the choices you made.

All payoffs are in cash, and are in addition to the \$7.50 show-up fee that you receive just for being here. The only other task today is for you to answer some demographic questions. Your answers to those questions will not affect your payoffs.

Are there any questions?

# Bibliography

- Abdellaoui, Mohammed, Aurélien Baillon, Laetitia Placido and Peter P. Wakker. “The Rich Domain of Uncertainty: Source Functions and Their Experimental Implementation.” *American Economic Review*, 101 no. 2 (April 2011): 695-723.
- Aese, Knut. “Wealth Effects in the Demand for Insurance.” *NHH Dept. Of Finance and Management Science Discussion Paper No. 2007/6*, 2007.
- Ahn, David, Syngjoo Choi, Douglas Gale and Shachar Kariv “Estimating Ambiguity Aversion in a Portfolio Choice Experiment.” *Working Paper*, Department of Economics, University of California at Berkeley, 2011.
- Akerlof, George A. “The Market for ‘Lemons’: Quality Uncertainty and the Market Mechanism.” *Quarterly Journal of Economics*, 84 no. 3 (August 1970): 488-500.
- Alary, David, Christian Gollier and Nicolas Treich. “The Effect of Ambiguity Aversion on Insurance Demand.” *Working Paper*, Toulouse School of Economics, 2010.
- Allen, Franklin. “Discovering Personal Probabilities When Utility Functions Are Unknown.” *Management Science*, 33 no. 4 (April 1987): 452-454.
- Andersen, Steffen, John Fountain, Glenn W. Harrison and E. Elisabet Rutström. “Estimating Subjective Probabilities.” *Working Paper 2010-06*, Center for the Economic Analysis of Risk, Robinson College of Business, Georgia State University, 2010.
- Andersen, Steffen, John Fountain, Glenn W. Harrison, Arne Risa Hole and E. Elisabet Rutström. “Inferring Beliefs as Subjectively Imprecise Probabilities.” *Theory and Decision*, (2012), *forthcoming*.
- Andersen, Steffen, Glenn W. Harrison, Morten I. Lau and E. Elisabet Rutström. “Eliciting Risk and Time Preferences.” *Econometrica*, 76 no. 3 ( May 2008): 583-618.
- Andersen, Steffen, Glenn W. Harrison, Morten I. Lau and E. Elisabet Rutström. “Discounting Behavior and the Magnitude Effect.” *CEAR Working Paper 2011-01*, Center for the Economic Analysis of Risk, Robinson College of Business, Georgia State University, 2011a.

- Andersen, Steffen, Glenn W. Harrison, Morten I. Lau and E. Elisabet Rutström. "Discounting Behavior: A Reconsideration." *CEAR Working Paper 2011-03*, Center for the Economic Analysis of Risk, Robinson College of Business, Georgia State University, 2011b.
- Andersen, Steffen, Glenn W. Harrison, Morten I. Lau and E. Elisabet Rutström. "Multiattribute Utility, Intertemporal Utility and Correlation Aversion." *CEAR Working Paper 2011-04*, Center for the Economic Analysis of Risk, Robinson College of Business, Georgia State University, 2011c.
- Angrist, Joshua D. and Jörn-Steffen Pischke *Mostly Harmless Econometrics: An Empiricist's Companion*. Princeton: Princeton University Press, 2009.
- Arrow, Kenneth J. "Alternative Approaches to the Theory of Choice in Risk-Taking Situations." *Econometrica*, 19 no. 4 (October 1951): 404-437.
- Arrow, Kenneth J. "Uncertainty and the Welfare Economics of the Medical Care." *American Economic Review*, 53 no. 5 (December 1963): 941-969.
- Arrow, Kenneth J. "The Role of Securities in the Optimal Allocation of Risk-Bearing." *Review of Economic Studies*, 31 no. 2 (April 1964): 91-96.
- Arrow, Kenneth J. "Insurance, Risk and Resource Allocation." In *Essays in the Theory of Risk Bearing*. Amsterdam: North Holland Publishing Company, 1970.
- Arrow, Kenneth J. *Essays in the Theory of Risk Bearing*. Chicago: Markham Publishing Company, 1971.
- Athey, Susan C. "Monotone Comparative Statics Under Uncertainty." *Quarterly Journal of Economics*, 117 no.1 (2002): 187-223.
- Babbel, David F. and Craig B. Merrill. "Rational Decumulation." *Working Paper 06-14*, Working Paper Series, Wharton Financial Institutions Center, University of Pennsylvania, 2006.
- Barron, Hutton. "Behavioral Decision Theory: A Topical Bibliography for Management Scientists." *Interfaces*, 5 no. 1 (November 1974): 56-62.
- Beattie, Jane and Graham Loomes. "The Impact of Incentives Upon Risky Choice Experiments." *Journal of Risk and Uncertainty*, 14 no 2 (March 1997): 149-162.
- Becker, Selwyn W. and Fred O. Brownson. "What Price Ambiguity? Or the Role of Ambiguity in Decision-Making." *Journal of Political Economy*, 72 no, 1 (February 1964): 62-73.
- Becker, Gordon M., Morris H. DeGroot and Jacob Marschak. "Measuring Utility By A Single-Response Sequential Method." *Behavioral Science*, 9 no. 3 (July 1964): 226-232.

- Bell, David E. "Regret in Decision Making under Uncertainty." *Operations Research*, 30 no. 5 (September-October 1982): 961-981.
- Benarjee, Abhijit V. and Esther Duflo. "The Experimental Approach to Development Economics." *Annual Review of Economics*, 1 no. 1 (May 2009): 151-178.
- Benartzi, Shlomo. "Excessive Extrapolation and the Allocation of 401(k) Accounts to Company Stock." *Journal of Finance*, 56 NO 5 (October 2001): 1747-1764.
- Benartzi, Shlomo, Alessandro Previtro and Richard H. Thaler. "Annuitization Puzzles." *Journal of Economic Perspectives*, 25 no. 4 (Fall 2011): 143-164.
- Benartzi, Shlomo and Richard H. Thaler. "Myopic Loss Aversion and the Equity Premium Puzzle." *Quarterly Journal of Economics*, 110 no. 1 (February 1995): 73-92.
- Berg, Joyce E., Lane A. Daley, John W. Dickhaut and John R. O'Brien. "Controlling Preferences for Lotteries on Units of Experimental Exchange." *Quarterly Journal of Economics*, 101 no 2 (May 1986): 281-306.
- Berg, Joyce E., Thomas A. Rietz and John W. Dickhaut "On the Performance of the Lottery Procedure for Controlling Risk Preferences. In *Handbook of Experimental Economics Results*, edited by Charles R. Plott and Vernon L. Smith. New York: Elsevier Press, 2008.
- Bernoulli, Daniel. "Specimen Theories Novae de Mensura Sorties." *Commentarii Academiae Scientiarum Imperialis Petropolitanae*, Tomus V, Papers of the Imperial Academy of Sciences in Petersburg, 1 (1738): 175-192; translated as "Exposition of a New Theory on the Measurement of Risk," *Econometrica*, 22 (1954): 23-36.
- Binswanger, Hans P. "Attitudes Toward Risk: Experimental Measurement in Rural India." *American Journal of Agricultural Economics*, 62 no 3 (August 1980): 395-407.
- Bone, John, John Hey and John Suckling. "Are Groups More (or Less) Consistent Than Individuals." *Journal of Risk and Uncertainty*, 18 no. 1 (April 1999), 63-81.
- Bone, John, John Hey and John Suckling. "A Simple Risk-Sharing Experiment." *Journal of Risk and Uncertainty*, 28 no. 1 (January 2004): 23-28.
- Borch, Karl. "Equilibrium in a Reinsurance Market." *Econometrica*, 30 no. 3 (July 1962): 424-444.
- Borch, Karl. "Introduction." In *Risk and Uncertainty: Proceedings of a Conference held by the International Economic Association*, edited Karl Borch and Jan Mossin. New York: St. Martin's Press, 1968.
- Boyle, Phelim P., Lorenzo Garlappi, Raman Uppal and Tan Wang. "Keynes Meets Markowitz: The Tradeoff Between Familiarity and Diversification." *Management Science*, 58 no 2 (February 2012): 253-272.

