Cointegration Based Optimisation of Currency Portfolios

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Abstract

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Cointegration is a technique that has been used for some time to optimise equity portfolios, but there is limited evidence of its application in managing currency portfolios. This research examines whether there is any value to be gained by using cointegration based strategies to optimise currency portfolios that are U.S. dollar, Euro and Sterling based respectively. We build 'major currency pair (MCP) tracking' portfolios to replicate the classic index tracking strategy commonly applied to equity portfolios, overcoming the lack of an 'index' for currencies by using the most frequently traded currency pair for each portfolio, namely the EUR/USD for the U.S. dollar and Euro portfolios, whilst the GBP/USD is used for the sterling portfolios. We compare the out-of-sample performances of these portfolios to simple benchmark techniques of optimisation. The results are encouraging, with the detection of long-run relationships adding value, particularly for the sterling portfolios. Because of the generally low volatile nature of the cointegration portfolios, they could be leveraged to match the returns being offered to investors by the benchmarks, as they generally offered better risk-adjusted returns. They also enhance portfolio stability.

There are several side issues that are also discussed, such as whether profits can still be derived from applying simple trading rules to currency portfolios, and if so what are the best currencies to invest in and how far should an investor internationally diversify their portfolios. Some of the results found are encouraging, and point towards the added value given by 'lesser' currencies particularly the Scandinavian currencies, with the Swedish Krona appearing to boost the risk-adjusted returns of portfolios. Technical analysis is shown to still have a role to play in currency trading.

Keywords: Cointegration, portfolio optimisation, index tracking, trend-following strategies, trading rules, diversification, currency trading benchmark

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1. Introduction

The Foreign Exchange Market (Forex) is the biggest single market in the world with an estimated average $3.2 trillion dollars being exchanged on a daily basis according to the last BIS Triennial Survey (2007)

The appeals of the Forex market lie in its 24 hour accessibility and high liquidity. Amongst the currency pairs that are traded, the EUR/USD is far and away the most frequent and therefore the most liquid, with approximately a 27% share of the market. Hence the EUR/USD is the backbone of many currency trading portfolios. The next most popular traded currency pairs are: the USD/JPY, GBP/USD and USD/CHF. The reason these are the most commonly traded pairs is that only the most economically/politically stable and liquid currencies are demanded in sufficient quantities, with the U.S. dollar being the most traded because it has traditionally attracted risk averse investors. However, part of the motivation of this research is to discover whether diversifying a portfolio with lesser traded currency pairs and altering the risk/return landscape of currency portfolios in this way could provide greater risk adjusted profits.

It appears that there is already a natural progression of diversification away from the major currencies. In 2007, the four most traded currencies, the US Dollar, the Euro, the Japanese Yen and Sterling were involved in 8 percentage points fewer foreign exchange transactions than they were in 2004 (BIS survey 2007), which shows increasing diversification. Combined with the fact that the percentage share invested in the three major currency pairs, the EUR/USD, USD/JPY and USD/GBP respectively, dropped from 59% in 2004 to 52% in 2007, this indicates that the monopoly of the 'major' currencies in the market may be ceasing, as investors show increasing interest in high yielding currencies such as the Australian dollar and 'fringe' currencies like the Swedish Krona. If this is the case, then it would be pragmatic to show which of the 'lesser' currencies perform best in portfolios and if the U.S. dollar is the most traded currency because it is the most available, most liquid and therefore the most convenient. By constructing portfolios based solely on Euro and Sterling currency pairs respectively, it would also be of particular interest to see whether these portfolios could outperform those based on the U.S. Dollar.

Correlation analysis has long been used as a technique for portfolio optimisation, and it is still used today to model interdependencies between financial assets. To be an effective Forex trader, understanding an overall portfolio's sensitivity to market volatility is important. Due to currencies being priced in pairs, no single pair trades completely independently of the others. One reason for interdependence of currency pairs is immediately conspicuous: when trading the GBP/JPY, for example, an investor is effectively trading a kind of derivative of the GBP/USD and USD/JPY pairs; therefore, GBP/JPY must be somewhat correlated to one if not both of these other currency pairs[1]. Correlation analysis can be used to control a portfolio's exposure to market movements. However, many prominent risk analysts have questioned the use of correlation as a financial tool, highlighting its inherent instability as one of its major shortcomings. Amongst its most vehement critics, Taleb (2007) cites that testing for correlation between the same two time series over consecutive time periods may give you completely different results. In terms of

[1] However, the relationship between currency pairs is much more difficult to ascertain than this and owes its links to complex market forces.

portfolio analysis, this would mean frequent rebalancing of portfolios constructed using this technique.

Cointegration is a concept that has been introduced over the last few decades and is increasingly used in financial econometrics. It is related to correlation in terms of establishing patterns between financial time series, but the major distinction is that cointegration can measure long term co-movements of data in levels, whereas correlation measures only short term co-movements of stationary data.. Applying this to currency management, it could technically be used to construct a portfolio where constant rebalancing is not required, therefore decreasing the reliance of portfolio currency managers on newly acquired information in a market which is deemed the most efficient. Another main advantage of cointegration analysis, as compared to correlation, is that it enables the use of the entire information set comprised in level financial variables (Alexander and Dimitriu 2002), meaning that it has the ability to detect trends in prices.

There are a plethora of studies involving the use of cointegration to construct equity based portfolios, but a seeming reluctance to transfer this technique over to the currency markets. The most obvious reason for this is the lack of a standardised benchmark or index of any sort. Indeed Lequeux and Acar (1998) mention that whilst most of the other markets have indices to explain their performances and respond to the portfolio analysis needs of the fund manager, very few are available in the currency markets. Whilst stocks have their respective indices by which to perform cointegration based trading strategies such as the 'index tracking strategy' or the 'long/short market neutral strategy' with lower complexity, exchange rates do not have the equivalent, making these strategies less easily applicable. The search for a benchmark has led Lequeux and Acar (1998) to create the AFX Index, a dynamic currency management benchmark which is based on using a basket of moving averages as a technical trading rule applied to a group of major currency pairs.

Additionally, as the EUR/USD takes up a majority share of the market, this could hypothetically be used as an 'index' for the U.S. Dollar and Euro based portfolios. The problem of finding an 'index' can therefore be overcome and an index tracking strategy can be implemented for the purposes of this research. Consequently 'major currency pair (MCP) tracking' portfolios will be constructed, where the portfolio is designed to track the performance of the most frequently traded currency pair contained within it. The performance of all the portfolios will be contrasted to the performance of their respective 'index' to see whether it is more profitable to just invest in the MCP, rather than add several currency pairs to the portfolio, all of which would require rebalancing at further transaction costs. We use two simple benchmark models and compare them against the cointegration based portfolio optimisation techniques to see if value can be added by the detection of long-run relationships.

The main motivation of this research is to see which techniques are most effective in constructing a currency portfolio that can be used over an investment horizon to produce consistent returns. A second research question is to see whether diversifying a portfolio beyond the major currency pairs produces better risk adjusted returns than simply trading the main currencies, particularly the EUR/USD, and to see which of the lesser traded currency pairs add the most value to the portfolios. A related motivation is also to show whether portfolio optimisation by cointegration can diversify risk as well as, or better than, portfolios which are not cointegrated.

Additionally, the AFX index, namely a weighted average of 32-, 67- and 121-day moving averages will be used as a benchmark, so it will be interesting to see if they perform as strongly as they did in the Lequeux and Acar (1998) paper. More recent studies, such as Schulmeister (2009), have suggested that technical analysis utilising daily data no longer has the ability to create excess returns as markets become more efficient, so conclusions will be drawn here also.

The rest of this paper is organised as follows: Section 2 reviews literature relevant to this research. Section 3 describes the methodologies used in this study. Section 4 covers the derivation of data and the construction of the portfolios, whilst Section 5 presents the results provided by the out-of-sample portfolios in relation to their in-sample counterparts. Section 6 closes the research and provides a summary of conclusions.

2. Literature Review

Portfolio optimisation was initiated as a concept by Markowitz (1952), who was a pioneer of Modern Portfolio Theory which proposes how investors use diversification to optimise their portfolios as they try to balance obtaining the highest possible returns whilst reducing the amount of risk they are prepared to take on. This can be achieved by adding a certain combination of non-perfectly correlated stocks which can reduce the overall portfolio risk,. Markowitz (1952) states that, once fully diversified, there is a minimum variance portfolio that can be obtained at which point increasing its returns means increasing the risk the portfolio is subjected to.

This risk/return trade-off has meant that international diversification is now a growing trend for portfolio managers as investors look for new ways to diversify.

The use of cointegration itself as a financial tool has grown in the last few decades. Since the idea was introduced in the early 1980s, it has generated a lot of interest amongst econometricians and, more recently, investors. Thus, the literature concerned with the theory and providing applications is rapidly expanding. Engle and Granger (1987) made a significant breakthrough in the methodology as they suggested a two-step method of investigating long-run relationships between time series which involved least squares estimation. This testing was for bivariate processes so could only have zero or one cointegrating vectors. Engle and Yoo (1987) develop this idea to discuss cointegration tests with more than two variables, whilst Stock and Watson (1988) present tests for multiple cointegrating vectors and observe that cointegrated variables share common stochastic trends. Johansen (1988) unifies these lines of argument by developing a maximum likelihood approach to testing for and estimating multiple cointegration vectors.