- Boyle, Phelim P. and Jennifer Mao. "An Exact Solution for the Optimal Stop Loss Limit," *Journal of Risk and Uncertainty*, 50 no. 4 (December 1983): 719-726.
- Braun, Michael and Alexander Muermann. "The Impact of Regret on the Demand for Insurance." *Journal of Risk and Insurance*, 71 no. 4 (2004): 737-767.
- Braunstein, Yale M. and Andrew Schotter. "Labor Market Search: An Experimental Study." *Economic Inquiry*, 20 no 1 (January 1982): 133-144.
- Brewer, K. R. W. "Decisions under Uncertainty: Comment." *Quarterly Journal of Economics*, 77 no. 1 (February 1963): 159-161.
- Brewer, K. R. W. and William Fellner "The Slanting of Subjective Probabilities - Agreement on Some Essentials." *Quarterly Journal of Economics* 79 no. 4 (November 1965): 657-663.
- Briys, Eric, Georges Dionne and Louis Eeckhoudt. "More on Insurance as a Giffen Good." *Journal of Risk and Uncertainty*, 2 no. 4 (December 1989): 415-420.
- Briys, Eric and Henri Loubergé. "On the Theory of Rational Insurance Purchasing: A Note." *Journal of Finance*, 40 no. 2 (June 1985): 577-581.
- Briys, Eric and Harris Schlesinger. "Risk Aversion and the Propensities for Self-Insurance and Self-Protection." *Southern Economic Journal*, 57 no. 2 (October 1990): 458-467.
- Briys, Eric, Harris Schlesinger and J. Matthias G. Schulenburg. "Reliability of Risk Management: Market Insurance, Self-Insurance and Self-Protection." *The Geneva Papers on Risk and Insurance Theory*, 16 no. 1 (June 1991): 45-58.
- Brown, Jeffrey R. "Rational and Behavioral Perspective on the Role of Annuities in Retirement Planning." *Working Paper 13537*, NBER Working Paper Series, 2007.
- Burke, Michael, John R. Carter, Robert D. Gominiak and Daniel F. Ohl. "An Experimental Note on the Allais Paradox and Monetary Incentives." *Empirical Economics*, 21 no. 4 (December 1996): 612-632.
- Camerer, Colin F. "Recent Tests of Generalizations of Expected Utility Theory." In *Utility: Theories Measurement, and Applications*, edited by W. Edwards. Norwell, MA: Kluwer, 1992.
- Camerer, Colin F., and Robin Hogarth. "The Effects of Financial Incentives in Experiments: A Review and Capital-Labor Framework." *Journal of Risk and Uncertainty*, 19 no. 1-3 (December 1999): 7-42.
- Camerer, Colin F. and Martin Weber. "Recent Developments in Modeling Preferences: Uncertainty and Ambiguity." *Journal of Risk and Uncertainty*, 5 no. 4 (October 1992): 325-370.

- Cameron, Trudy. "A New Paradigm for Valuing Non-market Goods Using Referendum Data: Maximum Likelihood Estimation by Censored Logistic Regression." *Journal of Environmental Economics and Management*, 15 no. 3 (September 1988): 355-379.
- Campbell, John Y. "Household Finance." *The Journal of Finance*, 61 no 4 (August 2006):1553-1604.
- Carroll, Christopher D., and Miles S. Kimball. "Liquidity Constraints and Precautionary Saving." *Working Paper*, Department of Economics, Johns Hopkins University, 2005.
- Cherbonnier, Frédéric and Christian Gollier. "Decreasing Aversion under Ambiguity." *Working Paper*, Toulouse School of Economics, 2011.
- Chen, Hua and Reza S. Mahani. "Optimal Demand for Insurance with Consumption Commitment." *Asia-Pacific Journal of Risk and Insurance*, 2011, forthcoming.
- Chesney, Marc and Henri Loubergé. "Risk Aversion and the Composition of Wealth in the Demand for Full Insurance Coverage." *Schweizerische Zeitschrift fr Volkswirtschaft und Statistik*, 122 no. 3 (September 1986): 359-370.
- Chetty, Raj and Adam Szeidl. "Consumption Commitments and Risk Preferences." *Quarterly Journal of Economics*, 122 no. 2 ( May 2007): 831-877.
- Chiappori, Pierre-André, Krislert Samphantharak, Sam Schulhofer-Wohl and Robert M. Townsend. "Heterogeneity and Risk Sharing in Village Economies." *Working Paper No. 683*, Research Department, Federal Reserve Bank of Minneapolis, 2011.
- Clarke, Daniel J. "A Theory of Rational Demand for Index Insurance." *Discussion Paper No. 572*, Department of Economics, University of Oxford, October 2011.
- Clarke, Kevin A. "Nonparametric Model Discrimination in International Relations." *Journal of Conflict Resolution*, 47 no (February 2003): 72-93.
- Clarke, Kevin A. "A Distribution-Free Test for Nonnested Model Seleaction." *Political Analysis*, 15 no 3 (Summer 2007): 347-363.
- Cochrane, John H. "Portfolio Theory." *Working Paper*, University of Chicago, Graduate School of Business, 2007.
- Collard, Fabrice, Sujoy Mukerji, Kevin K. Sheppard and Jean-Marc Tallon. "Ambiguity and Historical Equity Premium." *Working Paper No. 550*, Department of Economics Discussion Paper Series, Oxford University, 2012.
- Coller, Maribeth; Glenn W. Harrison and E. Elisabet Rutström. "Latent Process Heterogeneity in Discounting Behavior." *Oxford Economic Papers*, 64 no. 2 (April 2012): 375-391.

- Coller, Maribeth and Melonie B. Williams. "Eliciting Individual Discount Rates." *Experimental Economics*, 2 no. 2 (December 1999): 107-127.
- Conlisk, John. "Three Variants on the Allais Example." *The American Economic Review*, 79 no. 3 (June 1989): 392-407.
- Cooper, Russell, Douglas V. DeJong, Robert Forsythe and Thomas W. Ross. "Communication in the Battle of the Sexes Game: Some Experimental Results." *Rand Journal of Economics*, 20 no. 4 (Winter 1989): 568-587.
- Cooper, Russell, Douglas V. DeJong, Robert Forsythe and Thomas W. Ross. "Selection Criteria in Coordination Games: Some Experimental Results." *American Economic Review* 80 no. 1 (March 1990): 218-233.
- Cooper, Russell, Douglas V. DeJong, Robert Forsythe and Thomas W. Ross. "Communication in Coordination Games." *Quarterly Journal of Economics*, 107 no. 2 (May 1992): 739-771.
- Cooper, Russell, Douglas V. DeJong, Robert Forsythe and Thomas W. Ross. "Forward Induction in the Battle-of-Sexes Games." *American Economic Review*, 83 no. 5 (December 1993): 1303-1316.
- Courbage, Christophe. "Self-Insurance, Self-Protection and Market Insurance within the Dual Theory of Choice." *The Geneva Papers on Risk and Insurance Theory*, 26 no. 1 (June 2001): 43-56.
- Cox, James C. and Ronald L. Oaxaca. "Inducing Risk-Neutral Preferences: Further Analysis of the Data." *Journal of Risk and Uncertainty*, 11 no. 1 (July 1995): 65-79.
- Cox, James C., Vjollca Sadiraj and Ulrich Schmidt. "Paradoxes and Mechanism for Choice under Risk." *CEAR Working Paper 2011-12*, Center for the Economic Analysis of Risk, Robinson College of Business, Georgia State University, 2011.
- Cox, James C., Vernon L. Smith and James M. Walker. "Experimental Development of Sealed-Bid Auction Theory: Calibrating Controls for Risk Aversion." *American Economic Review*, 75 no 2 (May 1985): 160-165.
- Cubitt, Robin P., Chris Starmer and Robert Sugden. "On the Validity of the Random Lottery Incentive System." *Experimental Economics*, 1 no 2 (September 1998a): 115-131.
- Cubitt, Robin P., Chris Starmer and Robert Sugden. "Dynamic Choice and Common Ration Effect: An Experimental Investigation." *The Economic Journal*, 108 no 450 (September 1998b): 1362-1380.
- Cummins, J. David and Olivier Mahul. "The Demand for Insurance with an Upper Limit on Coverage." *Journal of Risk and Insurance*, 71 no. 2 (June 2004): 253-264.