Applications of these techniques are now commonplace. Choudhury (1997) uses cointegration tests to analyse long-run relationships between six Latin American stock markets and the U.S market, whilst Corhay, Rad and Urbain (1993) conduct a similar study across the European stock markets. Moving away from stock market linkages, Kroner and Sultan (1993) argue for the use of cointegration as a tool in the dynamic hedging of foreign currency futures to good effect, whilst Lai and Lai (1991) use cointegration as a test for market efficiency by analysing the spot and forward rates for five major currencies against the U.S. Dollar. Andrade, Clare and Thomas (1991) use the Johansen (1988) procedure to test whether a domestic investor in the U.S., U.K., Germany and Japan respectively can benefit by internationally diversifying their bonds and equities portfolios. Interestingly they found that all but a Japanese investor could benefit by doing so. More recent applications include Kucukcolak (2008), who tests for integration between the Turkish stock market index with other EU indices for diversification benefits, whilst Alexander and Dimitriu (2004) use cointegration to try and build optimised equity portfolios, with varying results. It remains of interest to see whether internationally diversified currency portfolios can perform as well as their counterparts in the bonds and equities markets, and if sterling or euro investors can benefit as much, or more, than U.S. dollar investors by internationally diversifying their portfolios.

In terms of portfolio optimisation by means of cointegration, Dunis and Ho (2005) use cointegration to construct European equities portfolios, adopting the classic index tracking strategy which is modified in this paper. They do this on the understanding premise that an equity index is by definition a weighted sum of its constituents so that there should a sufficiently large basket of component equities which is cointegrated with the index (Pindyck and Rothemberg 1992). Alexander and Dimitriu (2002) also build index tracking cointegration portfolios in their paper for the Dow Jones Industrial Average Index. These papers have the advantage of a clear benchmark to use as a dependent variable when conducting an OLS regression and analysing the cointegration equation residuals, which is not afforded to currency portfolios. No literature has been found which applies these methods to currencies, so it remains to be seen how the major currency pairs to the U.S. Dollar, Euro and Sterling respectively perform as an 'index' in these tests.

There is justifiable scepticism at making money by predicting price changes in any given market as markets are meant to fully integrate all new information. Although the forex market is one of the quicker ones to absorb new information, efficient market hypothesis relies on rational profit motivated investors (Arnott and Pham, 1993). And as Lequeux (2001) mentions, the two largest participants in the foreign exchange market have no direct profit motive, those being international corporations trying to hedge currency risk and central banks trying to dampen fluctuations in the currency rate to control inflation. This implies that there is added value to be had by active currency management as investors can use technical analysis to exploit this intervention. Additionally, Arnott and Pham (1993) state that currency markets have statistically significant 'trends'. As trading managers use similar technical forecasts to trigger their positions in the financial markets, most managed currency funds exhibit a common factor. It was upon this basis that Lequeux and Acar (1998) created the AFX Index, formed by a set of trading rules being applied to a portfolio of weighted, frequently traded currency pairs, in the thinking that this could create a dynamic benchmark for actively managed currency trading portfolios. Whilst planning to construct our portfolios by means of varying techniques, we shall be employing the trading rules that govern the AFX Index as they produced solid results.[2]

[2] Using the AFX Index as a benchmark for the portfolios was considered here. However, with the Index being created in 1984, the currency weightings were no longer considered relevant for this investment horizon. Also,

As Dunis and Levy (2002) suggest, active trading strategies based on trend following techniques presume that abnormal gains can be made from financial markets which are greater than a random walk would suggest. Lequeux (2001) advocates this by stating there is some statistical evidence that active management of currencies can significantly add value over time. Other evidence on the profitability of trend following strategies in the forex markets are found in papers by Arnott and Pham (1993), Schulmeister (2006), Levich and Thomas (1993), amongst many others. In particular, Schulmeister's results suggest that the use of various short-term Moving Averages (MAs) as technical trading rules result in profitable trading strategies even after adjusting for interest expense and transaction costs. Menkhoff and Taylor (2007) also advocate technical analysis, suggesting that it may be an efficient form of information processing and that it may provide information on non-fundamental influences on foreign exchange movements. However some more recent studies indicate that the profitability of using trading rules to create profits in the foreign exchange is declining. Schulmeister (2009) conducts a further study and concludes that the excess returns created by technical analysis have ceased to exist after the late 1990's boom due to markets becoming more efficient, whilst Neely *et al.* (2006) and Levich and Pojarliev (2008) adopt similar conclusions. Instead investors are using alternative methods of forecasting like Neural Networks, as shown effectively by Pasquier *et al.* (2007).

Dunis and Miao (2005) use the moving average convergence and divergence system (MACD) in the context of trying to find an optimal trading frequency over a given investment horizon. They test several MACD systems and interestingly find that those with short-term MAs outperform those with long-term MAs, as well as suggesting that MACDs do not perform well in markets with high volatility. However, they also note that MACDs tend to perform better in currency markets than in bond or commodity markets. It is hoped that the weighted average of a 1D-32D, 1D-61D, and 1D-117D MAs will help reduce portfolio risk and, as Lequeux and Acar (1998) mention, reduce the transaction costs that implementing shorter-term MA strategies will imply.

3. Methodology and Investment Strategies

3.1 Cointegration Models

A time series is said to be stationary if it does not contain any unit root. If this is the case then it is said to be 'integrated to order 0' or 'I(0)'. Engle and Granger (1987) state that a linear combination of two or more non-stationary series may itself be stationary and, if such a stationary linear combination exists, the non-stationary time series are said to be cointegrated. So, if Y_t and X_t are I(1) series and cointegrated, there exists a linear relation

$$u_t = Y_t - \alpha - \beta X_t \qquad (1)$$

which is I(0). $Y_t = \alpha + \beta X_t$ can be interpreted as an equilibrium or long-run relationship between these series and u_t is referred to as the error-correction term (ECT) since it gives

this index is created for 'trend' following portfolios and, as yet, there is no evidence to suggest that cointegrating currency portfolios are 'trend' followers.

the 'error' value in $Y_t = \alpha + \beta X_t$. As u_t is I(0), it has a constant mean of zero, which makes sense as it reflects the deviation from the equilibrium, which in the long-term is zero for cointegrated variables. With respect to foreign exchange markets, if Y_t and X_t were two exchange rates, country specific market forces may cause them to drift apart in the short term, but if a cointegrating relation exists, then they will move together in the long-run. The regression equation $Y_t = \alpha + \beta X_t + u_t$ would be suitable in terms of finding cointegrating coefficients for portfolio construction. Engle and Granger (1987) and Engle and Yoo (1987) propose a 2-step estimation method, where the first step consists of estimating a long-run equilibrium relationship and then estimating a dynamic error-correction relationship using lagged residuals.
The Engle-Granger procedure is described in terms of a bivariate system. This procedure can only identify one cointegration vector and the problem of finding cointegration becomes more complicated when the number of variables in the model is greater than two. The Johansen (1988) method for multiple cointegration allows testing for a number of cointegrating vectors at the same time. It relies on estimating a vector autoregression (VAR) model in differences, such as:

$$\Delta X_t = \mu + \Gamma_1 \Delta X_{t-1} + \Gamma_2 \Delta X_{t-2} + ... + \Gamma_p \Delta X_{t-p-1} + \Pi\, X_{t-p-1} + u_t \qquad (2)$$

Here X is an (mX1) vector of I(1) variables, such as exchange rates, the Γ_j and Π are (mXm) matrices of unknown parameters. M is the number of variables in X and p is the maximum number of lags in the model. Π gives the long run relationships between the X variables, which represent our cointegrating vectors and Γ_j gives the short term effects. The number of cointegrated vectors is determined by the rank of Π. If Π has zero rank, no stationary linear combination can be identified and the variables in X_t are not cointegrated. The number of lags to be included within the model is determined by minimising Akaike's error criterion.

For the MCP tracking strategy, the choice of dependent variable is not necessarily obvious and is an original feature of this research. We have decided upon using the most frequently traded currency to each of the U.S. Dollar, Sterling and Euro when constructing the U.S. dollar, sterling and euro based portfolios respectively. This then presents the next problem that the 'major' currency pair will inevitably form part of the portfolio also, so the weighting that it is attributed within the portfolio will have to be decided. This can be done arbitrarily and actually gives scope to better the portfolio results.

Both methods described above would be appropriate to find cointegration equations but we select the Johansen procedure as it can estimate more than one cointegrating vector in the model and the Johansen system can contain both I(0) and I(1) variables, which the Engle-Granger approach does not allow.

3.2 Index Tracking Strategy

The investment strategy selected in this paper is a replication of the classic index tracking strategy, which is usually applied to equities, and aims at replicating the index in terms of returns and volatility using cointegration rather than correlation. As Alexander and Dimitriu (2002) suggest, this allows us to make use of the full information contained in the exchange rate prices and base the portfolio weights on the long run behaviour of exchange rates, consequently bringing stability to the portfolio and reducing the need for rebalancing. Note that, here, the portfolios will be tracking the major currency pair (MCP) contained in each portfolio, rather than a traditional index, and are henceforth called MCP tracking portfolios.

The selection of portfolio constituents would usually be an important stage of the process if the paper concerned stocks, for example, as there would most likely be several to choose from. However, with exchange rates, there are not as many which are frequently traded, making this less of a concern. In this paper, they will be selected based upon market trading volume, with the most frequently traded currency pairs being added to the portfolio first and lesser currency pairs being added from that point on. The constituent selection usually heavily impacts on the quality of the 'index' tracking, but, as the 'index' is also contained in the portfolios, the tracking will also be affected by what proportions of the portfolio are afforded to the 'index'. The portfolio holdings are then determined by the cointegration optimisation technique, with the weights in each portfolio based on the ordinary least square coefficients of the cointegration equation that regresses the index log price on the portfolio exchange rates log prices over the selected in-sample period.

$$\log(\text{index}_t) = c_0 + \sum_{k=1}^{N} c_{k+1} * \log(P_{k,t}) + \varepsilon_t \qquad (3)$$

As mentioned above, the 'index' used here will also be part of the portfolio and its weighting shall be decided arbitrarily. $P_{k,t}$ is the constituent exchange rate at a given time t and the c_k's give the coefficients that will form the portfolio weights. It is noted here that the residuals from the above regression must be stationary, otherwise the OLS coefficients will be inconsistent and the relationship will be invalid. As Alexander and Dimitriu (2002) point out, the log transformation produces a more harmonious series and can be done on the premise that, if the level variables are cointegrated, their logarithms will also be. Additionally, Dunis and Ho (2005) mention that using log prices has the advantage that the tracking error ε_t is in return format and the c_k coefficients are portfolio weights.

Using this cointegrating relationship, as a matter of interest we also estimate the daily prices for each of the portfolios, to be used in 'notional cointegration' portfolios, and apply technical trading rules to this portfolio price series. It should be mentioned here that this is a marginal concept as there is currently no exchange-traded funds (ETF) to trade a currency portfolio as a whole in this fashion. This concept would rely on changing the position of the portfolio by making changes to each of its constituent parts. So if the MA signal states that the position on the portfolio should change from, say, long to short, then the position on all the portfolio currency pairs will also change from long to short, with the amounts dependent on their weightings within the portfolio. This process would take place whenever the whole portfolio produces a signal for a change of position from the MAs. This mode of construction differs from the MCP tracking portfolios, where each currency pair produces their own signals. Here, the currency pairs only change their position when the portfolio as a whole says to do so.. Additionaly, these portfolios aggregate the currency pairs together when the data is still in levels, so should retain the information contained in the actual prices.

The calculations for the portfolio price are fairly simple. Again we use the coefficients from the least squares regression equation. The exchange rate prices are kept in levels and on any given trading day, these coefficients are multiplied by their respective exchange rate prices. The sum of these factors creates a portfolio price series. We use the following equation for the portfolio price calculation:

$$\Pi_t = (\Sigma^{n}_{k=1} c_k * P_{k,t})/(\Sigma^{n}_{k=1} c_k) \quad (4)$$

Π_t is the portfolio price at time t, c_k is the cointegrating coefficient for the exchange rates respectively and $P_{k.t}$ is the exchange rate price at time t. Once the portfolio price has been calculated for each portfolio, the same technical trading rule will be applied to it that will be applied to each individual currency pair, and a set of portfolio returns will be generated in this way.

Additionally, as Alexander and Dimitriu (2002) explain when they use this technique applied to stocks in the Dow Jones Industrial Average Index, the use of this method would mean daily rebalancing of the portfolio to account for the effect of price changes to the stock weights and they deemed the transaction costs this would require to be prohibitive, although they acknowledge it would reduce the tracking errors. Whilst accepting their rebalancing concerns, the transaction costs in the forex market are considerably lower so this would be an interesting strategy to implement. If the results are good, it may advocate a niche in the market for trading these types of portfolios.

3.3 Moving Average Convergence and Divergence Systems (MACDs)

One of the simplest technical trading strategies to use is the single moving average which says: when an exchange rate, for example, penetrates from below (above) a moving average, calculated from that exchange rate's most recent prices, of a given length *m*, a buy (sell) signal is generated. For a daily trading frequency, if the current price is above the *m*-moving average, then it is left long for the next 24 hours, otherwise it is held short. The rate of return generated by a simple moving average of order m has been calculated as $R_t = B_{t-1}X_t$ where $Xt = \ln(P_t/P_{t-1})$ the underlying logarithmic return, P_t is the asset price at time t, and B_{t-1} is the signal triggered by the trading rule at time t-1 defined by:

$$B_{t-1} = 1 \text{ if } P_{t-1} > \frac{1}{m}\sum_{t=1}^{m} P_{t-1} \quad \text{(long position)} \quad (5)$$

$$B_{t-1} = -1 \text{ if } P_{t-1} < \frac{1}{m}\sum_{t=1}^{m} P_{t-1} \quad \text{(short position)} \quad (6)$$

Each day moving averages are calculated for each currency pair involved in the portfolios. These averages are then compared to the current price of the currency pairs. Should the currency price be greater than the moving average, the investor assumes that a long position is to be initiated or held for the next 24 hours, otherwise the reverse will apply. As we replicate the Lequeux and Acar (1998) AFX Index technical trading rules for this paper, we calculate 1D-32D, 1D-61D, and 1D-117D MAs respectively (Question-32/61/117 is mentioned above). The return on each currency pair is the equally weighted average of the returns earned by the set of 3 moving averages on each currency pair, and these will then be aggregated across the portfolio depending on the weighting of each currency pair within the portfolio. This trading rule is also applied to each portfolio price series as defined above.

4. Data Collection and Portfolio Construction

4.1 Data Collection

The data used in this paper is exchange rate time series using the USD, EUR and GBP as base currencies. All the exchange rate series used in this paper are taken from an historical exchange rate database provided by DataStream. The databank spans from 1st January 1999 till 31st December 2008, a total of 9 years worth of data with 2348 observations in total.

4.1.1 Data Selection and Modification

The Augmented Dickey-Fuller (ADF) test was conducted on each of the exchange rate prices to confirm that they are all non-stationary[3] but are I(1), and therefore qualify for cointegration testing.

For the purposes of this research, it was necessary to establish the most traded currency pairs that include the U.S. dollar, euro and sterling. This is not an easy task in a market that is so vastly traded with many different locations of trade. However, every three years, the Bank for International Settlements coordinates a global central bank survey of foreign exchange and derivatives markets activity. The last survey was held in April 2007, where central banks and monetary authorities from 54 countries and jurisdictions collected data on turnover in traditional foreign exchange markets, with the final results being published in September 2007. The most traded currency pairs are shown in Table 4.1, with the most traded currencies being shown in Table 4.2. The U.S. dollar is the most traded currency and is involved in the top seven most frequently traded pairs, with the euro being the next most traded currency. Table 4.1 shows that it is hard to attain the exact details of the most frequently traded market pairs, making portfolio construction by means of market turnover uncertain after this point. Therefore, to select the currency pairs to be considered for the portfolios, we take the ten most traded currencies shown in Table 4.2, regardless of whether they are displayed by Table 4.1 to be part of the most traded currency pairs involving the base currencies, and use their pairings with the U.S. dollar, euro and sterling respectively as the basis of our portfolios[4]. So for the U.S. Dollar based portfolios, the first portfolio contains the USD/EUR, USD/JPY and USD/GBP. Then, for the second portfolio, we add the USD/CHF on the basis that the Swiss Franc is the next most traded currency as demonstrated in Table 4.2, and continue this addition of U.S. dollar currency pairs based on market share according to Table 4.2 until reaching the USD/NOK. At this point, we would have seven U.S. Dollar portfolios, with the seventh one containing nine exchange ratesThe Sterling and Euro based portfolios are constructed in the same way. This results in twenty-one portfolios being constructed for further testing.[5]

A further point to be mentioned here is that, when collecting the exchange rate data, it is necessary for the Euro, U.S. Dollar and Sterling to be the base currencies of the currency pairs. For instance, using the EUR/USD as an example, the first listed currency to the left is known as the base currency (in this example, the Euro), while the second one on the right is called the counter currency (in this example, the U.S. Dollar). So the exchange rates have to

[3] The results of these tests will not be produced here to conserve space, but are available from the authors.
[4] Whilst acknowledging that this is not the most ideal methodology for portfolio construction, the lack of data on this matter renders these assumptions necessary and is deemed appropriate option for this research.
[5] A full listing of portfolio evolution can be found in Appendix 1

be quoted as 'the amount of counter currency purchased by one unit of the respective base currencies'. This is necessary to maintain fair testing and so the results follow a unilateral pattern. The Euro and Sterling currency pairs are both quoted in this format, as are most of the USD currency pairs. However, there are three exceptions to this rule, namely the EUR/USD, GBP/USD and AUD/USD. For these pairs the U.S. dollar is not quoted as the base currency, with a rising quote meaning the US dollar is weakening. To counteract this we taking the reciprocal of these time series to mean 'the number of counter currency per dollar'.