- d'Albis, Hippolyte and Emmanuel Thibault. "Ambiguous Life Expectancy and Optimal Annuitization." *Unpublished Manuscript*, Toulouse School of Economics, 2009.
- Davidoff, Thomas, Jeffrey R. Brown and Peter A. Diamond. "Annuities and Individual Welfare." *American Economic Review*, 95 no. 5 (December 2005): 1573-1590.
- Davis, B. Donald and M. Elisabeth Paté-Cornell. "A Challenge to the Compound Lottery Axiom: A Two-Stage Normative Structure and Comparison to Other Theories." *Theory and Decision*, 37 no. 3 (November 1994): 267-309.
- Deaton, Angus S. "Saving in Developing Countries: Theory and Review." *World Bank Economic Review*, (1990): 61-96.
- Deaton, Angus S. "Saving and Liquidity Constraints." *Econometrica*, 59 no. 5 (September 1991): 1221-1248.
- Deaton, Angus S. "Instruments, Randomization and Learning about Development." *Journal of Economic Literature*, 48 no. 2 (June 2010): 424-455.
- Deck, Cary A. "A Test of Game-Theoretic and Behavioral Models in Exchange and Insurance Environments." *American Economic Review*, 91 no.5 (December 2001): 1546-1555.
- de Finetti, Bruno. "Foresight: Its Logical Laws, Its Subjective Sources." Reprint in *Studies in Subjective Probability*, edited by Kyburg, Henry E. and Howard E. Smokler. New York: Wiley and Sons, 1964.
- de Finetti, Bruno. "Logical Foundations and Measurement of Subjective Probability." *Acta Psychologica*, 34 (December 1970): 129-145
- de Finetti, Bruno. "Sulla Preferibilita." *Giornal degli Econmomisti e Analli di Economia*, 11 (1952): 685-709.
- Dekel, Eddie. "Asset Demand Without the Independence Axiom." *Econometrica* 57 no. 1 (January 1989): 163-169.
- Di Mauro, Carmela and Anna Maffioletti. "An Experimental Investigation of the Impact of Ambiguity on the Valuation of Self-Insurance and Self-Protection." *Journal of Risk and Uncertainty*, 13 no. 1 (July 1996): 53-71.
- Dionne, Georges and Louis Eeckhoudt. "Self-Insurance, Self-Protection and Increased Risk Aversion." *Economics Letters* 17 no. 1-2 (1985): 39-42.
- Dionne, Georges and Neil A. Doherty. "Adverse Selection in Insurance Markets: A Selective Survey." In *Contributions to Insurance Economics*, edited by Georges Dionne. Boston: Kluwer Academics Publishers, 1992.

- Dionne, Georges, Neil A. Doherty and Nathalie Fombaron. "Adverse Selection in Insurance Markets." In *Handbook of Insurance*, edited by Georges Dionne. Boston: Kluwer Academic Publishers, 2000.
- Doherty, Neil A. "Portfolio Efficient Insurance Buying Strategies." *Journal of Risk and Insurance*, 51 no. 2 (June 1984): 205-224.
- Doherty, Neil A. and Andreas Richter. "Moral Hazard, Basis Risk and Gap Insurance." *Journal of Risk and Insurance*, 69 no. 1 (March 2002): 9-24.
- Doherty, Neil A. and Harris Schlesinger. "The Optimal Deductible for an Insurance Policy When Initial Wealth is Random." *Journal of Business*, 56 no. 4 (October 1983): 555-565.
- Doherty, Neil A. and Harris Schlesinger. "Rational Insurance Purchasing: Consideration of Contract Nonperformance." *Quarterly Journal of Economics*, 105 no. 1 (February 1990): 243-253.
- Dorflleitner, Gregor and Michael Krapp. "On Multiattributive Risk Aversion: Some Clarifying Results." *Review of Managerial Science*, 1 no. 1 (2007): 47-63.
- Dufo, Esther. "Field Experiments in Development Economics." In *Advances in Economics and Econometrics: Theory and Applications*, edited by Richard Blundell, Whitney K. Newey and Torsten Persson. New York: Cambridge University Press, Vol. 2, 2006.
- Dufo, Esther and Michael Kremer "Use of Randomization in the Evaluation of Development Effectiveness." In *Evaluating Development Effectiveness*, edited by George K. Pitman, Osvaldo N. Feinstein, and Gregory K. Ingram. New Brunswick, NJ: Transaction Publishers, 2005.
- Dufo, Esther, Rachel Glennesster and Michael Kremer. "Using Randomization in Developing Economics Research: A Toolkit." In *Handbook of Development Economics*, Vol. 4, edited by T. Paul Schultz and John A. Strauss. NY: North-Holland, 2007.
- Duncan, George T. "A Matrix Measure of Multivariate Local Risk Aversion." *Econometrica*, 45 no. 4 (May 1977): 895-903.
- Ebert, Sebastian and Daniel Wisen. "Testing for Prudence and Skewness Seeking." *Management Science*, 57 no. 7 (July 2011): 1334-1349.
- Eeckhoudt, Louis and Miles Kimball. "Background Risk, Prudence and Demand for Insurance." In *Contributions to Insurance Economics*, edited by Georges Dionne. Boston: Kluwer Academic Publishers, 1992.
- Eeckhoudt, Louis and Christian Gollier. "The Effects of Changes in Risk on Risk Taking: A Survey." *Handbook of Insurance*, edited by Georges Dionne. Boston: Kluwer Academic Publishers, 2000.

- Ehrlich, Isaac and Gary S. Becker. "Market Insurance, Self-Insurance and Self-Protection." *Journal of Political Economy*, 80 no. 4 (July-August 1972): 623-648.
- Ellsberg, Daniel. "Risk, Ambiguity and the Savage Axioms." *Quarterly Journal of Economics*, 75 no.4 (November 1961): 643-669.
- Ellsberg, Daniel. *Risk, Ambiguity and Decision*. New York: Garland Publishing Inc., 2001.
- Enz, Rudolf. "The S-Curve Relation Between Per-Capita Income and Insurance Penetration." *Geneva Papers on Risk and Insurance*, 25 no. 3 (July 2000): 396-406.
- Epstein, Larry G. and Martin Schneider. "Recursive Multiple-Prior." *Journal of Economic Theory*, 113 no. 1 (November 2003): 1-31.
- Epstein, Larry G. and Stephen M. Tanny. "Increasing Generalized Correlation: a Definition and Some Economic Consequences." *Canadian Journal of Economics*, 13 no. 1 (February 1980): 16-34.
- Ergin, Haluk and Faruk Gul. "A Theory of Subjective Compound Lotteries." *Journal of Economic Theory*, 144 no. 3 (May 2009): 899-929.
- Fama, Eugene F. "Multifactor Portfolio Efficiency and Multifactor Asset Pricing." *Journal of Financial and Quantitative Analysis*, 31 no 4 ( December 1996): 441-465
- Fan, Chinn-Ping. "Allais Paradox in the Small." *Journal of Economic Behavior and Organization*, 49 no. 3 (November 2002): 411-421.
- Fehr, Ernst, and Lorenz Goette. "Do Workers Work More if Wages Are High? Evidence from a Randomized Field Experiment." *American Economic Review*, 97 no. 1 (March 2007): 298-317.
- Fellner, William. "Distortion of Subjective Probabilities as Reaction to Uncertainty." *Quarterly Journal of Economics*, 48 no 5 (November 1961): 670-689.
- Fellner, William. "Slanted Subjective Probabilities and Randomization: Reply to Howard Raiffa and K. R. W. Brewer." *Quarterly Journal of Economics*, 77 no 4 (November 1963): 676-690.
- Fiore, Stephen M., Glenn W. Harrison, Charles E. Hughes and E. Elisabet Rutström. "Virtual Experiments and Environmental Policy." *Journal of Environmental Economics and Management*, 57 no. 1 (January 2009): 65-86.
- Flemming, J. S. "The Utility of Wealth and the Utility of Windfalls." *Review of Economic Studies*, 36 no. 1 (January 1969): 55-66.
- Frederick, Shane, George Loewenstein and Ted O'Donoghue. "Time Discounting and Time Preference: A Critical Review." *Journal of Economic Literature*, 40 no. 2 (June 2002):351-401.

- Frenkel-Brunswik, Else. "Intolerance of Ambiguity as an Emotional and Perceptual Personality Variable." *Journal of Personality*, 18 no. 1 (September 1949): 108-143.
- Friedman, Milton and Leonard J. Savage. "The Utility Analysis of Choices Involving Risk." *Journal of Political Economy*, 56 no. 4 (August 1948): 279-304.
- Foncel, Jérôme and Nicolas Treich. "Insurance as a Normal Good: Empirical Evidence for a Puzzle." *Working Paper*, Toulouse School of Economics, 2009.
- Fox, Craig R. and Amos Tversky. "Ambiguity Aversion and Comparative Ignorance." *Quarterly Journal of Economics*, 110 no 3 (August 1995): 585-603.
- Fuchs, Victor R. "From Bismarck to Woodcock: The 'Irrational' Pursuit of National Health Insurance." *Journal of Law and Economics*, 19 no. 2 (August 1976): 347-359.
- Ganderton, Philip T., David S. Brookshire, Michael McKee, Steve Steward and Hale Thurston. "Buying Insurance for Disaster-Type Risks: Experimental Evidence." *Journal of Risk and Uncertainty*, 20 no. 3 (May 2000): 271-289.
- Gigliotti, Gary and Barry Sopher. "A Test of Generalized Expected Utility Theory." *Theory and Decision*, 35 no 1 (July 1993): 75-106.
- Gilboa, Itzhak and David Schmeidler. "Maximin Expected Utility with a Non-unique Prior." *Journal of Mathematical Economics*, 18 no. 2 (April 1989): 745-780.
- Ghirardato, Paolo, Fabio Maccheroni and Massimo Marinacci. "Differentiating Ambiguity and Ambiguity Attitude." *Journal of Economic Theory*, 118 no. 2 (October 2004): 133-173.
- Goetzmann, William N. and Alok Kumar "Equity Portfolio Diversification." *Review of Finance*, 12 no. 3 (2008): 433-463.
- Gollier, Christian. "Economic Theory of Risk Exchanges: A Review." In *Contributions to Insurance Economics*, edited by Georges Dionne. Boston: Kluwer Academic Publishers, 1992.
- Gollier, Christian. "The Comparative Statics of Changes in Risk Revisited." *Journal of Economic Theory*, 66 no. 2 (August 1995): 522-536.
- Gollier, Christian. "A Note on Portfolio Dominance." *Review of Economic Studies*, 64 no. 1 (January 1997): 140-150.
- Gollier, Christian. "Optimal Insurance Design: What Can We Do With and Without Expected Utility." In *Handbook of Insurance*, edited by Georges Dionne. Boston: Kluwer Academic Publishers, 2000.
- Gollier, Christian. *The Economics of Risk and Time*. Cambridge, MA: MIT Press, 2001.

- Gollier, Christian. "Portfolio Choices and Asset Prices: The Comparative Statics of Ambiguity Aversion." *Working Paper*, Toulouse School of Economics, 2009.
- Gollier, Christian and Louis Eeckhoudt. "The Effects of Changes in Risk on Risk Taking: A Survey." *Handbook of Insurance*, edited by Georges Dionne. Boston: Kluwer Academic Publishers, 2000.
- Gollier, Christian and Alexander Muermann. "Optimal Choice and Beliefs with Ex Ante Savoring and Ex Post Disappointment." *Management Science*, 56 no. 8 (August 2012): 1272-1284.
- Gollier, Christian and John W. Pratt. "Risk Vulnerability and the Tempering Effect of Background Risk." *Econometrica*, 64 no. 5 (September 1996): 1109-1123
- Gollier, Christian and Harris Schlesinger. "Second-Best Insurance Contract Design in an Incomplete Market." *Scandinavian Journal of Economics*, 97 no. 1 (March 1995): 123-135.
- Gould, John P. "The Expected Utility Hypothesis and the Selection of Optimal Deductibles for a Given Insurance Policy." *Journal of Business*, 42 no. 2 (April 1969): 143-151.
- Grether, David M. "Testing Bayes Rule and the Representativeness Heuristic: Some Experimental Evidence." *Journal of Economic Behavior and Organization*, 17 no. 1 (January 1992): 31-57.
- Hamilton, Vernon. "Perceptual and Personality Dynamics in Reactions to Ambiguity." *The British Journal of Psychology*, 48 no. 3 (August 1957): 200-215.
- Hakansson, Nils H. "Friedman-Savage Utility Functions Consistent with Risk Aversion." *Quarterly Journal of Economics*, 84 no. 3 (August 1970): 472-487.
- Halevy, Yoram. "Ellsberg Revisited: An Experimental Study," *Econometrica*, 75 no. 2 (March 2007): 503-536.
- Harless, David W. "Predictions about Indifference Curves inside the Unit Triangle: a Test of Variants of Expected Utility." *Journal of Economic Behavior and Organization*, 18 no. 3 (August 1992): 391-414.
- Harrison, Glenn W. "Theory and Misbehavior of First-Price Auctions." *American Economic Review*, 79 no. 4 (September 1989): 749-762.
- Harrison, Glenn W. "Theory and Misbehavior of First-Price Auctions: Reply." *American Economic Review*, 82 no. 5 (December 1992): 1426-1443.
- Harrison, Glenn W. "Expected Utility Theory and the Experimentalists." *Empirical Economics*, 19 no. 2 (April 1994): 223-253.

- Harrison, Glenn W. "Making Choice Studies Incentive Compatible." In *Valuing Environmental Amenities Using Stated Choice Studies: A Common Sense Guide to Theory and Practice*, edited by Barbara J. Kanninen, Boston: Kluwer, 2006, 65-108.
- Harrison, Glenn W. "Experimental Methods and the Welfare Evaluation of Policy Lotteries." *European Review of Agricultural Economics*, 38 no. 3 (2011a): 335-360.
- Harrison, Glenn W. "Randomisation and Its Discontents." *Journal of African Economies*, 20 no. 4 (2011b): 626-652.
- Harrison, Glenn W. "The Rich Domain of Uncertainty: Comment." *CEAR Working Paper 2011-13*, Center for the Economic Analysis of Risk, Robinson College of Business, Georgia State University, 2011.
- Harrison, Glenn W., Stephen J. Humphrey, and Arjan Verschoor. "Choice under Uncertainty: Evidence from Ethiopia and Uganda." *Economic Journal*, 120 no. 543 (March 2010): 80-104.
- Harrison, Glenn W., Eric Johnson, Melayne M. McInnes and E. Elisabet Rutström. "Measurement With Experimental Controls." In *Measurement in Economics: A Handbook*, edited by Marcel Boumans. San Diego, CA: Elsevier, 2007.
- Harrison, Glenn W., Morten I. Lau, and Melonie B. Williams. "Estimating Individual Discount Rates in Denmark: A Field Experiment." *American Economic Review*, 92 no. 5 (December 2002): 1606-1617.
- Harrison, Glenn W., Morten I. Lau and E. Elisabeth Rutström. "Theory, Experimental Design and Econometrics are Complementary (And So are Lab and Field Experiments)." In *The Methods of Modern Experimental Economics*, edited by Guillaume Fréchette and Andrew Schotter New York: Oxford University Press, 2012.
- Harrison, Glenn W. and John List. "Field Experiments." *Journal of Economic Literature*, 42 (December 2004): 1009-155.
- Harrison, Glenn W. and John List and Charles A. Towe. "Naturally Occurring Preferences and Exogenous Laboratory Experiments: A case Study of Risk Aversion." *Econometrica*, 75 no. 2 (March 2007): 433-458.
- Harrison, Glenn W. and Kevin McCabe. "Testing Noncooperative Bargaining Theory in Experiments. In *Research in Experimental Economics*, edited by R.M. Isaac. Greenwich: JAI Press, Volume 5, 1992.
- Harrison, Glenn W. and Kevin McCabe. "Expectations and Fairness in a Simple Bargaining Experiment." *International Journal of Game Theory*, 25 no 3 (September 1996): 303-327.