Table 4.1: BIS Triennial Survey (2007): most traded currency pairs

Reported foreign exchange market turnover by currency pair[1]

Daily averages in April, in billions of US dollars and per cent

	2001		2004[2]		2007	
	Amount	% share	Amount	% share	Amount	% share
US dollar/euro	354	30	503	28	840	27
US dollar/yen	231	20	298	17	397	13
US dollar/sterling	125	11	248	14	361	12
US dollar/Australian dollar	47	4	98	5	175	6
US dollar/Swiss franc	57	5	78	4	143	5
US dollar/Canadian dollar	50	4	71	4	115	4
US dollar/Swedish krona[3]	…	…	…	…	56	2
US dollar/other	195	17	295	16	572	19
Euro/yen	30	3	51	3	70	2
Euro/sterling	24	2	43	2	64	2
Euro/Swiss franc	12	1	26	1	54	2
Euro/other	21	2	39	2	112	4
Other currency pairs	26	2	42	2	122	4
All currency pairs	1,173	100	1,794	100	3,081	100

[1] Adjusted for local and cross-border double-counting. [2] Data for 2004 have been revised. [3] The US dollar/Swedish krona pair could not be separately identified before 2007 and is included in "other". Table B.5

Table 4.2: BIS Triennial Survey (2007): most traded currencies

Currency distribution of reported foreign exchange market turnover[1]
Percentage shares of average daily turnover in April 2007

	2001	2004[2]	2007
US dollar	90.3	88.7	86.3
Euro	37.6	36.9	37.0
Yen	22.7	20.2	16.5
Pound sterling	13.2	16.9	15.0
Swiss franc	6.1	6.0	6.8
Australian dollar	4.2	5.9	6.7
Canadian dollar	4.5	4.2	4.2
Swedish krona	2.6	2.3	2.8
Hong Kong dollar	2.3	1.9	2.8
Norwegian krone	1.5	1.4	2.2
New Zealand dollar	0.6	1.0	1.9
Mexican peso	0.9	1.1	1.3
Singapore dollar	1.1	1.0	1.2
Won	0.7	1.2	1.1
Rand	1.0	0.8	0.9
Danish krone	1.2	0.9	0.9
Rouble	0.4	0.7	0.8
Zloty	0.5	0.4	0.8
Indian rupee	0.2	0.3	0.7
Renminbi	0.0	0.1	0.5
New Taiwan dollar	0.3	0.4	0.4
Brazilian real	0.4	0.2	0.4
All currencies	200.0	200.0	200.0
Emerging market currencies[3]	16.9	15.4	19.8

[1] Because two currencies are involved in each transaction, the sum of the percentage shares of individual currencies totals 200% instead of 100%. Adjusted for local and cross-border double-counting. [2] Data for 2004 have been revised. [3] Defined as the residual after accounting for the top eight currencies, the Norwegian krone, the New Zealand dollar and the Danish krone. Table B.6

4.2 Portfolio Construction

Once the constituents for the portfolios were decided upon, in-sample portfolios were constructed for the period of 13th June 2000 to 31st December 2007. The entire databank was used from 2nd January 2000 onwards but the in-sample period started from 13th June of that year so the 1D-121D MA had enough previous data from which to calculate the trading decision for the first day of the in-sample period.. The out-of-sample period for each of the portfolios was then constructed from 2nd January 2008 till 31st December 2008. The trading decision of the MAs for the first day of the out-of-sample period was a continuation from the previous sufficient amount of data taken from the in-sample period, so up to 121 trading days worth of data from the in-sample period were involved in the calculations for the opening day of the out-of-sample period.

The seven U.S. Dollar portfolios were labelled portfolios 1-7 (P1-P7), with the P1 consisting of the first three major U.S. dollar currency pairs. The next most frequent currency was then paired to the U.S. Dollar and added to the P2. The currency pairs were added one by one to the portfolios until the USD/NOK was added to P7. The Euro portfolios were constructed in the same way and were numbered portfolios 8-14 (P8-P14), with the Sterling portfolios being portfolios 15-21 (P15-P21).

As stated, the exchange rates are tested for stationarity by the ADF test. Then, if suitable for cointegration testing, the logs of the exchange rates are taken, as explained in the methodology, and the cointegration test applied to them[6], with the number of lags to be included within the model being determined by minimising Akaike's error criterion. Also Enders (1995) states that we should be very wary of a result indicating that the variables are

[6] The results from this testing are not shown here to reserve space but are available upon request.

cointegrated using only one dependent variable for the testing, advocating further testing to find a model's strength. With this in mind, and as the dependent variable in the cointegrating equations are not immediately obvious, we conduct the cointegration tests again using an arbitrarily random ordering of the variables in the portfolio (with a different dependent variable other than the chosen index). This will ensure that any cointegration found amongst the exchange rates is genuine and that any explicit trend amongst the portfolios is not caused by them having the same dependent variable in each cointegration portfolio. However, the final portfolio cointegrating equation shall come from the original ordering of the variables (based on the currency pairs' market share) and the dependent variable will be the pre-selected portfolio index (for example the USD/EUR for the U.S. dollar portfolios).

As mentioned before, the cointegration equation will then allow the portfolio weights to be determined, using the regression coefficients and normalizing their sum to one. The benchmark of each portfolio will be allocated a 50% share in each portfolio. If any of the portfolios do not display cointegration in either ordering of the variables, then each currency pair within the portfolio will simply be allocated an equal weight and its returns shall be calculated from there.

For each of the currency pairs involved with the portfolios, 32, 61 and 117 day MAs are used as a trading rule to calculate their individual returns. An equally-weighted average of these MAs is then taken to determine the finalised returns for each pair. These are then used in accordance with the cointegration portfolio weights to determine the portfolio returns. There are four simple applications to each of the twenty-one portfolios (presence of cointegration permitting), which are described below.

4.2.1 Equal Weights Portfolios

This is the first and most simple of the benchmark portfolios. The returns of each currency pair is calculated individually by the equally-weighted average of the MA returns. Each currency pair is given an equal weighting in the portfolio, so each constituent weight will equal 1/n, where n is the number of exchange rates in the portfolio, and the returns are aggregated this way. These portfolios are labelled portfolios 1A-21A (P1A–P21A).

4.2.2 Historical Returns Portfolios

This is the second benchmark used and has a bit more relevance as it is based on the returns of each currency pair in the in-sample period. The basis of these portfolios is to address the performance of each currency pair within the portfolio. For instance, if the USD/EUR performs particularly well in-sample, then more weighting should be allocated to this currency pair out-of-sample. The cumulative returns of each currency pair shall be found from the in-sample investment horizon and these will be summed together. The out-of-sample portfolio weights[7] will then be calculated as the fraction the in-sample cumulative return for each currency pair takes up of this total. So if the total cumulative return of a portfolio is 50% and the cumulative return of the EUR/USD within this portfolio is 10%, then the EUR/USD shall be allocated a portfolio weight of 20% for the out-of-sample period. The equation for this is:

[7] The constituent weights for these portfolios are available upon request from the authors.

$$\Omega_k = (\chi_k) / (\Sigma^n_{k=1}\chi_k) \quad (7)$$

Where Ω_k is the portfolio weight of the currency pair k, χ_k is the in-sample cumulative return of the currency pair k and n defines the number of currency pairs contained in the portfolio. This method is designed to decipher a continuing trend from the in-sample returns to the out-of-sample returns in the hope of gaining greater profits. These portfolios are labelled portfolios 1B-21B (P1B-P21B).[8]

There seemed little point in applying the portfolio weights over the in-sample period as the portfolio weights for these portfolios were directly derived from the performance of each currency pair over the in-sample period, making these results pointless as a basis for comparison.

4.2.3 Major Currency Pair (MCP) Tracking Cointegration Portfolios

Only those portfolios which show cointegration shall be constructed in this manner. Once cointegration has been established, the portfolio weights[9] of each constituent currency pair will be determined from the least squares regression equation 3 above. The returns of each currency pair are calculated individually by the equally-weighted average of the MA returns, and these are aggregated together, using the portfolio constituent weights, to give the portfolio return. These portfolios are labelled portfolios 1C-21C (P1C – P21C).

4.2.4 Notional Cointegration Based Portfolios

As mentioned earlier, this is a marginal concept based on cointegration which we are monitoring out of interest. Again only those portfolios which show cointegration are constructed in this way. They are constructed by taking each cointegration equation coefficient within a portfolio and multiplying them by their respective daily exchange rate 'price', as shown in equation 4 above[10]. The sum of these then gives the portfolio price for each given day and the basket of MAs is then applied to the portfolio price series. A set of portfolio returns are generated from the equally-weighted average of the returns from the MAs. These portfolios are labelled portfolios 1D-21D (P1D – P21D).