- Harrison, Glenn W., Jimmy Martínez-Correa; and J. Todd Swarthout. "Inducing Risk Neutral Preferences with Binary Lotteries: A Reconsideration." *CEAR Working Paper 2012-02*, Center for the Economic Analysis of Risk, Robinson College of Business, Georgia State University, 2012.
- Harrison, Glenn W., Jimmy Martínez-Correa; and J. Todd Swarthout. "Reduction of Compound Lotteries with Objective Probabilities; Theory and Evidence." *CEAR Working Paper 2012-04*, Center for the Economic Analysis of Risk, Robinson College of Business, Georgia State University, 2012.
- Harrison, Glenn W., Jimmy Martínez-Correa; and J. Todd Swarthout. "Reduction of Compound Lotteries with Objective Probabilities; Theory and Evidence." *CEAR Working Paper 2012-04*, Center for the Economic Analysis of Risk, Robinson College of Business, Georgia State University, 2012.
- Harrison, Glenn W., Jimmy Martínez-Correa; and J. Todd Swarthout. "Eliciting Subjective Probabilities with Binary Lotteries." *Working Paper*, Department of Risk Management and Insurance, Robinson College of Business, Georgia State University, 2012c..
- Harrison, Glenn W. and E. Elisabet Rutström. "Trade Wars, Trade Negotiations, and Applied Game Theory." *Economic Journal*, 101 no 406 (May 1991): 420-435.
- Harrison, Glenn W. and E. Elisabet Rutström. "Experimental Evidence on the Existence of Hypothetical Bias in Value Elicitation Experiments." In *Handbook of Experimental Economics Results*, edited by Charles R. Plott and Vernon L. Smith, New York: Elsevier Press, 2008.
- Harrison, Glenn W. and E. Elisabet Rutström. "Expected Utility Theory and Prospect Theory: One Wedding and a Decent Funeral." *Experimental Economics*, 12 no. 2 (June 2009): 133-158.
- Harrison, Glenn, and J. Todd Swarthout. "The Independence Axiom and the Bipolar Behaviorist." *CEAR Working Paper 2012-01*, Center for the Economic Analysis of Risk, Robinson College of Business, Georgia State University, 2012.
- Heath, Chip and Amos Tversky. "Preference and Belief: Ambiguity and Competence in Choice under Uncertainty." *Journal of Risk and Uncertainty*, 4 no 1 (January 1991): 5-28.
- Herrero, Carmen, Josefa Tomás and Antonio Villar. "Decision Theories and Probabilistic Insurance: An Experimental Test." *Spanish Economic Review*, 8 no. 1 (March 2006): 35-52.
- Heckman, James J. "Building Bridges between Structural and Program Evaluation Approaches to Evaluating Policy." *Journal of Economic Literature*, 48 no. 2 (June 2010): 356-398.

- Hershey, John C. and Paul J. H. Schoemaker “Risk Taking and Problem Context in the Loss Domain: An Expected Utility Analysis,.” *Journal of Risk and Insurance*, 47 no. 1 (March 1980): 111-132.
- Hey, John, Gianna Lotito and Anna Maffioletti. “The Descriptive and Predictive Adequacy of Decision Making under Uncertainty,” *Journal of Risk and Uncertainty*, 41 no. 2 (October 2010): 81-111.
- Hey, John D. and Chris Orme. “Investigating Generalizations of Expected Utility Theory Using Experimental Data.” *Econometrica*, 62 no 6 (November 1994): 1291-1326.
- Hiebert, L. Dean. “Optimal Loss Reduction and Increases in Risk Aversion.” *Journal of Risk and Insurance*, 56 no. 2 (June 1989): 300-305.
- Hirshleifer, Jack. *Investment, Interest and Capital* Englewood-Cliffs, NJ: Prentice-Hall, 1970.
- Holt, Charles A. “Preference Reversals and the Independence Axiom.” *American Economic Review*, 76 no 3 (June 1986): 508-515
- Holt, Charles A. and Angela M. Smith. “An Update on Bayesian Updating.” *Journal of Economic Behavior and Organization*, 69 no. 2 (February 2009): 125-134.
- Hossain, Tanjim and Ryo Okui. “The Binarized Scoring Rule.” *Working Paper*, University of Toronto, August 2011.
- Hoy, Michael and Arthur J. Robson. “Insurance as a Giffen Good.” *Economics Letters*, 8 (1981): 47-51.
- Hu, Wei-Yun, and Jason S. Scott. “Behavioral Obstacles to the Annuity Market.” *Pension Research Council Working Paper 2007-10*, Wharton School, University of Pennsylvania, 2007.
- Hurwicz, Leonid. “Optimality Criteria for Decision Making Under Ignorance.” *Cowles Commission Paper 370*, 1951a.
- Hurwicz, Leonid. “Some Specification Problems and Applications to Econometric Models,” *Econometrica*, 19 no. 3 (July 1951b), 343-344.
- Huang, Rachel J., Alexander Muermann and Larry Y Tzeng. “Hidden Regret and Advantageous Selection in Insurance Markets.” *WPS 07-09*, PARC Working Paper Series, University of Pennsylvania, 2007.
- Imbens, Guido W. “Better LATE than Nothing: Some Comment on Deaton (2009) and Heckman and Urzua (2009).” *Journal of Economic Literature*, 48 no. 2 (June 2010): 399-423.

- Ju, Nengjiu and Jianjun Miao. "Ambiguity, Learning and Asset returns," *Econometrica*, forthcoming, 2012.
- Kadane, Joseph B. "Healthy Skepticism as an Expected-Utility Explanation of the Phenomena of Allais and Ellsberg." *Decision Making Under Risk and Uncertainty: New Models and Empirical Findings*, edited by John Geweke. Boston: Kluwer, 1992.
- Kadane, Joseph B. and Robert L. Winkler. "Separating Probability Elicitation from Utilities." *Journal of the American Statistical Association*, 83 no 402 (June 1988): 357-363.
- Kahneman, Daniel and Amos Tversky. "Prospect Theory: An Analysis of Decision under Risk." *Econometrica*, 47 no. 2 (May 1979): 263-292.
- Karni, Edi. "On Multivariate Risk Aversion." *Econometrica*, 47 no. 6 (November 1979): 1391-1401.
- Karni, Edi. *Decision Making under Uncertainty*. Cambridge, MA: Harvard University Press, 1985.
- Karni, Edi. "A Mechanism for Eliciting Probabilities." *Econometrica*, 77 no. 2 (March 2009): 603-606.
- Keeney, Ralph L. "Risk Independence and Multiattributed Utility Functions." *Econometrica*, 41 no. 1 (January 1973): 27-34.
- Keane, Michael P. "Structural vs. Atheoretic Approaches to Econometrics." *Journal of Econometrics*, 156 no. 1 (May 2010): 3-20.
- Kihlstrom, Richard E. and Leonard J. Mirman. "Risk Aversion with Many Commodities." *Journal of Economic Theory*, 8 no. 3 (July 1974): 361-388.
- Kim, Young Chin. "Choice in the Lottery-Insurance Situation Augmented-Income Approach." *Quarterly Journal of Economics*, 87 no. 1 (February 1973): 148-156.
- Kimball, Miles S. "Precautionary Saving in the Small and in the Large." *Econometrica*, 58 no. 1 (January 1990): 53-73.
- Klibanoff, Peter; Massimo Marinacci and Sujoy Mukerji. "A Smooth Model of Decision Making under Ambiguity." *Econometrica*, 73 no. 6 (November 2005): 1849-1892.
- Klibanoff, Peter; Massimo Marinacci and Sujoy Mukerji. "Recursive Smooth Preferences," *Journal of Economic Theory*, 144 (2009): 930-976.
- Knight, Frank H. *Risk, Uncertainty and Profit*. Boston: Houghton Mifflin Company, 1921.
- Konrad, Kai A. and Stergios Skaperdas. "Self-Insurance and Self-Protection: A Nonexpected Utility Analysis." *The Geneva Papers on Risk and Insurance Theory*, 18 no. 2 ( December 1993): 131-146.

- Kszegi, Botond, and Mathew Rabin. "Choices, Situations and Happiness." *Journal of Public Economics*, 92 (2008): 1821-1832.
- Kunreuther, Howard and Mark Pauly. "Neglecting Disaster: Why Dont People Insure Against Large Losses?" *Journal of Risk and Uncertainty*, 28 no. 1 (January 2004): 5-21.
- Kunreuther, Howard and Mark Pauly. "Insurance Decision-Making and Market Behavior." *Foundations and Trends in Microeconomics*, 1 no.2 (2005): 63-127.
- Kunreuther, Howard, Ralph Ginsberg, Louis Miller, Phillip Sagi and Paul Slovic. *Disaster Insurance Protection: Public Policy Lessons*. New York: Wiley Interscience, 1978.
- Kyburg, Henry E. and Howard E. Smokler. *Studies in Subjective Probability*. New York: Wiley and Sons, 1964.
- Laury, Susan K., Melayne M. McInnes and J. Todd Swarthout. "Insurance Decisions for Low-Probability Losses." *Journal of Risk and Uncertainty*, 39 no. 1 (August 2009): 17-44.
- Lee, Hyong J. and George E. Pinches. "On Optimal Insurance Purchasing." *Journal of Risk and Insurance*, 55 no. 1 (March 1988): 145-149.
- Leland, Hayne E. "Saving and Uncertainty: The Precautionary Demand for Saving." *Quarterly Journals of Economics* 82 no. 3 (August 1968): 465-473.
- Li, Shou-Hua, Richard M. Simon and John J. Gart. "Small Sample Properties of the Mantel-Haenszel Test." *Biometrika*, 66 no. 1 (April 1979): 181-183.
- Loewenstein, George, and Drazen Prelec. "Anomalies in Intertemporal Choice: Evidence and Interpretation." *Quarterly Journal of Economics*, 107 no. 2 (1992): 573-592.
- Loomes, Graham and Robert Sugden. "Regret Theory: An Alternative Theory of Rational Choice under Uncertainty." *The Economic Journal*, 92 (December 1982) 805-824.
- Loomes, Graham and Robert Sugden. "Some Implications of a More General Form of Regret Theory." *Journal of Economic Theory*, 41 no. 2 (April 1987) 270-287.
- Loubergé, Henri. "Developments in Risk and Insurance Economics: the Past 25 Years." *Handbook of Insurance*, edited by Georges Dionne. Boston: Kluwer Academic Publishers, 2000.
- Luce, R. Duncan and A.A.J.Marley. "On Elements of Chance." *Theory and Decision*, 49 (2000): 97-126.
- Maccheroni, Fabio, Massimo Marinacci and Doriana Ruffino. "Alpha as Ambiguity: Robust Mean-Variance Portfolio Analysis." *Working Paper*, Università Bocconi, 2011a.
- Maccheroni, Fabio, Massimo Marinacci and Doriana Ruffino. "Does Uncertainty Vanish in the Small? The Smooth Ambiguity Case." *Working Paper*, Università Bocconi, 2011b.

- Machina, Mark. "Expected Utility Analysis without the Independence Axiom." *Econometrica*, 50 no. 2 (March 1982): 277-323.
- Machina, Mark. "Non-Expected Utility and the Robustness of the Classical Insurance Paradigm." *The Geneva Paper on Risk and Insurance Theory*, 20 no. 1 (June 1995): 9-50.
- Machina, Mark. "The Robustness of the Classical Insurance Paradigm." In *Handbook of Insurance*, edited by Georges Dionne. Boston: Kluwer Academic Publishers, 2000.
- Machina, Mark. "Ambiguity Aversion with Three or More Outcomes." Paper presented at the CEAR Workshop Rank-Dependent Models of Risky Behavior: A 30th Birthday Party, Georgia State university, September 22-23, 2011.
- Machina, Mark, and David Schmeidler. "A More Robust Definition of Subjective Probability." *Econometrica*, 60 no. 4 (July 1992): 745-780.
- Machina, Mark, and David Schmeidler. "Bayes without Bernoulli: Simple Conditions for Probabilistically Sophisticated Choice." *Journal of Economic Theory*, 67 no. 1 (October 1995): 106-128.
- McKelvey, Richard D. and Talbot Page. "Public and Private Information: An Experimental Study of Information Pooling." *Econometrica*, 58 no. 6 (November 1990): 1321-1339.
- Markowitz, Harry. "The Utility of Wealth." *Journal of Political Economy*, 60 no. 2 (April 1952): 151-158.
- Martínez-Correa, Jimmy. "Insurance Decisions under Ambiguity." Ph.D. diss., Department of Risk Management and Insurance, Georgia State University, 2012.
- Massa, Massimo and Andrei Simonov. "Hedging, Familiarity and Portfolio Choice." *Review of Financial Studies*, 19 no 2 (Summer 2006): 633-685.
- Mathieson, James E. and Robert L. Winkler. "Scoring Rules for Continuous Probability Distributions." *Management Science*, 22 no. 10 (June 1976): 1087-1096.
- Mayers, David and Clifford W. Smith Jr. "The Interdependence of Individual Portfolio Decisions and the Demand for Insurance." *Journal of Political Economy*, 91 no. 2 (April 1983): 304-311.
- McClelland, Gary H., William D. Schulze and Don L. Coursey. "Insurance for Low-Probability Hazards: A Bimodal Response to Unlikely Events" *Journal of Risk and Uncertainty*, 7 no. 1 (August 1993): 95-116.
- Merton, Robert C. "Operation and Regulation in Financial Intermediation: A Functional Perspective." *Operation and Regulation of Financial Markets*, edited by Peter Englund. The Economic Council, Stockholm, 1993.