4.3 Performance Analysis

The final stage is the computation and analysis of portfolio results. For each portfolio, annualised return, annualised standard deviation, information ratio[11], maximum drawdown and correlation of the portfolio returns with the 'MCP' returns are calculated to gauge portfolio performance. Once the daily returns of the portfolio are calculated, they are compared to the daily returns of the 'MCP' using a tracking error. This is calculated as the average absolute difference between the daily returns of the portfolio and benchmark over the investment horizon. For the MCP tracking portfolios, the tracking error is expected to be

[8] We acknowledge that the Markowitz approach for portfolio optimisation could have been an option here, but as this is just a simple benchmark and not the main focus of this study, we feel this approach is suitable for this research.

[9] A full listing of each portfolio's constituent weights is given in Appendix 2.

[10] The constituent weights here are essentially the same as in the MCP tracking portfolios.

[11] The information ratio is the average annual return of an investment strategy divided by its annualised standard deviation.

low, especially if the ‘major currency pair’ takes up a significant proportion of the portfolio. This is also done for the notional cointegration portfolios, where the frequent rebalancing of each currency pair will most likely reduce the tracking error. Finally, the equal weights and historical returns portfolios are expected to be the least accurate ‘MCP’ trackers, mainly because the constituent weights are not decided by the cointegration equation and there will be less weighting allocated to the index in these portfolios. This calculation is less relevant for these portfolios but is calculated nonetheless.

4.4 Transaction Costs

We follow Lequeux and Acar (1998) and set transaction costs at 0.03% per round-trip transaction for all exchange rates in the portfolios.

5. Results and Analysis

As the ADF test indicated that all time series contained a unit root, all cointegration tests were conducted. We used the Johansen test for multiple cointegration, as it allows us to test for a number of cointegrating vectors, and gave preference to the trace test over the maximum eigenvalue test, if both tests contradict each other. Our results indicate that, at least, there exists a single cointegrating vector in each of the models. All portfolios considered exhibit a common stochastic trend (i.e. there is a long-run relationship between the tested exchange rates). This is partly because 50% of the MCP is contained in the portfolios also. However, cointegration testing with a random ordering of the variables (and no constraint on the MCP weighting), rather than by market share, also displayed cointegration for all of the portfolios, showing that there is a long-run link between the exchange rates in all of the portfolios considered[12]. At first sight, this could suggest that diversification using lesser exchange rates may no longer be beneficial for international investors. However, cointegration cannot test for a gradual move towards or away from a closer relationship. It would therefore be wrong to conclude from our cointegration analysis that international diversification is no longer beneficial before seeing the results obtained for each portfolio. Also, this means that there are no non-cointegrated portfolios to analyse against the cointegrated ones to see which are better in terms of diversifying risk.

The results for the in-sample portfolios are generally poor. All the equal weights portfolios generate negative returns whilst this is the case for most of the cointegration portfolios. However, even here value can be seen by using the cointegration based optimisation methods, which steadily improve the portfolio returns for the Euro and U.S. Dollar portfolios. They also produce remarkably low tracking errors and high correlations to the MCP, which is encouraging. Whilst acknowledging this, the in-sample period was mainly used to produce optimum constituent weights for the out-of-sample portfolios. Thus, we focus our analysis on the out-of-sample portfolio results. The in-sample portfolio results are not shown here to conserve space but are available upon request. We order the results by currency, which allows us to draw comparisons between the optimisation techniques for each currency, after which we can analyse the performance of each currency. We start with the U.S. Dollar out-of-sample portfolios.

[12] The results of these tests are not shown here to conserve space but can be requested from the authors.

5.1 U.S. Dollar Out-of-Sample Portfolios

As table 5.1 below shows, the cointegration based portfolios are generally less volatile, as we would expect, and this leads to lower tracking errors. The equal weights portfolios have low volatility up until the point where they are diversified too much, which contradicts the idea of international diversification. It should be noted that the global financial crisis began to really take hold in financial markets late 2008, so the benefits of international diversification may have been nullified as the effects of the crisis became global. Having said this, the equal weights portfolios information ratios get better as they become more diversified. These results show cointegration optimisation, particularly for the MCP tracking portfolios, works in the respect of minimising volatilities. This stability is also demonstrated by their low maximum drawdowns, with a best of just -4.59% for portfolio 4C.

The historical returns portfolios generally have large weightings attached to most of the currency pairs, which in turn gives large returns and volatilities. These effects could be counteracted by leveraging the other portfolios. For instance, portfolio 4B produces a higher return than portfolio 4C but portfolio 4C could be leveraged to double the amount, which would double the returns offered to almost equal those of portfolio 4B and the volatility would still be less for portfolio 4C. In terms of risk-adjusted returns, there is little difference between the cointegration techniques and benchmarks, with portfolio 4B marginally having the best information ratio of 0.95. However, there are no information ratios over one here, although this seems to be frequently attainable by currency managers.

Equal Weights	Benchmark	Portfolio 1A	Portfolio 2A	Portfolio 3A	Portfolio 4A	Portfolio 5A	Portfolio 6A	Portfolio 7A
Cumulative Return	2.43%	3.38%	5.20%	4.35%	9.04%	10.18%	9.98%	**11.33%**
Ann. Return	2.35%	3.28%	5.04%	4.21%	8.76%	9.87%	9.68%	**10.98%**
Ann. St. Dev.	6.49%	6.25%	**6.23%**	7.17%	10.26%	10.88%	10.83%	11.37%
Information Ratio	0.36	0.52	0.81	0.59	0.85	0.91	0.89	**0.97**
Avg. Correlation to Benchmark	--	**0.41**	0.29	0.20	0.14	0.13	0.12	0.11
Tracking Error	--	**0.34%**	0.38%	0.42%	0.49%	0.51%	0.51%	0.54%
Maximum Drawdown	-7.00%	-7.71%	-7.66%	-6.87%	-6.60%	-6.67%	**-6.45%**	-6.60%
Historical Returns	**Benchmark**	**Portfolio 1B**	**Portfolio 2B**	**Portfolio 3B**	**Portfolio 4B**	**Portfolio 5B**	**Portfolio 6B**	**Portfolio 7B**
Cumulative Return	2.43%	36.88%	13.94%	11.05%	16.31%	15.38%	**89.62%**	42.55%
Ann. Return	2.35%	35.75%	13.51%	10.71%	15.81%	14.91%	**86.87%**	41.24%
Ann. St. Dev.	**6.49%**	70.79%	15.82%	14.41%	16.58%	28.73%	317.97%	94.34%
Information Ratio	0.36	0.51	0.85	0.74	**0.95**	0.52	0.27	0.44
Avg. Correlation to Benchmark	--	-0.41	-0.26	-0.22	**-0.12**	-0.21	-0.23	-0.23
Tracking Error	--	3.54%	0.90%	**0.82%**	**0.82%**	1.34%	13.51%	4.13%
Maximum Drawdown	**-7.00%**	-108.58%	-19.96%	-15.79%	-11.65%	-27.91%	-361.72%	-105.26%
Index Tracking	**Benchmark**	**Portfolio 1C**	**Portfolio 2C**	**Portfolio 3C**	**Portfolio 4C**	**Portfolio 5C**	**Portfolio 6C**	**Portfolio 7C**
Cumulative Return	2.43%	-0.57%	4.28%	4.62%	**8.03%**	7.64%	5.36%	5.17%
Ann. Return	2.35%	-0.55%	4.15%	4.48%	**7.78%**	7.41%	5.20%	5.01%
Ann. St. Dev.	6.49%	6.41%	**5.65%**	5.79%	7.22%	6.97%	7.73%	7.44%
Information Ratio	0.36	-0.09	0.73	0.77	**1.08**	1.06	0.67	0.67
Avg. Correlation to Benchmark	--	0.53	**0.56**	0.54	0.46	0.47	0.44	0.45
Tracking Error	--	0.29%	**0.27%**	**0.27%**	0.32%	0.32%	0.32%	0.32%
Maximum Drawdown	-7.00%	-8.09%	-5.75%	-5.04%	**-4.59%**	-4.77%	-7.27%	-6.74%
Notional Cointegration	**Benchmark**	**Portfolio 1D**	**Portfolio 2D**	**Portfolio 3D**	**Portfolio 4D**	**Portfolio 5D**	**Portfolio 6D**	**Portfolio 7D**
Cumulative Return	2.43%	10.93%	5.10%	5.96%	**25.57%**	8.79%	5.79%	5.17%
Ann. Return	2.35%	10.63%	4.96%	5.79%	**24.88%**	8.55%	5.63%	5.03%
Ann. St. Dev.	**6.49%**	17.50%	8.58%	10.36%	36.65%	10.60%	11.70%	11.57%
Information Ratio	0.36	0.61	0.58	0.56	0.68	**0.81**	0.48	0.43
Avg. Correlation to Benchmark	--	**0.16**	0.10	0.06	0.03	0.03	0.01	0.01
Tracking Error	--	0.83%	**0.49%**	0.56%	1.36%	0.55%	0.59%	0.59%
Maximum Drawdown	**-7.00%**	-12.09%	-9.13%	-12.40%	-22.26%	-7.81%	-7.76%	-7.92%