- Mitchell, Olivia and Jose Ruiz. "Pension Payouts in Chile: Past, Present, and Future Prospects." *WPS 09-02*, PARC Working Paper Series, Wharton School, 2009.
- Mitchell, Olivia S. and Stephen P. Utkus. "Lessons from Behavioral Finance for Retirement Plan Design." In *Pension Design and Structure: New Lessons from Behavioral Finance*, edited by Olivia S. Mitchell and Stephen P. Utkus. New York: Oxford University Press, 2004.
- Modigliani, Franco. "Life Cycle, Individual Thrift, and the Wealth of Nations." *American Economic Review*, 76 no. 3 (June 1986): 297-313.
- Moffet, Denis. "Optimal Deductible and Consumption Theory." *Journal of Risk and Insurance*, 44 no. 4 (December 1977): 669-682.
- Moore, Evan and Catherine Eckel. "Measuring Ambiguity Aversion." *Unpublished Manuscript*, Department of Economics, Virginia Polytechnic University, 2003.
- Mossin, Jan. "Aspects of Rational Insurance Purchasing." *Journal of Political Economy*, 76 no. 4 (July-August 1968): 553-556
- Mosteller, Frederick and Philip Noguee. "An Experimental Measurement of Utility." *Journal of Political Economy*, 59 no. 5 (October 1951): 371-404.
- Myung, Noah. "Ambiguity Aversion in Asset Market: Experimental Study of Home Bias." *Working Paper No. 1306*, California Institute of Technology, Division of Humanities and Social Sciences, 2009.
- Nau, Robert. "Uncertainty Aversion with Second-Order Utilities and Probabilities." *Management Science*, 52 no. 1 (January 2006): 136-145.
- Ng, Yew-Kwang. "Why do People Buy Lottery Tickets-Choices Involving Risk and the Indivisibility of Expenditure." *Journal of Political Economy*, 73 no. 5 (October 1965): 530-535.
- Nielsen, William S. "A Simplified Axiomatic Approach to Ambiguity Aversion." *Journal of Risk and Uncertainty*, 41 no. 2 (October 2010): 113-124.
- Ochs, Jack, and Alvin E. Roth. "An Experimental Study of Sequential Bargaining." *American Economic Review*, 79 no 3 (June 1989): 355-384.
- Offerman, Theo, Joep Sonnemans, Gijs van de Kuilen and Peter P. Wakker. "A Truth-Serum for Non-Bayesians: Correcting Proper Scoring Rules for Risk Attitudes." *Review of Economic Studies*, 76 no. 4 (October 2009): 1461-1489.
- Pashigian, B. Peter, Lawrence L. Schkade and George H. Menefee. "The Selection of an Optimal Deductible for a Given Insurance Policy." *Journal of Business*, 39 no. 1 (January 1966): 35-44.

- Peijnenburg, Kim, Theo Nijman and Bas J.M. Werker. "Optimal Annuitization with Incomplete Annuity Markets and Background Risk During Retirement." *Discussion Paper 01/2010 - 003*, Network for Studies on Pensions, Aging and Retirement, 2010.
- Pratt, John W. "Risk Aversion in the Small and the Large." *Econometrica*, 32 no. 1/2 (January-April 1964): 122-136.
- Pye, Gordon. "Present Values for Imperfect Capital Markets." *Journal of Business*, 39 no. 1 (January 1966): 45-51.
- Quiggin, John. "A Theory of Anticipated Utility." *Journal of Economic Behavior and Organization*, 3 no. 4 (December 1982): 323-343.
- Quiggin, John. "Comparative Statics for Rank-Dependent Expected Utility Theory." *Journal of Risk and Uncertainty*, 4 no. 4 (December 1991): 338-350.
- Quiggin, John. "Risk, Self-Protection and Ex Ante Economic Value-Some Positive Results." *Journal of Environmental Economics and Management*, 23 no. 1 (July 1992): 40-53.
- Quiggin, John. *Generalized Expected Utility Theory: Rank-Dependent Model*. Boston: Kluwer Academic Publishers, 1993.
- Quiggin, John. "Regret Theory with General Choice Sets." *Journal of Risk and Uncertainty*, 8 no. 2 (March 1994): 153-165.
- Ramsey, Frank P. *The Foundations of Mathematics and Other Logical Essays*. New York: Harcourt Brace and Co, 1926.
- Raviv, Artur. "The Design of an Optimal Insurance Policy." *American Economic Review*, 69 no. 1 (March 1979): 84-96.
- Read, Daniel. "Monetary Incentives, What Are They Good For?" *Journal of Economic Methodology*, 12 no. 2 (2005): 265-276.
- Rabin, Mathew. "Risk Aversion and Expected-Utility Theory." *Econometrica*, 68 no. 5 (September 2000): 1281-1292.
- Rao, J. N. K. and A. J. Scott. "On Chi-squared Tests for Multiway Contingency Tables with Cell Proportions Estimated from Survey Data." *Annals of Statistics*, 12 no. 1 (March 1984): 46-60.
- Razin, Assaf. "Rational Insurance Purchasing", *Journal of Finance*, 31 no. 1 (March 1976): 133-137.
- Richard, Scott F. "Multivariate Risk Aversion, Utility Independence and Separable Utility Functions." *Management Science*, 22 no. 1 (September 1975): 1221.
- Rietz, Thomas A. "Implementing and Testing Risk Preference Induction Mechanisms in Experimental Sealed Bid Auctions." *Journal of Risk and Uncertainty*, 7, 1993, 199-213.

- Roth, Alvin E. and Michael W. K. Malouf. "Game-Theoretic Models and the Role of Information in Bargaining." *Psychological Review*, 86 no 6 (1979): 574-594.
- Rothschild, Michael and Joseph E. Stiglitz. "Increasing Risk I: A Definition." *Journal of Economic Theory*, 2 no. 3 (September 1970): 225-243.
- Rothschild, Michael and Joseph E. Stiglitz. "Increasing Risk II: Its Economic Consequences." *Journal of Economic Theory* 3 no. 1 (March 1971): 66-84.
- Rothschild, Michael and Joseph E. Stiglitz. "Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information." *Quarterly Journal of Economics*, 90 no. 4 (November 1976): 629-649.
- Rosenzweig, Mark R. and Kenneth I. Wolpin. "Natural 'Natural Experiments' in Economics," *Journal of Economics Literature*, 38 no. 4 (December 2000): 827-874.
- Rutström, E. Elisabet and Nathaniel T. Wilcox. "Stated Beliefs versus Inferred Beliefs: A Methodological Inquiry and Experimental Test." *Games and Economic Behavior*, 67 no. 2 (November 2009): 616-632.
- Samuelson, Paul A. "Probability, Utility, and the Independence Axiom." *Econometrica*, 20 no. 4 (October 1952): 670-678.
- Sandmo, Agnar. "The Effect of Uncertainty on Saving Decisions." *Review of Economic Studies*, 37 no. 3 (July 1970): 353-360.
- Sarin, Rakesh K. and Peter Wakker. "Dynamic Choice and NonExpected Utility." *Journal of Risk and Uncertainty* 17 no. 2 (November 1998): 87-119.
- Sarin, Rakesh K. and Martin Weber. "Effects of Ambiguity in Market Experiments." *Management Science*, 39 no. 5 (May 1993): 602-615.
- Savage, Leonard J. "The Theory of Statistical Decision." *Journal of the American Statistical Association*, 46 no. 253 (March 1951): 55-67.
- Savage, Leonard J. "Elicitation of Personal Probabilities and Expectations." *Journal of the American Statistical Association*, 66 (December 1971): 783-801.
- Savage, Leonard J. *The Foundations of Statistics*. New York: John Wiley and Sons. Reprint, New York: Dover Publications, Inc., 1972.
- Schneeweiss, Hans. "The Ellsberg Paradox from the Point of View of Game Theory." *Inference and Decision*, 1 (1973): 65-78
- Schlag, Karl H. and Joël van der Weele. "Eliciting Probabilities, Means, Medians, Variances and Covariances without Assuming Risk-neutrality." *Working Paper*, Universitat Pompeu Fabra, Barcelona, 2009.