Table 5.1 U.S. Dollar Out-of-sample Portfolios

5.2 Euro Out-of-Sample Portfolios

Equal Weights	Benchmark	Portfolio 8A	Portfolio 9A	Portfolio 10A	Portfolio 11A	Portfolio 12A	Portfolio 13A	Portfolio 14A
Cumulative Return	**10.51%**	10.14%	8.25%	7.74%	8.55%	9.02%	7.76%	8.45%
Ann. Return	**10.35%**	9.98%	8.12%	7.62%	8.42%	8.88%	7.64%	8.32%
Ann. St. Dev.	12.06%	9.41%	7.93%	7.73%	8.44%	7.78%	6.81%	**6.66%**
Information Ratio	0.86	1.06	1.02	0.98	1.00	1.14	1.12	**1.25**
Avg. Correlation to Benchmark	--	**0.79**	0.75	0.75	0.66	0.65	0.65	0.62
Tracking Error	--	**0.33%**	0.35%	0.35%	0.39%	0.40%	0.40%	0.42%
Maximum Drawdown	-7.22%	-6.25%	-5.98%	-5.69%	-5.28%	-4.59%	-4.02%	**-3.85%**
Historical Returns	Benchmark	Portfolio 8B	Portfolio 9B	Portfolio 10B	Portfolio 11B	Portfolio 12B	Portfolio 13B	Portfolio 14B
Cumulative Return	10.51%	**10.62%**	8.64%	8.18%	9.40%	9.89%	9.46%	9.06%
Ann. Return	10.35%	**10.45%**	8.50%	8.05%	9.25%	9.74%	9.31%	8.92%
Ann. St. Dev.	12.06%	12.27%	10.26%	9.28%	10.65%	9.25%	**8.88%**	9.35%
Information Ratio	0.86	0.85	0.83	0.87	0.87	**1.05**	1.05	0.95
Avg. Correlation to Benchmark	--	0.27	0.28	**0.35**	0.33	**0.35**	**0.35**	0.34
Tracking Error	--	0.65%	0.58%	**0.54%**	-7.99%	**0.54%**	**0.54%**	0.55%
Maximum Drawdown	-7.22%	-10.49%	-9.32%	-7.99%	-6.84%	-5.72%	**-5.49%**	-5.77%
Index Tracking	Benchmark	Portfolio 8C	Portfolio 9C	Portfolio 10C	Portfolio 11C	Portfolio 12C	Portfolio 13C	Portfolio 14C
Cumulative Return	**10.51%**	9.80%	8.34%	8.09%	7.75%	7.53%	5.17%	5.18%
Ann. Return	**10.35%**	9.64%	8.21%	7.96%	7.63%	7.42%	5.09%	5.09%
Ann. St. Dev.	12.06%	9.53%	8.43%	8.24%	8.59%	9.10%	**6.44%**	6.45%
Information Ratio	0.86	**1.01**	0.97	0.97	0.89	0.82	0.79	0.79
Avg. Correlation to Benchmark	--	0.89	0.92	0.94	0.92	0.90	**0.99**	**0.99**
Tracking Error	--	0.25%	**0.23%**	**0.23%**	0.24%	0.25%	0.24%	0.24%
Maximum Drawdown	-7.22%	-6.82%	-6.46%	-6.18%	-6.66%	-7.16%	**-4.22%**	**-4.22%**
Notional Cointegration	Benchmark	Portfolio 8D	Portfolio 9D	Portfolio 10D	Portfolio 11D	Portfolio 12D	Portfolio 13D	Portfolio 14D
Cumulative Return	10.51%	**14.20%**	13.15%	12.67%	12.68%	12.30%	3.65%	3.12%
Ann. Return	10.35%	**13.98%**	12.95%	12.47%	12.49%	12.10%	3.60%	3.07%
Ann. St. Dev.	12.06%	15.45%	14.98%	14.80%	15.30%	16.02%	**9.39%**	9.43%
Information Ratio	0.86	**0.90**	0.86	0.84	0.82	0.76	0.38	0.33
Avg. Correlation to Benchmark	--	0.39	0.39	0.39	0.39	0.37	**0.41**	**0.41**
Tracking Error	--	0.69%	0.67%	0.67%	0.69%	0.72%	**0.50%**	**0.50%**
Maximum Drawdown	**-7.22%**	-8.78%	-8.76%	-8.79%	-9.58%	-10.48%	-9.50%	-9.92%

Table 5.2 Euro Out-of-sample Portfolios

A notable feature from table 5.2 above is the remarkably low tracking errors and high average correlations with the MCP of the MCP tracking portfolios, with portfolios 13C and 14C, demonstrating a near perfect correlation with the MCP. Again the volatility is mostly minimised by the MCP tracking portfolios and the maximum drawdowns show stability, although the equal weights portfolios produce almost as good a performance in terms of volatility.

Interestingly, for both the cointegration based methods, the volatility is minimised through increased diversification but returns are also, so there seems no value in adding 'lesser' currency pairs here. Just investing in the EUR/USD seems to make the most sense here as it garners the highest return and makes for simplicity of portfolio. However, for both benchmarks, the opposite applies, with the best results coming from the more diversified portfolios. Equal weights portfolio gives best risk-adjusted performance, with an information ratio high of 1.25 for portfolio 14A, but the difference when compared to the MCP portfolios is marginal. The maximum drawdowns for the equal weights portfolios are also good, in particular a result of -3.85% for portfolio 14A. None of these portfolios however stand out as satisfactory investment vehicles.

5.3 Sterling Out-of-sample Portfolio

It is interesting to note here that, whilst the U.S. Dollar and euro cointegration portfolios produced fairly consistent portfolio weights, the Sterling portfolios produced cointegration coefficients which allocated little weighting to the GBP/USD, probably due to the GBP/USD being quite volatile over the in-sample period. With 50% of all the cointegration portfolios allocated to the GBP/USD, the result is highly leveraged positions in certain currency pairs to counteract this, which produces large returns and volatilities in these portfolios. This would also explain the higher tracking errors and lower correlations to the GBP/USD, which are natural results as less weighting is attributed to the GBP/USD in these portfolios.

The Sterling portfolios actually produce the best risk-adjusted returns out-of-sample, especially the performances of the notional portfolios 20D and 21D, which may advocate a market for these types of portfolios to be traded. The risk-adjusted performances of portfolios 19C-21C and 17D-21D stand out and would support the suggestion that cointegration adds value to optimising currency portfolios as they outperform the benchmarks here. The information ratios of 1.46 and 1.44 for portfolios for portfolios 20D and 21D respectively, in particular, represent good investments. The only drawback is the high volatilities and maximum drawdowns for the cointegration based portfolios, but this is due to the highly leveraged positions of the currency pairs involved in these portfolios as discussed and could be reduced accordingly. This also meant that the tracking errors are generally greater for all these portfolios and the average correlations to the MCP are also quite low for MCP tracking portfolios. In contrast to the Euro and U.S. Dollar portfolios, here better results are produced through diversification of the cointegration based portfolios.

Equal Weights	Benchmark	Portfolio 15A	Portfolio 16A	Portfolio 17A	Portfolio 18A	Portfolio 19A	Portfolio 20A	Portfolio 21A
Cumulative Return	9.83%	12.49%	**12.54%**	8.34%	8.76%	6.08%	5.67%	3.60%
Ann. Return	9.64%	12.25%	**12.29%**	8.18%	8.59%	5.96%	5.56%	3.53%
Ann. St. Dev.	11.84%	10.35%	10.34%	8.98%	9.25%	8.54%	8.25%	**7.82%**
Information Ratio	0.81	1.18	**1.19**	0.91	0.93	0.70	0.67	0.45
Avg. Correlation to Benchmark	--	**0.80**	0.73	0.73	0.67	0.62	0.58	0.52
Tracking Error	--	**0.31%**	0.36%	0.35%	0.39%	0.41%	0.42%	0.45%
Maximum Drawdown	-15.18%	-10.04%	-10.56%	-10.16%	-8.13%	**-7.70%**	-7.95%	-10.04%
Historical Returns	**Benchmark**	**Portfolio 15B**	**Portfolio 16B**	**Portfolio 17B**	**Portfolio 18B**	**Portfolio 19B**	**Portfolio 20B**	**Portfolio 21B**
Cumulative Return	9.83%	**13.78%**	13.32%	12.97%	12.46%	7.92%	7.33%	5.12%
Ann. Return	9.64%	**13.51%**	13.06%	12.72%	12.21%	7.77%	7.18%	5.02%
Ann. St. Dev.	11.84%	11.04%	11.04%	10.89%	11.07%	9.79%	9.39%	**8.89%**
Information Ratio	0.81	**1.22**	1.18	1.17	1.10	0.79	0.76	0.56
Avg. Correlation to Benchmark	--	**0.69**	0.62	0.62	0.57	0.51	0.48	0.43
Tracking Error	--	**0.38%**	0.44%	0.43%	0.48%	0.48%	0.49%	0.50%
Maximum Drawdown	-15.18%	-10.73%	-11.76%	-11.71%	-8.43%	**-8.13%**	-8.44%	-8.44%
Index Tracking	**Benchmark**	**Portfolio 15C**	**Portfolio 16C**	**Portfolio 17C**	**Portfolio 18C**	**Portfolio 19C**	**Portfolio 20C**	**Portfolio 21C**
Cumulative Return	9.83%	-64.51%	-36.80%	**136.07%**	58.30%	88.75%	68.99%	67.91%
Ann. Return	9.64%	-63.26%	-36.08%	**133.42%**	57.17%	87.03%	67.65%	66.59%
Ann. St. Dev.	**11.84%**	56.31%	36.58%	154.69%	88.34%	98.05%	63.96%	63.06%
Information Ratio	0.81	-1.12	-0.99	0.86	0.65	0.89	**1.06**	**1.06**
Avg. Correlation to Benchmark	--	-0.49	-0.38	**0.54**	0.50	0.53	0.51	0.51
Tracking Error	--	2.65%	**1.69%**	6.05%	3.33%	3.83%	2.46%	2.43%
Maximum Drawdown	**-15.18%**	-123.26%	-70.19%	-119.06%	-83.75%	-96.61%	-65.92%	-65.11%
Notional Cointegration	**Benchmark**	**Portfolio 15D**	**Portfolio 16D**	**Portfolio 17D**	**Portfolio 18D**	**Portfolio 19D**	**Portfolio 20D**	**Portfolio 21D**
Cumulative Return	9.83%	-27.45%	-28.89%	**24.49%**	23.09%	24.40%	21.60%	21.22%
Ann. Return	9.64%	-26.91%	-28.33%	**24.01%**	22.64%	23.92%	21.18%	20.81%
Ann. St. Dev.	**11.84%**	18.46%	18.55%	18.50%	18.64%	19.13%	14.46%	14.50%
Information Ratio	0.81	-1.46	-1.53	1.30	1.21	1.25	**1.46**	1.44
Avg. Correlation to Benchmark	--	-0.65	-0.65	**0.65**	**0.65**	**0.65**	0.64	0.64
Tracking Error	--	1.15%	1.16%	0.62%	0.63%	0.65%	**0.50%**	**0.50%**
Maximum Drawdown	-15.18%	-34.21%	-34.35%	-15.66%	-16.09%	-16.32%	**-12.95%**	-13.26%

Table 5.3 Sterling Out-of-sample Portfolios

It should be mentioned that the benchmarks produced good risk-adjusted performances also and it appears that the sterling portfolios performed the best in the out-of-sample period.

5.4 Overall Analysis

Chart 5.1 offers an interesting comparison between the information ratios of the U.S. Dollar and Sterling out-of-sample portfolios. The information ratios steadily increase as we diversify the U.S. dollar portfolios, while they decline when we diversify the sterling counterparts. This suggests that it is worthwhile for U.S. investors to diversify their currency portfolio, whilst a U.K. investor should stick with the 'major' currency pairs when using this technique.

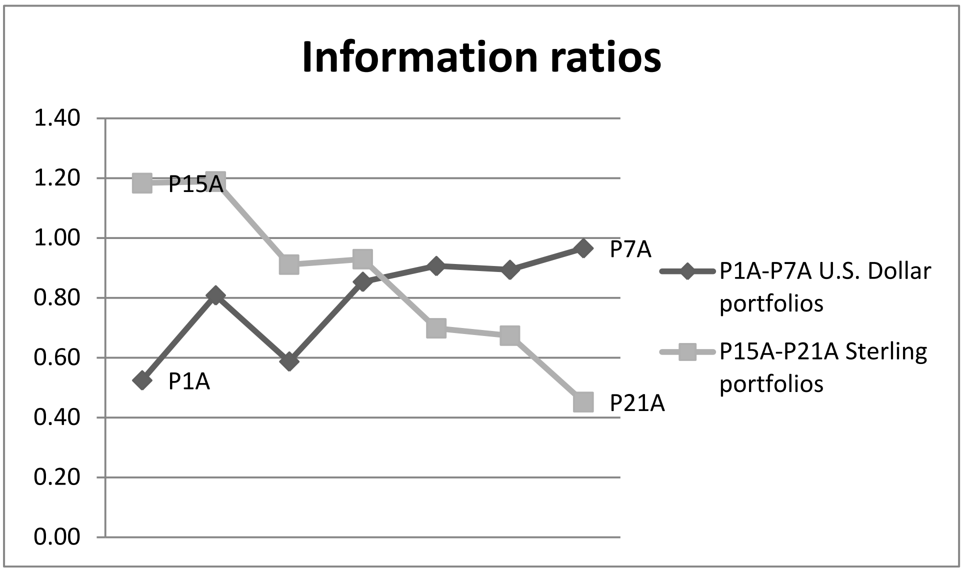

Chart 5.1 Out-of-sample information ratios for U.S. dollar and Sterling equal weights portfolios

This shows there is some evidence to support diversification, particularly in the out-of-sample periods for U.S. and European investors, although all the investors could feel the benefits of diversification in some circumstances. However, it would also be detrimental in others. The traditional diversification approaches may not have achieved intended benefits due to extreme levels of contagion as all countries became affected by the financial crisis and decreasing confidence levels.

The lack of consistency between the in-sample and the out-of-sample periods was disappointing, but reflected the increase in volatility of the global markets at this time. It would be interesting to conduct this study over a more stable period, although there is a clear possibility that these results were made better by the increased volatility. The results were invariably better over the out-of-sample period, which suggests the trading rules selected perform better in periods of high volatility. In particular, longer term MAs seem to be the best performing over these periods, where positions can be held for longer and losses are not made due to short-term price swings, something already underlined by Dunis and Miao (2005). Different trading rules could have been investigated but this was not the purpose for this paper. Some of the portfolios yield good results, which suggest that technical analysis still has a role to play when it comes to trading currencies. The portfolios perform poorly in the in-sample period and the results suggest that the in-sample period may be too long, which means that the relationships established here are less relevant when it comes to the out-of-sample period. This is particularly so when considering that all the out-of-sample period portfolios perform strongly.

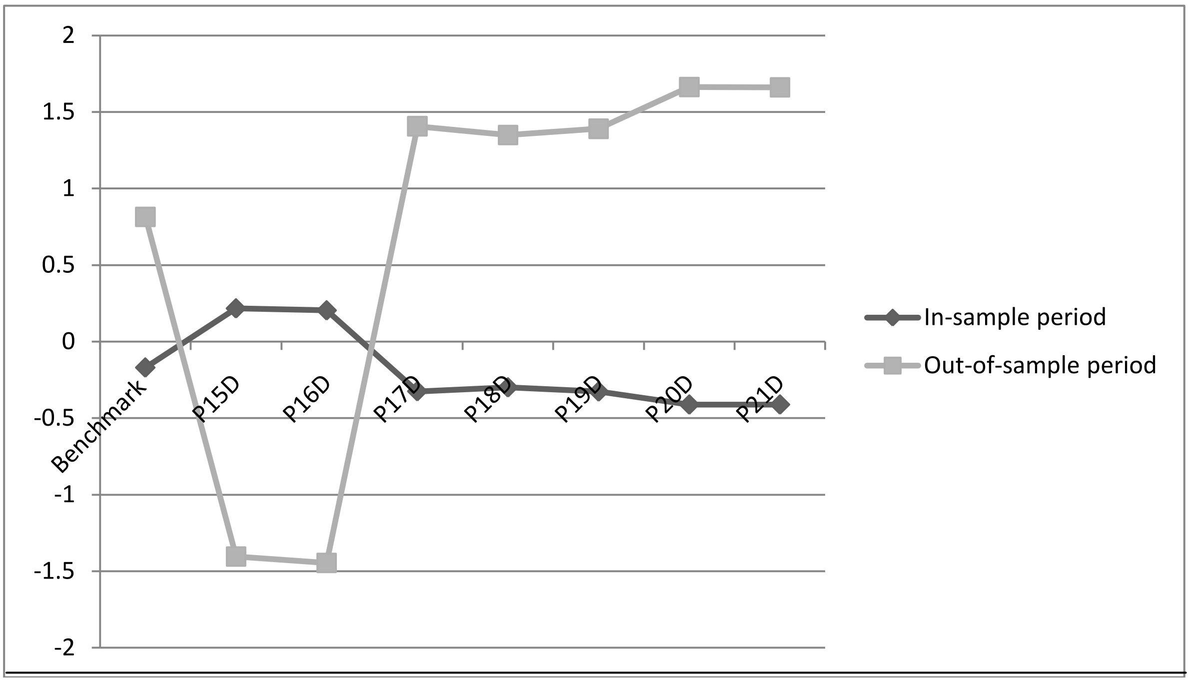

Chart 5.2 Information ratios for notional cointegration sterling portfolios 15D-21D

This point is illustrated in chart 5.2 by the difference between the results for the notional cointegration portfolios exhibited for the two investment periods. It also shows that diversification for a British investor was of benefit during this out-of-sample period.

As suggested by Alexander and Dimitriu (2002), transaction costs for the rebalancing required for the notional cointegration portfolios is prohibitive, maybe due to the volatility of that period. This means that the returns from these portfolios, especially the more diversified ones, suffered as a result. However, some of the portfolios produce good returns, especially the sterling portfolios, which suggest that there could be favourable conditions for trading whole portfolios in this manner.