- Schlesinger, Harris. "The Optimal Level of Deductibility in Insurance Contracts." *Journal of Risk and Insurance*, 48 no. 3 (September 1981): 465-481.
- Schlesinger, Harris. "Choosing a Deductible for Insurance Contracts: Best or Worst Insurance Policy?" *Journal of Risk and Insurance*, 52 no. 3 (September 1985): 465-481.
- Schlesinger, Harris. "The Theory of Insurance Demand." *Handbook of Insurance*, edited by Georges Dionne. Boston: Kluwer Academic Publishers, 2000.
- Schlesinger, Harris and Neil A. Doherty. "Incomplete Markets for Insurance: An Overview." *Journal of Risk and Insurance*, 52 no. 3 (September 1985): 402-423.
- Schmidt, Ulrich. "Insurance Demand and Prospect Theory." *Working Paper No. 1750*, Kiel Institute for the World Economy, 2012.
- Schoemaker, Paul J.H. and Howard C. Kunreuther. "An Experimental Study of Insurance Decisions." *Journal of Risk and Insurance*, 46 no. 4 (December 1979): 603-618.
- Schmeidler, David. "Subjective Probability and Expected Utility without Additivity." *Econometrica*, 57 no. 3 (May 1989) 571-587.
- Segal, Uzi. "The Ellsberg Paradox and Risk Aversion: An Anticipated Utility Approach." *International Economic Review*, 28 no. 1 (February 1987): 178-202.
- Segal, Uzi. "Probabilistic Insurance and Anticipated Utility." *Journal of Risk and Insurance*, 55 no. 2 (June 1988a), 287-297.
- Segal, Uzi. "Does the Preference Reversal Phenomenon Necessarily Contradict the Independence Axiom?" *American Economic Review*, 78 no. 1 (March 1988b): 233-236.
- Segal, Uzi. "Two-Stage Lotteries Without the Reduction Axiom." *Econometrica*, 58 no. 2 (March 1990): 349-377.
- Segal, Uzi. "The Independence Axiom Versus the Reduction Axiom: Must We Have Both?" In *Utility Theories: Measurements and Applications*, edited by Ward Edwards. Boston: Kluwer Academic Publishers, 1992.
- Selten, Reinhard, Abdolkarim Sadrieh and Klaus Abbink. "Money Does Not Induce Risk Neutral Behavior, but Binary Lotteries Do even Worse." *Theory and Decision*, 46 no 3 (June 1999): 211-249.
- Sheskin, David J. *Handbook of Parametric and Nonparametric Statistical Procedures*. Boca Raton: Chapman and Hall, Third Edition, 2004.
- Shogren, Jason F. "The Impact of Self-Protection and Self-Insurance on Individual Response to Risk." *Journal of Risk and Uncertainty*, 3 no. 2 (1990): 191-204.

- Shogren, Jason F. and Thomas D. Crocker. "Risk, Self-Protection and Ex Ante Economic Value." *Journal of Environmental Economics and Management*, 20 no. 1 (January 1991): 1-15.
- Shogren, Jason F. and Thomas D. Crocker. "Risk and its Consequences." *Journal of Environmental Economics and Management*, 37 no. 1 (January 1999): 44-51.
- Slovic, Paul, Baruch Fischhoff, Sarah Lichtenstein, Bernard Corrigan and Barbara Combs. "Preference for Insuring Against Probable Small Losses: Insurance Implications." *Journal of Risk and Insurance*, 44 no. 2 (June 1977): 237-258.
- Smith, Cedric A.B. "Consistency in Statistical Inference and Decision." *Journal of the Royal Statistical Society*, 23 no 1 (1961): 1-25.
- Smith, Vernon L. "Optimal Insurance Coverage." *Journal of Political Economy* 76 no. 1 (January-February 1968): 68-77.
- Smith, Vernon L. "Measuring Nonmonetary Utilities in Uncertain Choices: The Ellsberg Urn." *Quarterly Journal of Economics*, 83 no. 2 (May 1969): 324-329.
- Snow, Arthur. "Ambiguity Aversion and the Propensities for Self-Insurance and Self-Protection." *Journal of Risk and Uncertainty*, 42 no 1 (February 2011): 27-43.
- Sorenson, Roy A. *Thought Experiments* New York: Oxford University Press, 1992.
- Starmer, Chris. "Developments in Non-Expected Utility Theory: The Hunt for a Descriptive Theory of Choice under Risk." *Journal of Economic Literature*, 38 no 2 (June 2000): 332-382.
- Starmer, Chris, and Robert Sugden. "Does the Random-Lottery Incentive System Elicit True Preferences? An Experimental Investigation." *American Economic Review*, 81 no 4 (September 1991): 971-978.
- Sugden Robert. "An Axiomatic Foundation for Regret Theory." *Journal of Economic Theory*, 60 no. 1 (June 1993): 159-180.
- Sweeney, George H. and T. Randolph Beard. "The Comparative Statics of Self-Protection." *Journal of Risk and Insurance*, 59 no. 2 (June 1992): 301-309.
- Townsend, Robert M. "Risk and Insurance in Village India." *Econometrica*, 62 no. 3 (May 1994): 539-591.
- Trautmann, Stefan T. and Gijs van de Kuilen. "Belief Elicitation: A Horse Race among Truth Serums." *Center Working Paper 2011-117*, Tilburg University, November 2011.
- Turnbull, S. M. "Additional Aspect of Rational Insurance Purchasing." *Journal of Business*, 56 no. 2 (April 1983): 217-229.

- Tversky, Amos and Daniel Kahneman. "Rational Choice and the Framing of Decisions." *Journal of Business*, 59 no. 4 (October 1986): S251-S278.
- Tversky, Amos and Daniel Kahneman. "Advances in Prospect Theory: Cumulative Representation of Uncertainty," *Journal of Risk and Uncertainty*, 5 no. 4 (October 1992): 297-323.
- Tversky, Amos and Craig R. Fox. "Weighing Risk and Uncertainty." *Psychological Review*, 102 no. 2 (April 1995): 269-283.
- Vajda, Stefan. "Minimum Variance Reinsurance." *Astin Bulletin*, 1962.
- von Neumann, John and Oskar Morgenstern. *Theory of Games and Economic Behavior*. Princeton, NJ: Princeton University Press, 1953; Third Edition; Princeton University Paperback Printing, 1980.
- Vuong, Quang H. "Likelihood Ratio Tests for Model Selection and Non-Nested Hypothesis." *Econometrica*, 57 no 2 (March 1989): 307-333.
- Wakker, Peter P., Richard H. Thaler and Amos Tversky. "Probabilistic Insurance." *Journal of Risk and Uncertainty*, 15 no. 1 (October 1997): 7-28.
- Wakker, Peter, Daniëlle Timmermans and Irma Machielse. "The Effects of Statistical Information on Risk and Ambiguity Attitudes, and on Rational Insurance Decisions." *Management Science*, 53 no.11 (November 2007): 1770-1784.
- Walker, James M., Vernon L. Smith and James C. Cox. "Inducing Risk Neutral Preferences: An Examination in a Controlled Market Environment." *Journal of Risk and Uncertainty*, 3 no 1 (March 1990): 5-24.
- Weitzman, Martin. "Utility Analysis and Group Behavior: An Empirical Study." *Journal of Political Economy*, 73 no. 1 (February 1965): 18-26.
- Werner, Jan. "Participation in Risk Sharing under Ambiguity." *Preliminary Draft*, Department of Economics, University of Minnesota, 2011.
- Wilcox, Nathaniel T. "Stochastic Models for Binary Discrete Choice Under Risk: A Critical Primer and Econometric Comparison." In *Risk Aversion in Experiments*, edited by James C. Cox and Glenn W. Harrison. Bingley, UK: Emerald, Research in Experimental Economics, Volume 12, 2008.
- Wilcox, Nathaniel T. "A Comparison of Three Probabilistic Models of Binary Discrete Choice Under Risk." *Working Paper*, Economic Science Institute, Chapman University, March 2010.
- Wilcox, Nathaniel T. "'Stochastically More Risk Averse': A Contextual Theory of Stochastic Discrete Choice Under Risk." *Journal of Econometrics*, 162 no 1 (May 2011): 89-104.

- Winkler, Robert L. "Probabilistic Prediction: Some Experimental Results." *Journal of the American Statistical Association*, 66 no. 336 (December 1971): 675-685.
- Winkler, Robert L. And Allan H. Murphy. "Nonlinear Utility and the Probability Score." *Journal of Applied Meteorology*, 9 no 1 (February 1970): 143-148.
- Winter, Ralph. "Optimal Insurance under Moral Hazard. In *Handbook of Insurance*, edited by Georges Dionne. Boston: Kluwer Academics Publishers, 2000.
- Yaari, Menahem. "Uncertain Lifetime, Life Insurance, and the Theory of the Consumer." *Review of Economic Studies*, 32 no. 2 (April 1965a): 137-150.
- Yaari, Menahem. "Convexity in the Theory of Choice Under Risk." *Quarterly Journal of Economics*, 79 no. 2 (May 1965b), 278-290.