6.Conclusion

A main motivation for this research was to find the benefits, if any, of using cointegraton to diversify a currency trading portfolio. The traditional MCP tracking strategy portfolios displayed the strongest long-run relationships and were very stable in terms of volatility. The links between the in-sample and out-of-sample results, which is best demonstrated by the U.S. Dollar and Euro portfolios, were strong which suggests stable cointegration equation coefficients. The way that the volatilities of these portfolios remained low in a period of recognised market uncertainty is commendable. Also, for portfolios 13C and 14C, consisting of eight and nine assets respectively, to be perfectly correlated with the benchmark over the long in-sample period, and then remain nearly perfectly correlated over the out-of-sample period is an impressive result. The risk-adjusted returns of the U.S. Dollar MCP tracking portfolios are generally good, but the benchmarks outperform this method for the Euro portfolios. The sterling portfolios are the most volatile but produce the best risk-adjusted returns for each optimisation method, with particularly good results for the cointegration based methods. The out-of-sample notional cointegration portfolios produce strong results for the Sterling portfolios, with an information ratio high of 1.46 for portfolio

20D. The MCP tracking portfolios give an information ratio of 1.06 for portfolio 20C, which also represents a good investment. Overall cointegration appears to add value to currency portfolio optimisation, particularly for a U.K. and U.S. investor.

In terms of the best currency to invest into, the U.S. Dollar portfolios perform consistently well over the investment horizons and portfolio construction techniques, with the Euro not far behind. The sterling portfolios are erratic and highly volatile at times, but produce the strongest portfolios in terms of risk-adjusted returns. The issue of whether to diversify or not remains inconclusive and varies between techniques and investment horizons.

There was an experimental edge to this research about how the 'lesser' currencies would perform in these portfolios. A feature to be noted is the good performance of 'lesser' currencies, particularly the Swedish Krona, which adds value in most cases to the portfolios. The weightings attributed to this currency did not do its performance justice and it validates the reason why it has been more frequently traded as shown by the BIS triennial survey (2007). It performed well paired with the U.S. Dollar, as did the Australian Dollar and Danish Krone, and produced good out-of-sample results when paired with the Euro.

The results of these portfolios appear more favourable when compared to their counterparts in the equity markets. Whilst very few, if any, equity portfolios produced positive returns over 2008, the mostly positive currency portfolio returns shown here are encouraging.

The issue of which portfolio optimisation method works best to determine the portfolio weights yields mixed results here. The equal weighting method performs best for Euro portfolios and acquits itself well in the others. The marginal concept of the notional cointegration portfolios produce the strongest risk-adjusted returns in the Sterling portfolios, though admittedly this only works on the basis that there is a market to trade these portfolios in. The MCP tracking portfolios perform strongly also, giving good risk-adjusted returns for each currency, whilst the historical returns portfolios perform the worst, which again hints at the lack of cohesion between the in-sample and out-of-sample periods. In general, for these currencies over the selected investment horizon, cointegration-based optimisation strategies add value and should be in the armoury of currency fund managers but, as with all optimisation techniques, they should be used cautiously.

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Appendix 1.

U.S. Dollar Portfolios

'Index'	Portfolio 1	Portfolio 2	Portfolio 3	Portfolio 4	Portfolio 5	Portfolio 6	Portfolio 7
USD/EUR	USD/EUR	USD/EUR	USD/EUR	USD/EUR	USD/EUR	USD/EUR	USD/EUR
	USD/GBP	USD/GBP	USD/GBP	USD/GBP	USD/GBP	USD/GBP	USD/GBP
	USD/JPY	USD/JPY	USD/JPY	USD/JPY	USD/JPY	USD/JPY	USD/JPY
		USD/CHF	USD/CHF	USD/CHF	USD/CHF	USD/CHF	USD/CHF

			USD/CAD	USD/CAD	USD/CAD	USD/CAD	USD/CAD
				USD/AUD	USD/AUD	USD/AUD	USD/AUD
					USD/SEK	USD/SEK	USD/SEK
						USD/DKK	USD/DKK
							USD/NOK

Euro Portfolios

'Index'	Portfolio 8	Portfolio 9	Portfolio 10	Portfolio 11	Portfolio 12	Portfolio 13	Portfolio 14
EUR/USD	EUR/USD	EUR/USD	EUR/USD	EUR/USD	EUR/USD	EUR/USD	EUR/USD
	EUR/GBP	EUR/GBP	EUR/GBP	EUR/GBP	EUR/GBP	EUR/GBP	EUR/GBP
	EUR/JPY	EUR/JPY	EUR/JPY	EUR/JPY	EUR/JPY	EUR/JPY	EUR/JPY
		EUR/CHF	EUR/CHF	EUR/CHF	EUR/CHF	EUR/CHF	EUR/CHF
			EUR/CAD	EUR/CAD	EUR/CAD	EUR/CAD	EUR/CAD
				EUR/AUD	EUR/AUD	EUR/AUD	EUR/AUD
					EUR/SEK	EUR/SEK	EUR/SEK
						EUR/DKK	EUR/DKK
							GBP/NOK

Sterling Portfolios

'Index'	Portfolio 15	Portfolio 16	Portfolio 17	Portfolio 18	Portfolio 19	Portfolio 20	Portfolio 21
GBP/USD	GBP/USD	GBP/USD	GBP/USD	GBP/USD	GBP/USD	GBP/USD	GBP/USD
	GBP/EUR	GBP/EUR	GBP/EUR	GBP/EUR	GBP/EUR	GBP/EUR	GBP/EUR
	GBP/JPY	GBP/JPY	GBP/JPY	GBP/JPY	GBP/JPY	GBP/JPY	GBP/JPY
		GBP/CHF	GBP/CHF	GBP/CHF	GBP/CHF	GBP/CHF	GBP/CHF
			GBP/CAD	GBP/CAD	GBP/CAD	GBP/CAD	GBP/CAD
				GBP/AUD	GBP/AUD	GBP/AUD	GBP/AUD
					GBP/SEK	GBP/SEK	GBP/SEK
						GBP/DKK	GBP/DKK
							GBP/NOK

Table A1.1 Portfolio Evolution

Appendix 2

MCP tracking Portfolio Constituent Weights

	USD/EUR	USD/GBP	USD/JPY	USD/CHF	USD/CAD	USD/AUD	USD/SEK	USD/DKK	USD/NOK
Portfolio 1	50.00%	50.27%	-0.27%	--	--	--	--	--	--

Portfolio 2	50.00%	16.02%	0.93%	33.05%	--	--	--	--	--
Portfolio 3	50.00%	7.58%	2.66%	30.93%	8.83%	--	--	--	--
Portfolio 4	50.00%	4.12%	-0.53%	30.86%	4.19%	11.37%	--	--	--
Portfolio 5	50.00%	1.18%	-0.09%	33.54%	4.98%	7.05%	3.35%	--	--
Portfolio 6	50.00%	-1.44%	0.01%	-6.88%	3.40%	0.33%	-0.12%	54.70%	--
Portfolio 7	50.00%	-0.60%	0.00%	-3.34%	3.76%	0.04%	0.03%	50.67%	-0.56%
	EUR/USD	EUR/GBP	EUR/JPY	EUR/CHF	EUR/CAD	EUR/AUD	EUR/SEK	EUR/DKK	EUR/NOK
Portfolio 8	50.00%	31.41%	18.59%	--	--	--	--	--	--
Portfolio 9	50.00%	22.41%	8.40%	19.20%	--	--	--	--	--
Portfolio 10	50.00%	14.45%	7.17%	22.08%	6.30%	--	--	--	--
Portfolio 11	50.00%	18.73%	10.78%	20.45%	10.24%	-10.19%	--	--	--
Portfolio 12	50.00%	23.25%	14.99%	20.28%	10.97%	-11.08%	-8.41%	--	--
Portfolio 13	50.00%	3.69%	2.75%	2.32%	2.35%	-1.76%	-1.84%	42.49%	--
Portfolio 14	50.00%	3.69%	2.76%	2.32%	2.35%	-1.77%	-1.85%	42.48%	0.02%
	GBP/USD	GBP/EUR	GBP/JPY	GBP/CHF	GBP/CAD	GBP/AUD	GBP/SEK	GBP/DKK	GBP/NOK
Portfolio 15	50.00%	354.14%	-304.14%	--	--	--	--	--	--
Portfolio 16	50.00%	348.59%	-91.46%	-207.13%	--	--	--	--	--
Portfolio 17	50.00%	-1353.07%	283.63%	870.06%	249.39%	--	--	--	--
Portfolio 18	50.00%	-563.43%	214.60%	400.21%	203.20%	-204.58%	--	--	--
Portfolio 19	50.00%	-444.21%	279.70%	373.44%	205.12%	-210.21%	-153.84%	--	--
Portfolio 20	50.00%	-2828.81%	188.09%	165.25%	159.04%	-125.27%	-121.76%	2613.47%	--
Portfolio 21	50.00%	-2782.65%	186.61%	161.23%	156.49%	-123.75%	-121.02%	2570.56%	2.53%

Table A2.1 MCP Tracking Portfolios Constituent Weights